

Conditional preparation of maximal polarization entanglement

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A simple experimental setup consisting of a spontaneous parametric down-conversion source and passive linear optics is proposed for conditional preparation of a maximally entangled polarization state of two photons. Successful preparation is unambiguously heralded by coincident detection of four auxiliary photons. The proposed scheme utilizes the down-conversion term corresponding to the generation of three pairs of photons. We analyze imperfect detection of the auxiliary photons and demonstrate that its deleterious effect on the fidelity of the prepared state can be suppressed at the cost of decreasing the efficiency of the scheme.

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Pairs of photons in a maximally entangled polarization state constitute a basic constructional primitive in many protocols for quantum information processing, including teleportation, dense coding, and cryptography [1]. They have also been serving as a source of nonlocal correlations that violate Bell's inequalities, thus contradicting assumptions underlying local realistic theories. Despite enormous progress in generating entangled states of photons [2], starting from the initial experiments with atomic cascades [3], deterministic polarization entanglement in the photonic domain remains an elusive entity [4,5]. Indeed, the majority of current experiments is based on the production of photon pairs in the process of spontaneous parametric down-conversion [2], which is inherently random. Consequently, it is possible to determine whether a pair has been generated only by postselection, when looking *a posteriori* at the number of detected photons. This property is not essential in some applications such as tests of Bell's inequalities, but it becomes critical especially in experiments involving multiple photon pairs [4,6]. The random character of down-conversion sources may not be shared in the future by the solid-state sources of single photons or photon pairs that are presently being developed [7], though they will probably require operation at liquid-helium temperatures.

From a practical point of view, spontaneous parametric down-conversion in nonlinear crystals is a stable and robust process that requires modest experimental means to set up. An interesting problem is therefore whether the randomness of the parametric sources could be overcome by means of conditional detection. In such a scheme, detecting a number of auxiliary photons by trigger detectors would provide *a priori* information that an entangled photon pair has been generated, without destructive photodetection. Such a pair could be used in the event-ready manner, or possibly stored in a cavity [8] or an atomic system [9] for later use at any instant of time. The most natural approach to realize this idea would be to perform the procedure of entanglement swapping on two entangled pairs generated independently, one by each of the two crystals. The Bell measurement would then play a twofold role of collapsing the state of the remaining photons onto an entangled state as well as assuring their presence [10]. However, when the pairs are generated in parametric down-conversion, it is necessary to take into account other processes whose probability of occurrence is of

the same order of magnitude, such as generation of a double pair in one crystal and none in the second crystal [4]. This turns out to be a fundamental obstacle in the conditional preparation of maximal entanglement from four down-converted photons: it has been shown [5] that a maximally entangled state cannot be generated with a nonzero probability in any setup comprising down-converters and linear optics, based on detection of two auxiliary photons. This rules out the possibility of using the second-order term of the spontaneous down-conversion output, containing overall four photons, to produce maximally entangled pairs by means of conditional detection.

In this paper we show that the six-photon component of the spontaneous down-conversion output suffices to generate the maximally entangled polarization state of two photons in an event-ready manner. Specifically, we propose here an experimental setup based on a single nonlinear crystal producing two beams containing photons with pairwise correlated polarizations. We demonstrate that fourfold coincidence detection performed on fractions of the output beams picked off with nonpolarizing beam splitters leaves the remaining modes in a maximally entangled two-photon state. The use of the third-order term of spontaneous down-conversion places certainly more stringent requirements on the brightness of the necessary sources, but as we discuss later the present progress in down-conversion sources is likely to make this idea feasible in the near future. The proposed setup presents a substantial advancement over the scheme implied directly by the general methodology of quantum computing with linear optics [11], whose implementation with down-conversion sources would require in total four photon pairs [12].

The proposed setup, shown in Fig. 1, consists of one pumped nonlinear crystal generating pairs of down-converted photons entangled in their polarizations, and a passive optical circuit directing the down-converted photons into the output and detected modes. We label the modes with the corresponding annihilation operators. We assume that the conversion rate is low, which will allow us to describe the down-conversion process using the perturbative expansion in the number of the produced photons. Let $(\hat{a}_x, \hat{a}_y; \hat{b}_x, \hat{b}_y)$ be the polarization modes of the down-converted photons. The beam (\hat{a}_x, \hat{a}_y) is directed to a nonpolarizing beam splitter BS1 with the amplitude transmission coefficient $\cos \theta_a$,

