PHYSICAL REVIEW A **67**, 030701(R) (2003)

Screened Casimir force at finite temperatures: A possible role in nuclear interactions

B. W. Ninham* and M. Boström†

Department of Applied Mathematics, Research School of Physical Sciences and Engineering, Institute of Advanced Studies, Australian National University, Canberra, 0200 Australia

(Received 14 October 2002; published 10 March 2003)

We derive a simple asymptotic expression for the screened Casimir free energy that is valid in both the high-temperature limit and in the large-separation limit. Any finite (i.e., nonzero) plasma density fundamentally alters the long-range interaction. The similarity of the derived expression with the Yukawa potential of nuclear interactions encourages us to investigate the Casimir free energy between two nuclear particles in a sea of electrons and positrons. We use simple estimates to explore the possible role of screened Casimir interactions for nuclear interactions. The magnitude of the force and of the coupling constant indicates an intriguing possible interpretation in which the nuclear force is a screened Casimir force and the mesons can be viewed as plasmons in the electron-positron sea.

DOI: 10.1103/PhysRevA.67.030701 PACS number(s): 34.20.Cf, 21.10.Dr, 25.80.−e, 11.10.Wx

When two objects come close together the mutual electric polarizations of the material results in an attractive force. Casimir predicted already in 1948 an attractive interaction free energy between perfect metal surfaces at zero temperature [1]: $F(l) = -\pi^2\hbar c/720 l^3$. The Casimir and Lifshitz theories [1,2] have occupied such a vast literature that there should be little else to say. Many direct and indirect measurements of forces seem to have confirmed the theory [3]. Recent force measurements between metal surfaces claimed accuracies down to 1% [4–6]. But significant flaws in the theoretical framework have been revealed. As long ago as 1970 it was shown [7,8] that the complete Lifshitz free energy of interaction could be derived from Maxwell's equations plus the Planck quantization condition. This seemed mysterious since the original derivation involved complicated quantum electrodynamics and appeared quite general. The problem can be traced to a subtle error [2]. A nonlinear coupling constant integration in a formally exact Dyson integral equation for the dielectric susceptibility was replaced by a linear integration. So the formalism collapses to a semiclassical theory. One should observe that the self-energy shift of hydrogen atoms can be derived either using an extension of the Lifshitz formalism by Ninham *et al.* [9], or using a theory that essentially starts from the Klein-Gordon equation as outlined by Dyson [10] (some useful insights about vacuum polarization may be gained from Refs. [11–13]). The high-temperature asymptotic form of the Casimir interaction originally derived by Lifshitz for dielectric media has only recently been shown to be correct for real dissipative surfaces [14,15]. Other examples where classical quantum-mechanical results in the theory of molecular forces turn out be wrong are more surprising. The retarded Casimir-Polder force between two atoms has been shown to be correct only in the limit of zero temperature [16–18]. At any finite temperature a completely different form is obtained [$F(l) = -kT\zeta(3)/8\pi l^2$]. This form already built into Lifshitz theory [2], provides a remarkable demonstration of the correspondence principle. Retardation has a quite different interpretation to that in the books. It is not due to the finite velocity of light at all, but rather due to the quantization of light [17]. A further example, relevant to cold molecule formation and catalysis, is the retarded resonance interaction between excited-state–ground-state atoms. Here even the classical zero-temperature retarded form is wrong. The correct finite-temperature form is very different again and physically sensible [19]. The same remarks apply to the Föster interaction for long-range photon transfer between two different excited-state–ground-state molecules. There are further surprises in physical chemistry. For over 50 years the Deryaguin-Landau-Verwey-Overbeek (DLVO) theory of interactions has not been questioned. For interactions in and across electrolytes the theory separates electrostatic double layer forces from van der Waals–Lifshitz forces. The first is a nonlinear theory, the second (Lifshitz, appropriately extended to include salt) is linear. It has been proved that this ansatz violates both the Gibbs adsorption equation, and the gauge condition on the electromagnetic field [20]. This is not an esoteric matter and is a main reason the physical sciences have not contributed as well as they might to modern biology. When the theory is done correctly a large number of previously inexplicable phenomena to do with Hofmeister-specific ion effects, pH, buffers, and electrochemistry fall into place, and predictively [21,22]. The same is true for the theory of bulk electrolytes and interfacial tension at salt-water interfaces, which has profoundly important consequences for interpretation of potential at interfaces [23]. It has been shown too that the classical temperature-dependent zero frequency contribution to the (linear) Lifshitz theory is precisely equivalent to the linearized version of the Onsager-Samaris theory [24] for the change in interfacial tension with dissolved salt. The latter "works" at best only up to $10^{-4} M$; the linear form of Lifshitz theory is equally invalid. Now it is routine to invoke the DLVO ansatz, i.e., separate electrostatic and Casimir-Lifshitz forces in interpreting experiments aiming to measure the Casimir forces between metal plates, and this must be incorrect whenever any intervening plasma occurs, as it always must

