Scheme for the implementation of a universal quantum cloning machine via cavity-assisted atomic collisions in cavity QED

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We propose a scheme to implement the $1\rightarrow 2$ universal quantum cloning machine of Buzek and Hillery [Phys. Rev. A **54**, 1844 (1996)] in the context of cavity QED. The scheme requires cavity-assisted collision processes between atoms, which cross through nonresonant cavity fields in the vacuum states. The cavity fields are only virtually excited to face the decoherence problem. That's why the requirements on the cavity quality factor can be loosened.

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In the last decade, considerable progress in the field of quantum information processing has been made. New prospects in computation and communication technology are very challenging. Basic questions on this kind of information transfer have been raised. Quantum information differs from classical information in a fundamental way. For instance, it is not possible to construct a device that produces an exact copy of the state of a simple quantum system [1]. This statement is a consequence of the linearity of quantum mechanics. It constitutes one of the most significant differences between classical information and quantum information. The seminal paper of Buzek and Hillery [2] put a strong impulse on quantum cloning. This problem was extensively studied in the example of discrete quantum variable systems, such as quantum qubits [3] or d-level systems [4]. Bounds on the maximum possible fidelity of the clones produced by universal quantum cloning machine were derived and an optimal universal quantum cloning transformation was discovered [3,4]. In order to make new applications in this field possible, an appropriate quantum system is needed, which can be very well isolated from the environment to suppress decoherence processes. Several physical systems were suggested to implement the concept of quantum information processing: cavity QED [5], trapped ion systems [6], and nuclear-magnetic-resonance (NMR) systems [7]. Cavity QED with Rydberg atoms, which cross superconducting cavities, are nearly ideal systems for this purpose. Various entangled states such as Einstein-Podolsky Rosen (EPR) pairs [8] and GHZ states [9] have been successfully produced by a successive interaction of a series of atoms with the cavity field. An experimental implementation of the quantum logic gate [10] and the absorption-free detection of a single photon [11] have been reported by using a resonant atom-cavity interaction. A number of schemes have been proposed for the teleportation of atomic states [12], the implementation of quantum algorithms [13,14], and the realization of entanglement purification [15]. Recently, quantum cloning of a single-photon state was demonstrated experimentally [16] by using the scheme, which was proposed in Ref. [17]. An alternative experimental implementation of the cloning network, which is based on the NMR system, has been reported [18]. More recently, a cavity QED scheme is proposed to implement a $1\rightarrow 2$ universal quantum cloning machine by using a resonant atom-cavity interaction [19]. In this scheme, cavities act as memories, which store the information of an electric system and transfer it back to this electric system after the conditional dynamics. But, the decoherence of the cavity field becomes one of the main obstacles for the experimental realization of the universal quantum cloning machine.

In this paper, we propose a scheme to implement the $1\rightarrow 2$ universal quantum cloning machine of Buzek and Hillery [2] within cavity QED. In contrast to the scheme proposed in Ref. [19], our scheme requires a cavity-assisted collisions between atoms [20]. This technique has been experimentally demonstrated [21]. In order to overcome the main problem of decoherence a virtual excitation of the cavity field is chosen. That's why this kind of implementation becomes essentially insensitive to cavity losses and to thermal cavity excitations. The cavity-assisted collision processes have been used for the generation of entangled atomic states [22] and the implementation of a quantum search algorithm [14].

At first, we consider the interaction of N two-level atoms with a single-mode cavity field. In the interaction picture the Hamiltonian is

$$H=g\sum_{j=1}^{N}\,(e^{-i\delta t}a^{\dagger}\sigma_{-}^{j}+e^{i\delta t}a\sigma_{+}^{j}), \tag{1}$$

where $\sigma_{-}^{j}=|g_j\rangle\langle e_j|$ and $\sigma_{+}^{j}=|e_j\rangle\langle g_j|$, with $|g_j\rangle$ and $|e_j\rangle$ $(j=1,\ldots,N)$ are the ground states and the excited states of the jth atom. The annihilation and creation operator of the cavity field are a and $a^{\dagger}$. We use g as the atom-cavity coupling strength and δ as the detuning between the atomic transition frequency and the cavity frequency. We consider the case $\delta\gg g\sqrt{\overline{n}+1}$, with the mean photon number $\overline{n}$ of the cavity field. With this restriction, it is convenient to consider the interaction (1) in terms of a coarse-grained Hamiltonian, which neglects the effect of rapidly oscillating terms. We use the time-averaging method of Ref. [23] to derive the effective Hamiltonian [20,22]