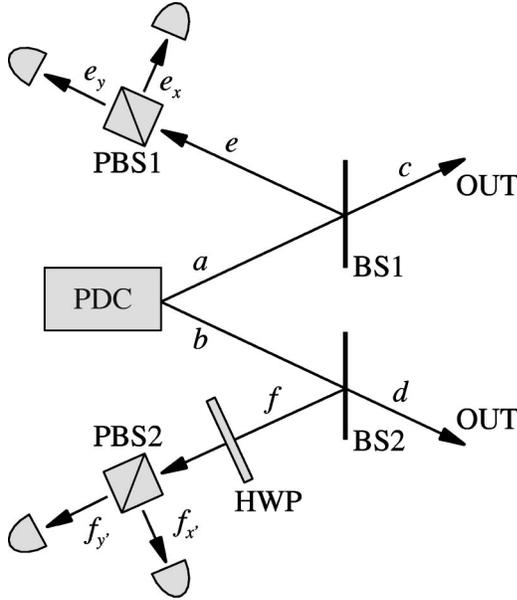


FIG. 1. Schematic representation of the setup for generating heralded entanglement. BS1, BS2, nonpolarizing beam splitters; HWP, half-wave plate; PBS1, PBS2, polarizing beam splitters. Lowercase letters label the beams.

where the transmitted modes (\hat{c}_x, \hat{c}_y) constitute the setup output, while the reflected modes (\hat{e}_x, \hat{e}_y) are detected locally. The other beam (\hat{b}_x, \hat{b}_y) is directed to another beam splitter BS2 with the amplitude transmission coefficient $\cos \theta_b$, where the transmitted modes (\hat{d}_x, \hat{d}_y) are the remaining two output modes of the setup, while the reflected modes (\hat{f}_x, \hat{f}_y) are analyzed in the $\pi/4$ rotated polarization modes $(\hat{f}_{x'}, \hat{f}_{y'})$, defined by the relations $\hat{f}_{x'} = (\hat{f}_x + \hat{f}_y)/\sqrt{2}$ and $\hat{f}_{y'} = (\hat{f}_x - \hat{f}_y)/\sqrt{2}$.

We assume that the photodetectors monitoring the auxiliary modes can perform the ideal projection onto the one-photon Fock state $|1\rangle\langle 1|$. We will be interested in events when each of the four auxiliary detectors registers exactly one photon. We shall demonstrate that the lowest order for which such an event can occur is when three pairs of down-converted photons are generated, and that in this order a fourfold coincident detection of single photons in the auxiliary modes $(\hat{e}_x, \hat{e}_y; \hat{f}_{x'}, \hat{f}_{y'})$ unambiguously heralds that the quadruplet of the output modes $(\hat{c}_x, \hat{c}_y; \hat{d}_x, \hat{d}_y)$ contains a pair of photons in a maximally entangled polarization state. This procedure prepares a maximally entangled state without the usual vacuum contribution, being consequently a source of event-ready entanglement in the photonic domain [10].

Let us now discuss the operation of the scheme in quantitative terms. The Hamiltonian governing the down-conversion process in the weak conversion regime describes two independent processes, corresponding to generation or deletion of a photon pair in modes $(\hat{a}_x; \hat{b}_y)$ or $(\hat{a}_y; \hat{b}_x)$, respectively. These two processes are added coherently with opposite probability amplitudes. Assuming that the effective dimensionless interaction time is r , we can decompose the

output state into terms that contain a fixed number of down-converted photons [4,13]:

$$|\Psi\rangle = \exp[r(\hat{a}_x^\dagger \hat{b}_y^\dagger - \hat{a}_x \hat{b}_y) - r(\hat{a}_y^\dagger \hat{b}_x^\dagger - \hat{a}_y \hat{b}_x)]|\text{vac}\rangle \\ = \sum_{n=0}^{\infty} \lambda_n |\Psi_n\rangle, \quad (1)$$

where

$$\lambda_n = \sqrt{n+1} (\tanh^n r / \cosh^2 r) \quad (2)$$

is the probability amplitude of generating n photon pairs, and the normalized n -pair component of the wave function takes the following form:

$$|\Psi_n\rangle = \frac{1}{\sqrt{n+1}} \sum_{m=0}^n (-1)^m |n-m, m; m, n-m\rangle \\ = \frac{1}{n! \sqrt{n+1}} (\hat{a}_x^\dagger \hat{b}_y^\dagger - \hat{a}_y^\dagger \hat{b}_x^\dagger)^n |\text{vac}\rangle. \quad (3)$$

The occupation numbers in the first expression for $|\Psi_n\rangle$ correspond to the ordering of the modes as $(\hat{a}_x, \hat{a}_y; \hat{b}_x, \hat{b}_y)$.

