Physical limitations on quantum nonlocality in the detection of photons emitted from positron-electron annihilation

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Recent experimental measurements of the time interval between detection of the two photons emitted in positron-electron annihilation have indicated that collapse of the spatial part of the photon's wave function, due to detection of the other photon, *does not occur*. Although quantum nonlocality actually occurs in photons produced through parametric down-conversion, the recent experiments give strong evidence *against* measurement-induced instantaneous spatial localization of high-energy γ photons. A quantum-mechanical analysis of the Einstein-Podolsky-Rosen problem is presented, which may help to explain the observed differences between photons produced through parametric down-conversion and photons produced through positron-electron annihilation. The results are found to concur with the recent experiments involving γ photons.

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Quantum nonlocality in measurements involving polarization and two-photon interference of correlated photons has been experimentally confirmed at many independent laboratories [1–4]. It has been generally postulated that nonlocal effects may also occur in regard to the spatial wave functions of the emitted photons. As an example, detection of one of the photons produced in parametric down-conversion is predicted to cause ''instantaneous'' localization of the other photon, subsequently eliminating any uncertainty in the time of arrival of the second photon at a second detector. Experimental support of this prediction has been reported by Hong *et al.* [4]. The two-photon interference method utilized in Ref. [4] indicates that the minimum time uncertainty, in the time interval between detection of the two down-converted photons, is less than 100 fs. This uncertainty in time is *much less* than the coherence time of the initial pump photons, which subsequently gives strong indication of nonlocal collapse of the photon wave function.

One may expect to observe similar nonlocal effects involving photons emitted from positron-electron annihilation. Recent high-resolution measurements of the time interval between the two photons emitted in positron-electron annihilation have been carried out by Irby [5]. The results of the measurements indicate that the *absolute minimum* uncertainty in detection time between arrival of the two photons is $\Delta t_{QM}=117\pm9$ ps, which surprisingly, agrees with the lifetime of positrons in bulk sodium (119 ps) predicted by quantum electrodynamics [6,7]. Although nonlocal effects are observed to occur in the case of down-converted photons, the experimental results give strong evidence *against* the instantaneous spatial localization of γ photons emitted from annihilation events.

In this paper, we present a quantum-mechanical analysis of the time interval between detection of correlated photons. The analysis is basically the same as that first presented by Einstein, Podolsky, and Rosen (EPR) in 1935 [8]. The main difference, however, is that we include time dependence in the quantum wave functions and also take into account restrictions on photon momenta due to energy conservation.

As in the original EPR paper, we assume that the total momentum before the particles interact (or are emitted) is zero. In addition, we assume that the particles interact at times $t<0$. The total wave function can then be written (for $t\geqslant0$)

$$\Psi(x_1,x_2,t)=\int_{-\infty}^{\infty}\psi_p(x_2,t)\,u_p(x_1,t)\,dp, \tag{1}$$

where $u_p(x_1,t)$ are eigenstates of particle one's momentum and energy

$$u_p(x_1,t)=e^{ipx_1/\hbar}e^{-iEt/\hbar}. \tag{2}$$

In order to conserve momentum, let us also assume

$$\psi_p(x_2,t)=e^{-ip(x_2+x_o)/\hbar}e^{-iEt/\hbar}, \tag{3}$$

which are eigenstates of particle two's momentum and energy. (x_o is an arbitrary constant introduced in the original EPR paper. In this case, however, since we are including the explicit time dependence, we will set $x_o=0$.) Note that if a measurement of particle one's momentum yields a value of p, the total wave function collapses to

$$\Psi(x_1,x_2,t)=\psi_p(x_2,t)u_p(x_1,t), \tag{4}$$

which has momentum eigenvalues of p and $-p$, respectively, for particles one and two. Before any measurement takes place, the total wavefunction is thus given by

$$\Psi(x_1,x_2,t)=\int_{-\infty}^{\infty}e^{ip(x_1-x_2)/\hbar}e^{-i2Et/\hbar}dp. \tag{5}$$