*Present address: Department of Chemistry and CSGI, University of Florence, 50019 Sesto Fiorentino, Italy. Electronic address: Barry.Ninham@anu.edu.au

†Electronic address: mtb110@rsphysse.anu.edu.au

to some extent. This has been explored for the metal case, at low temperatures [16,20]. The electrodynamic contributions modify the Casimir result and the classical image term ($n=0$) is exactly equivalent to that Onsager and Samaris, indicating for the reasons above, inadequacy of the whole theory.

We here extend that work to examine the effects of an intervening plasma between two perfect metal plates at high temperature or large distance for any plasma density. The result is nonanalytic insofar as the presence of even an infinitesimal plasma concentration fundamentally alters the long-range interaction asymptotic form at any finite temperature. We first derive a simple asymptotic expression for the free energy that is valid in both high-temperature and large-separation limits (for any finite plasma density). There is a fascinating similarity between the derived expression with the Yukawa potential of nuclear interactions. This inspires us to investigate the conditions under which the Casimir free energy between two nuclear particles in a sea of electrons and positrons could by itself accommodate the nuclear force (or lead to a QED correction to the nuclear interaction).

Consider now two perfectly conducting planar surfaces separated by a free-electron plasma. The model system is chosen for demonstrational purposes, but we expect that our conclusions will be relevant for the effect of intervening plasma on the free energy of interaction between particles in general. The frequency (ω) dependent dielectric susceptibility of a plasma is

$$\epsilon_2(\omega)=1-\frac{4\pi\rho e^2}{m\omega^2}, \tag{1}$$

where ρ is the number density of electrons (or charged particles) in the plasma, e is the unit electric charge, and m is the electron mass. For future convenience we define two help variables [16]: $\overline{\rho}=[\rho/(\pi m)][e\hbar/(kT)]^2$ and $x=(2kTl)/(\hbar c)$. Here $\hbar$ is the Planck's constant, c is the speed of light, k is the Boltzmann's constant, and T is the temperature. One of the main results of this Rapid Communication is that the high x (i.e., high temperature or large distances) asymptotic interaction energy for any finite plasma density can be written as

$$F=-\frac{kT}{2\pi}\int_\kappa^\infty dt\, t\ln(1-e^{-2lt})+F_{n>0}, \tag{2}$$

$$F_{n>0}=-\frac{(kT)^2}{l\hbar c}e^{-\pi\overline{\rho}x}e^{-2\pi x}+O(e^{-x^2}), \tag{3}$$

where $\kappa^2=\rho e^2/\epsilon_0 mc^2$. The first term is the $F_{n=0}$ term examined in some detail in a previous paper [16]. The second term, $F_{n>0}$, follows after some rather lengthy algebra that we, for clarity, outline in the Appendix. Equation (2) is equivalent to the linearized Onsager-Samaris result [9,20,24]. This means that in the presence of even an infinitesimal plasma concentration the Casimir result is invalid in that the expansion is nonanalytic in the density. We know that the linearized Onsager-Samaris result is an insufficient approximation for water-air surfaces (and so is the Casimir result with plasma). In any real system it is impossible to ignore some electron density in the gap from the surface and from real electron-positron pairs. So with a real system devoted to measuring Casimir forces, like that of Lamoreaux, rather than first subtracting off an electrostatic term we have to do the electrostatics properly, including the $n=0$ term in the Casimir free energy, in a nonlinear theory putting images up into a Gibbs adsorption isotherm [20–22]. The exception is that the thermal energy is now replaced in an intriguing way by mc^2.