Let us first show that any of the terms with $n < 3$ cannot give a fourfold coincidence on the auxiliary detectors. Obviously, the only term that could possibly give rise to such an event corresponds to $n=2$ and is explicitly given by

$$|\Psi_2\rangle = \frac{1}{\sqrt{3}} (|2,0;0,2\rangle - |1,1;1,1\rangle + |0,2;2,0\rangle). \quad (4)$$

The four-fold coincidence event implies that all the four photons have been reflected by the beam splitters BS1 and BS2. However, detection of two photons in the modes (\hat{e}_x, \hat{e}_y) means that we are observing the middle term of the sum in Eq. (4), and that the state of the remaining two photons is collapsed to $|1_{\hat{f}_{x'}}, 1_{\hat{f}_{y'}}\rangle$. This state is transformed by the half-wave plate to the form $(|2_{\hat{f}_{x'}}, 0_{\hat{f}_{y'}}\rangle - |0_{\hat{f}_{x'}}, 2_{\hat{f}_{y'}}\rangle)/\sqrt{2}$, which of course cannot give a coincidence on the detectors monitoring the modes $\hat{f}_{x'}$ and $\hat{f}_{y'}$. This is essentially the destructive two-photon interference effect observed first by Hong, Ou, and Mandel [14].

Having shown that $n=3$ is the lowest order that can contribute to the fourfold coincidence event in our scheme, let us now find the conditional state of the output modes provided that each of the auxiliary detectors has seen exactly one photon. The easiest way to approach this task is to use the second expression for the state $|\Psi_3\rangle$ given in Eq. (3) in terms of the creation operators:

$$|\Psi_3\rangle = \frac{1}{\sqrt{12}} (\hat{a}_x^\dagger \hat{b}_y^\dagger - \hat{a}_y^\dagger \hat{b}_x^\dagger)^3 |\text{vac}\rangle. \quad (5)$$

The linear circuit placed after the nonlinear crystal is described by the following transformation of the annihilation operators of the down-conversion modes:

$$\hat{a}_x = \hat{c}_x \cos \theta_a + \hat{e}_x \sin \theta_a, \quad (6a)$$

$$\hat{a}_y = \hat{c}_y \cos \theta_a + \hat{e}_y \sin \theta_a, \quad (6b)$$

$$\hat{b}_x = \hat{d}_x \cos \theta_b + (\hat{f}_{x'} + \hat{f}_{y'}) \sin \theta_b / \sqrt{2}, \quad (6c)$$

$$\hat{b}_y = \hat{d}_y \cos \theta_b + (\hat{f}_{x'} - \hat{f}_{y'}) \sin \theta_b / \sqrt{2}. \quad (6d)$$

After inserting the above representation into Eq. (5), we expand the resulting polynomial and isolate terms that contribute to the coincidence event of interest. These terms must contain the combination of the creation operators of the auxiliary modes in the form $\hat{e}_x^\dagger \hat{e}_y^\dagger \hat{f}_{x'}^\dagger \hat{f}_{y'}^\dagger$. The above rather lengthy procedure can be aided with a computer algebra system. Explicitly, the relevant terms are

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}} \sin^2 \theta_a \cos \theta_a \sin^2 \theta_b \cos \theta_b \frac{1}{\sqrt{2}} (\hat{c}_x^\dagger \hat{d}_x^\dagger + \hat{c}_y^\dagger \hat{d}_y^\dagger) \times \hat{e}_x^\dagger \hat{e}_y^\dagger \hat{f}_{x'}^\dagger \hat{f}_{y'}^\dagger |\text{vac}\rangle + \dots, \quad (7)$$

where “...” denote all the other components. It is thus seen that the coincident detection of four photons in each of the modes $\hat{e}_x, \dots, \hat{f}_{y'}$ nondestructively collapses the state of the output modes to the vector

$$|\Phi\rangle = \frac{1}{\sqrt{2}} (\hat{c}_x^\dagger \hat{d}_x^\dagger + \hat{c}_y^\dagger \hat{d}_y^\dagger) |\text{vac}\rangle, \quad (8)$$

which describes a maximally entangled state of two photons. This state can be of course transformed into any other Bell state with the help of phase shifters and polarization rotators.