The total wave function can also be written in terms of *instantaneous* position eigenstates $v_x(x_1,t)$ of particle one

$$\Psi(x_1,x_2,t)=\int_{-\infty}^{\infty}\phi_x(x_2,t)\,v_x(x_1,t)dx, \tag{6}$$

where particle two's wave function $\phi_x(x_2,t)$ is yet to be specified. Since the position eigenstates of particle one, measured at time t, are Dirac δ-functions $v_x(x_1,t)=\delta(x-x_1)$, with eigenvalues x_1, Eq. (6) reduces to

$$\Psi(x_1,x_2,t)=\phi_{x_1}(x_2,t), \tag{7}$$

where x_1 now represents an eigenvalue measured at time t. Therefore, the spatial wave function of particle two is dependent on the position measurement x_1 of particle one,

$$\phi_{x_1}(x_2,t)=\int_{-\infty}^{\infty} e^{ip(x_1-x_2)/\hbar}e^{-i2Et/\hbar}dp. \tag{8}$$

Let us first consider the case where the particles have nonzero rest mass. If particle one's position x_1 is measured at $t=0$, Eq. (8) reduces to

$$\phi_{x_1}(x_2,0)=\int_{-\infty}^{\infty} e^{ip(x_1-x_2)/\hbar}\,dp,$$
$$\phi_{x_1}(x_2,0)=\hbar\,\delta(x_1-x_2), \tag{9}$$

resulting in particle two being localized at $x_2=x_1$ (which is the same result as presented in the original EPR paper with $x_o=0$). If particle one's position x_1 is measured at a time other than zero, particle two's wave function at the measurement time t is then explicitly given by

$$\phi_{x_1}(x_2,t)=\int_{-\infty}^{\infty} e^{ip(x_1-x_2)/\hbar}e^{-i2\sqrt{(pc)^2+(mc^2)^2}t/\hbar}dp. \tag{10}$$

Thus, if particle one's position x_1 is measured at a time other than $t=0$, particle two *will not be localized*. Particle two is only localized at one instant, namely, $t=0$. Furthermore, regardless of *when* particle two is localized, particle two's wave function will always immediately and rapidly disperse as time progresses.

In contrast, since $E=pc$ for photons, dispersion no longer exists. The spatial wave function for photons is given by

$$\phi_{x_1}(x_2,t)=\int_{-\infty}^{\infty} e^{ip(x_1-x_2-2ct)/\hbar}dp,$$
$$\phi_{x_1}(x_2,t)=\hbar\,\delta(x_1-x_2-2ct). \tag{11}$$

Thus, after measurement of photon one's location x_1 at time t, photon two is instantaneously localized at $x_2=x_1-2ct$. In contrast with particles of nonzero rest mass, once photon two is localized, it will remain localized (propagating at c).

The result, given in Eq. (11) above, contradicts the experimental measurements. Localization of the second photon should *eliminate* any uncertainty in arrival time between the two photons. This glaring contradiction between theory and experiment can, however, be alleviated by properly taking into account necessary restrictions on photon momenta.

For the case of positron-electron annihilation, the emitted photons are restricted to a small range of possible momenta Δp centered at $p=mc$ in order for energy to be conserved. In order to take into account conservation of energy, let $|f(p)|^2dp$ be the probability of photons having momenta between p and $p+dp$. The total wave function is then given by

$$\Psi(x_1,x_2,t)=\int_{-\infty}^{\infty} f(p)\psi_p(x_2,t)u_p(x_1,t)dp. \tag{12}$$

Once photon one is detected, photon two's wave function is then given by

$$\phi_{x_1}(x_2,t)=\int_{-\infty}^{\infty} f(p)e^{ip(x_1-x_2-2ct)/\hbar}dp, \tag{13}$$

which is no longer equal to a Dirac δ-function. As Eq. (13) indicates, restrictions on emitted photon momenta *prohibit* instantaneous and complete localization of the second photon.

The prohibition on nonlocality indicated above may also be described in terms of partial entanglement [9,10]. As is well known in the quantum-optics community, if a particular observable is subject to physical restrictions, any other conjugate observable, associated with a noncommuting operator, will exhibit a corresponding restriction in terms of nonlocality. This can be more easily shown in terms of spin measurements.