The screened Casimir energy at high temperatures and large interparticle separations has a Yukawa form. We will now explore under which conditions it could have a possible role in the interaction between nuclear particles. If we know the details of the vacuum polarization, i.e., here the plasma density and effective temperature of the local electron-positron sea near two nuclear particles, we could estimate the screened Casimir interaction between nuclear particles. Since we do not know that, we start out from the Yukawa interaction between nuclear particles. This will give us an estimate of the electron-positron density and effective temperature required to give the "right" screening length. What is amazing is that this estimate gives the right values for the nuclear binding energy. Direct application of the Klein-Gordon equation gives the Yukawa potential between nuclear particles that is applicable at distances large compared to the screening length $l_\pi=\hbar/(m_\pi c)$ (m_π is the mass of the π mesons that mediate the interaction) [25],

$$V(l)\propto -e^{-l/l_\pi}. \tag{4}$$

We compare initially this expression with the $n=0$ part of the long-range screened Casimir interaction,

$$F_{n=0}=-\frac{kT\kappa^2}{2\pi}e^{-2l\kappa}\left[\frac{1}{2l\kappa}+\frac{1}{4l^2\kappa^2}\right]. \tag{5}$$

If the idea that we are presenting is correct, one should be able to extract the meson mass by taking the coefficients in the exponents to be equal:

$$m_\pi=\frac{2e\hbar}{c^2}\sqrt{(\rho_+ +\rho_-)/m\epsilon_0}. \tag{6}$$

Conversely, since we know that the meson mass is 135 MeV we can use this expression to estimate the density of electrons (ρ_-) and positrons (ρ_+). The screening length in the nuclear Yukawa potential is 1.458 fm and we find that this corresponds to a plasma density of 5.3×10^{41} m^{-3}. The equilibrium with respect to positron and electron production can at high temperatures be written [26] as $\rho_-=\rho_+=3\zeta(3)k^3T^3/(2\pi^2\hbar^3c^3)$. This means that the effective temperature of nuclear interaction via screened Casimir interaction would be 3.2×10^{11} K.

We know that the relevant length scales of nuclear interactions are around 1 fm and that the nuclear binding energy is around 8 MeV. The screened Casimir interaction energy between two plates (i.e., "nuclear particles") with a cross section of 1 fm^2, a distance 0.5 fm apart, receives 4.25 MeV from the $n=0$ term and 3.25 MeV from the $n>0$ terms. The total of around 7.5 MeV compares remarkably well with the binding energy of nuclear interactions. While the screening length of the $n=0$ term is defined above, it is interesting that the screening length of the $n>0$ terms also comes out about right; it is 0.56 fm. The nuclear interaction as a screened Casimir interaction would thus receive approximately equal contributions from the "classical" $n=0$ term and the "quantum" $n>0$ terms.

An important question is where the energy to generate this local electron plasma could come from. Feynman speculated that "high energy potentials could excite states corresponding to other eigenvalues, possibly thereby corresponding to other masses" [13]. It turns out that the low-temperature Casimir interaction, i.e., without an intervening plasma, by itself could be capable of generating the effective temperature required to obtain the plasma. For small values of x the Casimir interaction between perfectly conducting surfaces can, in the absence of an intervening plasma, be written as Eq. (35) of Ref. [16],

$$F(l,T)\simeq\frac{-\pi^2\hbar c}{720l^3}-\frac{(\rho_-+\rho_+)\hbar c}{6}+\frac{\pi^2 lk^4T^4}{45\hbar^3c^3}+\cdots, \quad (7)$$

where the first term is the zero-temperature Casimir energy, the third is the blackbody energy, and the second has been rewritten in terms of electron and positron densities as defined above. If we assume that the entire zero-temperature Casimir energy is transformed into blackbody energy (which at high temperatures can generate an electron-positron plasma) we could estimate the temperature as $T\approx\hbar c/2lk$. This would at a distance of 3.6 fm give the required effective temperature (at the distances discussed above the effective temperature is even larger, around 2.3×10^{12} K). It is intriguing that a cancellation of the Casimir zero point energy and the blackbody energy term, just like the cancellation of the $n=0$ term at low temperatures [16,17], gives the "right" result.