The efficiency of successful state preparation in our scheme can be defined as the probability of the fourfold coincidence event assuming that exactly three photon pairs have been generated in the down-conversion process. It is explicitly given by

$$P = \frac{1}{2} (\sin^2 \theta_a \cos \theta_a \sin^2 \theta_b \cos \theta_b)^2, \quad (9)$$

and it attains its maximum, equal to $(2/9)^3 \approx 0.011$, when the power transmission coefficients of the beam splitters are $\cos^2 \theta_a = \cos^2 \theta_b = 1/3$. In order to obtain the overall preparation efficiency, this value of P needs to be multiplied by the probability $(\lambda_3)^2$, defined by Eq. (2), of generating the six-photon state. As it would be beneficial to run the down-conversion process with the interaction parameter r as large as possible while staying within the perturbative regime, we have also estimated the contribution from the next leading order term given by the state $|\Psi_4\rangle$. The probability of producing a fourfold single-photon coincidence by this component is $\frac{13}{5} (\sin \theta_a \cos \theta_a \sin \theta_b \cos \theta_b)^4$, which for the choice of the transmission coefficients optimizing P gives approximately 6.3×10^{-3} .

The operation of the proposed scheme can be understood more intuitively using the Fock state representation of the state $|\Psi_3\rangle$:

$$|\Psi_3\rangle = \frac{1}{2} (|3,0;0,3\rangle - |2,1;1,2\rangle + |1,2;2,1\rangle - |0,3;3,0\rangle).$$

Observation of a twofold coincidence in the modes (\hat{e}_x, \hat{e}_y) means that one photon has been extracted from each of the modes \hat{a}_x and \hat{a}_y . This leaves us with the state of the remaining photons proportional to $-|1,0;1,2\rangle + |0,1;2,1\rangle$. Further, a twofold coincidence on the detectors in the lower arm

of the setup means that the state of the modes \hat{f}_x and \hat{f}_y (before the beam splitter BS2) has been collapsed to the coherent superposition $(|0,2\rangle - |2,0\rangle) / \sqrt{2}$. This means that two photons must have been removed either from the mode \hat{b}_x or \hat{b}_y by the beam splitter BS2. This gives the state of the remaining photons of the form $|1,0;1,0\rangle + |0,1;0,1\rangle$, which has been defined in Eq. (8).

We will now analyze the effect of imperfections of the auxiliary detectors in the proposed scheme. The most critical issue is undercounting photons, when the detectors give a fourfold single-photon coincidence even if more than two photons have been reflected by the beam splitters BS1 and/or BS2. In such a case the output state will be contaminated by additional terms containing a single photon or vacuum, which are orthogonal to the maximally entangled two-photon component. However, we shall demonstrate that the effect of detection losses can be suppressed to some extent at the cost of decreasing the efficiency of the state preparation.

We shall model detector losses in the standard way assuming the same detection efficiency equal to η for all the four detectors. The most convenient way to include losses in the previous calculations is to introduce additional tilded modes $\hat{e}_x, \hat{e}_y, \hat{f}_{x'}, \hat{f}_{y'}$ that are mixed with the fields monitored by the detectors [15]. This corresponds to replacing the operator \hat{e}_x in Eq. (6a) by a combination $\sqrt{\eta} \hat{e}_x + \sqrt{1-\eta} \hat{e}_x$, and similarly for $\hat{e}_y, \hat{f}_{x'}$, and $\hat{f}_{y'}$. The tilded operators $\hat{e}_x, \dots, \hat{f}_{y'}$ describe here photons that escape detection due to nonunit efficiency. After straightforward algebra we find that the component of the reduced density matrix for the output modes $(\hat{c}_x, \hat{c}_y; \hat{d}_x, \hat{d}_y)$ that is correlated with the observation of a fourfold single-photon coincidence on the trigger detectors has the following form:

$$\hat{\rho} = \frac{1}{4} \eta^4 \sin^8 \theta [2 \cos^4 \theta |\Phi\rangle \langle \Phi| + (1-\eta) \times \sin^2 \theta \cos^2 \theta (|1,0;0,0\rangle \langle 1,0;0,0| + |0,1;0,0\rangle \langle 0,1;0,0| + |0,0;1,0\rangle \langle 0,0;1,0| + |0,0;0,1\rangle \langle 0,0;0,1|) + 2(1-\eta)^2 \sin^4 \theta |\text{vac}\rangle \langle \text{vac}|], \quad (10)$$