Let us assume that two particles are emitted such that the total spin wave function, measured along the x axis, is given by

$$\Psi_x=a|\uparrow\rangle_1|\downarrow\rangle_2-b|\downarrow\rangle_1|\uparrow\rangle_2, \tag{14}$$

where $a^2+b^2=1$. If $a=b$, then individual spin measurements (along x) for either particle are unrestricted and completely uncertain. If $a\neq b$, there exists partial restriction. If either a or b is equal to zero, spin measurements are completely restricted. Note that the above wave function will always exhibit maximum nonlocality (for measurements along x) regardless of the values of a and b. A measurement of $|\uparrow\rangle_1$ will always yield $|\downarrow\rangle_2$.

Particle spin along the z axis, however, is a conjugate observable. For spin measurements along the z axis, the total wave function can easily be shown to be

$$\Psi_z=\frac{1}{\sqrt{2}}|\uparrow\rangle_1\left\{\frac{(a-b)}{\sqrt{2}}|\uparrow\rangle_2-\frac{(a+b)}{\sqrt{2}}|\downarrow\rangle_2\right\}+\frac{1}{\sqrt{2}}|\downarrow\rangle_1\left\{\frac{(a+b)}{\sqrt{2}}|\uparrow\rangle_2-\frac{(a-b)}{\sqrt{2}}|\downarrow\rangle_2\right\}. \tag{15}$$

If $a=b$, observables associated with spin measurements along x are unrestricted. For spin measurements along z, Eq. (15) reduces to (with $a=b$)

$$\Psi_z=-\frac{1}{\sqrt{2}}|\uparrow\rangle_1|\downarrow\rangle_2+\frac{1}{\sqrt{2}}|\downarrow\rangle_1|\uparrow\rangle_2. \tag{16}$$

In this case, spin measurements along the z axis exhibit maximum nonlocality, or maximum entanglement.

In contrast, let us assume $a=0.8$ and $b=0.6$. In this case, spin measurements along x are only slightly restricted. For spin measurements along the z axis, Eq. (15) reduces to

$$\Psi_z=\frac{1}{\sqrt{2}}|\uparrow\rangle_1\{0.141|\uparrow\rangle_2-0.989|\downarrow\rangle_2\}+\frac{1}{\sqrt{2}}|\downarrow\rangle_1\{0.989|\uparrow\rangle_2-0.141|\downarrow\rangle_2\}. \quad (17)$$

In this case, spin measurements along z no longer exhibit maximum nonlocality. If a measurement of particle one's spin yields $|\uparrow\rangle_1$, only 98% of the time will particle two yield $|\downarrow\rangle_2$.

On the other extreme, if $b=0$, the observables associated with spin measurements along x are maximally restricted. For spin measurements along the z axis, Eq. (15) reduces to (with $b=0$)

$$\Psi_z=\frac{1}{\sqrt{2}}|\uparrow\rangle_1\left\{\frac{1}{\sqrt{2}}|\uparrow\rangle_2-\frac{1}{\sqrt{2}}|\downarrow\rangle_2\right\}+\frac{1}{\sqrt{2}}|\downarrow\rangle_1\left\{\frac{1}{\sqrt{2}}|\uparrow\rangle_2-\frac{1}{\sqrt{2}}|\downarrow\rangle_2\right\}. \quad (18)$$

In this case, measurement of particle one's spin *does not in any way influence* the measurement of the spin of particle two. Nonlocality is erased.

As the above analysis indicates, measurements involving a particular observable *may or may not* exhibit nonlocality, depending upon the degree of physical restraints that may exist on conjugate observables. In the case of photons being emitted from positron-electron annihilation, photon momenta are, for all practical purposes, *maximally restricted*. This then essentially eliminates nonlocality in the conjugate position observables. Therefore $\triangle t_{QM}\neq 0$. (However, partial entanglement still, nonetheless, exists. In the limit of well-defined momenta, $\triangle t_{QM}\rightarrow\infty$ for the case of no entanglement.) In striking contrast, parametric down-converted photons exhibit a *much stronger correlation* in time than that of γ photons. This may be attributed to the fact that down-converted photons possess much larger uncertainties in emission energy than those of γ photons.

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