Since this clearly is an enormous conceptual leap and the treatment far from rigorous, it is appropriate to be very cautious. The result presented here is based on using a linear and scalar theory (rather than an appropriate nonlinear vector theory) and a nonrelativistic dielectric function (rather than a wave-vector-dependent retarded dielectric function along the lines of Davies and Ninham [16]). A more formal derivation would also need to take into account the spherical nature (see, for example, Ref. [27]) of the nuclear particles (we should not be surprised if the geometry influences the mass). It would also need to consider in detail the proposed transformation of Casimir energy into a screened Casimir energy with an electron-positron plasma of the form suggested here. However, we think that the results in themselves are so interesting and intriguing that they need to be discussed. The picture that would emerge naturally from a consistent continuation of the above discussion is that mesons could be interpreted as plasmons in the electron-positron sea. The π mesons would be the one-plasmon excitation, whereas, for example, K mesons would come out as multipole plasmon excitations. The "nuclear" version of the interaction between two molecules in an electrolyte (Eq. (7.6) of Ref. [9]) could, within the same framework, be viewed as a Klein-Gordon equation for the interaction between two nucleons (the resulting interaction energy comes out as above from Eq. (7.10) of Ref. [9]). An interesting aspect is that the plasmon lifetime [28] in the plasma, with electron and positron densities as given above, agrees qualitatively with the experimentally observed lifetime of π^0 mesons. We find this all very exciting and will return to it in more detail, and we also hope that it will inspire further discussion and investigation by others. We would like to end with the words of Dyson from his 1949 paper: "The future theory will be built, first of all upon the results of future experiments, and secondly upon an understanding of the interrelations between electrodynamics and mesonic and nucleonic phenomena" [11].

APPENDIX: EFFECT OF INTERVENING PLASMA ON CASIMIR FREE ENERGY

The purpose of this appendix is to very briefly indicate the main steps in the derivation, rather than giving a complete derivation, of the Casimir free energy between perfect metal surfaces with an intervening plasma at high x. The complete free interaction energy of the system [16] can, after some algebra, be written in the following form,

$$F(l,T)=-\frac{kTx^3}{4l^2}I, \quad (A1)$$

where

$$I\equiv\int_0^\infty dy\, e^{-\pi\bar{\rho}y}y^{-5/2}\bar{\omega}(x^2/y)[1+2\bar{\omega}(y)] \quad (A2)$$

and

$$\bar{\omega}(y)=\sum_{n=1}^{\infty}e^{-n^2\pi y}=\left[\frac{-1}{2}+\frac{y^{-1/2}}{2}+y^{-1/2}\bar{\omega}\left(\frac{1}{y}\right)\right]. \quad (A3)$$

We divide the integration range into three regions $\{0,1\}$, $\{1,x^2\}$, and $\{x^2,\infty\}$. By repeated use of Eq. (A2), and dropping terms $O(e^{-x^2})$, we find the asymptotic form for I,

$$I=I_1+I_2+O(e^{-x^2}), \quad (A4)$$

$$I_1=x^{-3}\int_0^{x^2}dy\, y^{1/2}e^{-\pi\bar{\rho}x^2/y}\bar{\omega}(y), \quad (A5)$$

$$I_2=2\int_1^{x^2}dy\, y^{-5/2}e^{-\pi\bar{\rho}y}\bar{\omega}(x^2/y)\bar{\omega}(y). \quad (A6)$$

I_1 can be shown (making use of the methods described by Ninham and Daicic [16]) to, except for a term $O(e^{-x^2})$, identically correspond to the $n=0$ term in the free-energy summation. In order to obtain our Eq. (2) we need to analyze I_2 in some detail. With n replaced with $n+1$ in Eq. (A3),

$$\overline{\omega}(y)=\sum_{n=0}^{\infty} e^{-(1+2n+n^2)\pi y}. \tag{A7}$$

This means that the following relations are valid for $\overline{\omega}(y)$:

$$e^{-\pi y}<\overline{\omega}(y)<\frac{e^{-\pi y}}{1-e^{-2\pi y}}. \tag{A8}$$

Using Eqs. (A6) and (A8) we find that I_2, apart from a very small uncertainty, $(1-\exp[-2\pi])$ not being identical to unity, is

$$I_2 \simeq 2x^{-3/2}\int_{1/x}^{x} dy\, y^{-5/2} e^{-\pi x(y+1/y)} e^{-\pi\overline{\rho} xy}. \tag{A9}$$