where we have assumed for simplicity the transmission coefficient to be equal for both the beam splitters: $\theta_a = \theta_b = \theta$, and the state $|\Phi\rangle$ has been defined in Eq. (8). The different terms are added incoherently in the above formula, as there is distinguishing information provided by the photons remaining in the undetected tilded modes modeling losses. We have kept the normalization of $\hat{\rho}$ implied by the complete multimode wave function. This allows us to calculate the probability of the coincidence event as the trace of $\hat{\rho}$:

$$P = \text{Tr} \hat{\rho} = \frac{1}{2} (\eta \sin^2 \theta)^4 (1 - \eta \sin^2 \theta)^2, \quad (11)$$

whereas the fidelity of the conditionally prepared state reads

$$F = \frac{\langle \Phi | \hat{\rho} | \Phi \rangle}{\text{Tr} \hat{\rho}} = \frac{\cos^2 \theta}{1 - \eta \sin^2 \theta}. \quad (12)$$

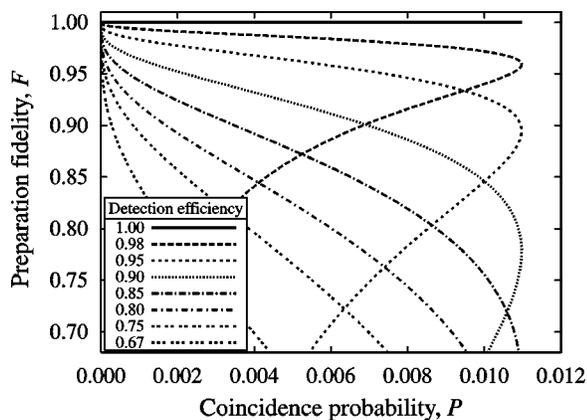


FIG. 2. The fidelity F of the prepared state versus the probability P of a fourfold single-photon coincidence for several values of the detection efficiency η . The curves are parametrized with the beam splitter coefficient θ running from 0 to $\pi/2$.

In Fig. 2 we depict parametric plots $[P(\theta), F(\theta)]$ parametrized with the beam splitter coefficient θ , for several values of the detection efficiency η . All the curves span from the same point ($P=0, F=1$) corresponding to $\theta=0$, i.e., fully transmitting beam splitters, to the maximum possible preparation efficiency [equal to the same value $(2/9)^3$ as long as $\eta \geq 2/3$], with the fidelity of the prepared state getting worse with decreasing η . However, it is clearly seen in Fig. 2 that the fidelity can be improved for any detection efficiency η by increasing the beam splitter transmission. In this regime, the probability of reflecting more than the minimum number of photons necessary to the trigger detectors becomes lower, and consequently the danger of undercounting the auxiliary photons is less important. This results in enhanced fidelity of the prepared state, though at the cost of lower preparation efficiency. This effect demonstrates that the proposed scheme is, in principle, robust to the effects of the nonunit efficiency of the trigger detectors.

In conclusion, we have proposed a conditional scheme

based on a spontaneous parametric down-conversion source that is capable of event-ready generation of polarization entangled photon pairs. The scheme utilizes the three-pair component of the down-conversion output, and the deleterious contribution from the lower order term vanishes due to the well-known destructive two-photon interference effect [14]. The proposed scheme can be considered as a next-order generalization of the earlier concept of entanglement swapping [10].

As our scheme relies basically on polarization interference of photons from one down-conversion source, the required experimental means appear to be moderate. The critical parameters of the source are its brightness and the preservation of the correlations in the number of produced photons. These characteristics seem to be most promising for currently studied sources based on nonlinear waveguides that offer improved control over the spatio-temporal structure of the produced photons [16]. Assuming that the probability of producing a single photon pair would be of the order of 5%, it is then easy to calculate that the probability of a fourfold coincidence in the proposed setup is 7×10^{-7} . This should give a few dozens of useful photon pairs per second for a pump laser with 100-MHz repetition rate, and this figure could be improved by an order or two of magnitude by using a multigigahertz mode-locked laser [17], provided that it can deliver enough pulse energy. These constraints, though certainly challenging, do not seem to be very far from the capabilities of current technology. It should also be noted that the visibility of the destructive two-photon interference occurring for the second-order down-conversion term needs to be high enough to make dominant the coincidence events generated by the next-order term. The experimental observation of the destructive interference for the second-order term should be feasible right now, as demonstrated in recent experiments by Lamas-Linares *et al.* [13,18].

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