This integral has a steep maximum, and in the asymptotic limit of large x,

$$I_2 \simeq 2x^{-3/2} e^{-\pi\overline{\rho} x} e^{-2\pi x}\int_{-\infty}^{\infty} dy\, e^{-\pi x y^2}. \tag{A10}$$

Our Eq. (2) follows from this expression.

[1] H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[2] E.M. Lifshitz, Zh. Éksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys. JETP **2**, 73 (1956)]; I.E. Dzyaloshinskii, E.M. Lifshitz, and L.P. Pitaevskii, Sov. Phys. Usp. **4**, 153 (1961) [Adv. Phys. **10**, 165 (1961)].
[3] J.N. Israelachvili and G.E. Adams, J. Chem. Soc., Faraday Trans. 1 **74**, 975 (1978); J.N. Israelachvili and D. Tabor, Proc. R. Soc. London, Ser. A **331**, 19 (1972).
[4] S.K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[5] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
[6] T. Ederth, Phys. Rev. A **62**, 062104 (2000).
[7] B.W. Ninham, V.A. Parsegian, and G.H. Weiss, J. Stat. Phys. **2**, 323 (1970).
[8] D.J. Mitchell, B.W. Ninham, and P. Richmond, Am. J. Phys. **40**, 674 (1972).
[9] J. Mahanty and B.W. Ninham, *Dispersion Forces* (Academic, London, 1976).
[10] F.J. Dyson, Phys. Rev. **73**, 617 (1948).
[11] F.J. Dyson, Phys. Rev. **75**, 486 (1949).
[12] F.J. Dyson, Phys. Rev. **75**, 1736 (1949).
[13] R.P. Feynman, Phys. Rev. **80**, 440 (1950).
[14] M. Boström and Bo E. Sernelius, Phys. Rev. Lett. **84**, 4757 (2000).
[15] I. Brevik, J.B. Aarseth, and J.S. Hoye, Phys. Rev. E **66**, 026119 (2002).
[16] B.W. Ninham and J. Daicic, Phys. Rev. A **57**, 1870 (1998). See also B. Davies and B.W. Ninham, J. Chem. Phys. **56**, 5797 (1972).
[17] H. Wennerström, J. Daicic, and B.W. Ninham, Phys. Rev. A **60**, 2581 (1999).
[18] M. Boström, J.J. Longdell, and B.W. Ninham, Phys. Rev. A **64**, 062702 (2001).
[19] M. Boström, J.J. Longdell, and B.W. Ninham, Europhys. Lett. **59**, 21 (2002); M. Boström, J.J. Longdell, D.J. Mitchell, and B.W. Ninham, Eur. Phys. J. D **22**, 47 (2003).
[20] B.W. Ninham and V. Yaminsky, Langmuir **13**, 2097 (1997).
[21] M. Boström, D.R.M. Williams, and B.W. Ninham, Phys. Rev. Lett. **87**, 168103 (2001).
[22] M. Boström, D.R.M. Williams, and B.W. Ninham, Langmuir **18**, 8609 (2002), and references therein.
[23] M. Boström, D.R.M. Williams, and B.W. Ninham, Langmuir **77**, 4475 (2001); W. Kunz and L. Belloni (private communication).
[24] L. Onsager and N.N.T. Samaris, J. Chem. Phys. **2**, 528 (1934).
[25] F.J. Yndurain, *Relativistic Quantum Mechanics and Introduction to Field Theory* (Springer, Berlin, 1996).
[26] L.D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1*, 3rd ed. (Butterworth-Heinemann, Oxford, 1999).
[27] P. Johansson and P. Apell, Phys. Rev. B **56**, 4159 (1997), and references therein.
[28] B.W. Ninham, C.J. Powell, and N. Swanson, Phys. Rev. **145**, 209 (1966).