$$H=\lambda\left[\sum_{j=1}^{N}\,(|e_j\rangle\langle e_j|aa^{\dagger}-|g_j\rangle\langle g_j|a^{\dagger}a)+\sum_{j,k=1}^{N}\,\sigma_{+}^{j}\sigma_{-}^{k}\right],$$

$$j\neq k, \tag{2}$$

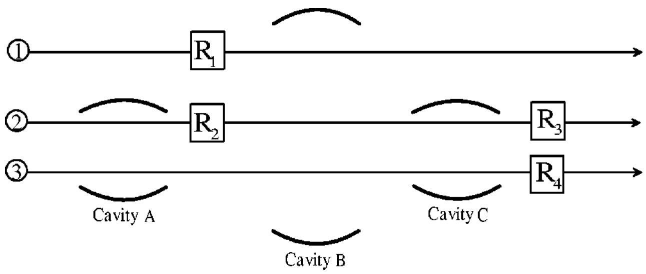

FIG. 1. This is the schematic diagram of the optimal $1\rightarrow2$ quantum cloning process, which copies the quantum state of the atom 1 to the atoms 2 and 3. Three cavities *A*, *B*, and *C* are involved, which are prepared in the vacuum state. The abbreviations R_i denote the Ramsey zones, in which a classical field rotates the atoms along the z axis by θ_i. At first the atoms 2 and 3 enter the cavity *A*, where they are prepared in a maximally entangled state. After the atoms 1 and 2 have been manipulated by classical fields, the three atoms are simultaneously sent into the cavity *B*, where they interact with each other via cavity-assisted atomic collision processes. After the atoms 2 and 3 have crossed through the cavity *B*, the cavity *C* and two classical fields are used to perform phase-shift operations on the quantum states of the atoms 2 and 3.

with the abbreviation $\lambda=g^2/\delta$. If the cavity field is initially in the vacuum state, the effective Hamiltonian reduces to

$$H=\lambda\left(\sum_{j=1}^{N}|e_j\rangle\langle e_j|+\sum_{j,k=1}^{N}\sigma_+^j\sigma_-^k\right),\quad j\neq k. \tag{3}$$

Now we present the scheme to implement the $1\rightarrow2$ universal quantum cloning machine of Buzek and Hillery [2]. The experimental setup, which we suggest to use, is depicted in Fig. 1. Three two-level atoms are needed to implement our scheme. Furthermore, we assume that all cavities are prepared in the vacuum state. The atoms interact with each other via cavity-assisted atomic collision processes [Eq. (3)]. We assume that atom 1 carries the quantum state, which will be cloned. It is prepared in an arbitrary pure state of the form

$$|\Psi_{in}\rangle=\alpha|g_1\rangle+\beta|e_1\rangle, \tag{4}$$

where α and β are complex coefficients. The other atoms 2 and 3 are prepared in the state $|e_2\rangle|g_3\rangle$. They are sent into the first cavity *A*. The effective interaction (3) between the atoms 2 and 3 can be written as

$$H=\lambda(|e_2\rangle\langle e_2|+|e_3\rangle\langle e_3|+\sigma_+^2\sigma_-^3+\sigma_-^2\sigma_+^3). \tag{5}$$

The evolution of the quantum state of the atoms 2 and 3 is given by

$$|e_2\rangle|g_3\rangle\rightarrow e^{-i\lambda t}[\cos(\lambda t)|e_2\rangle|g_3\rangle-i\sin(\lambda t)|g_2\rangle|e_3\rangle]. \tag{6}$$

If we choose the interaction time to satisfy $\lambda t=\pi/4$, the state of the system becomes

$$|\Psi_{23}\rangle=\frac{1}{\sqrt{2}}[|e_2\rangle|g_3\rangle-i|g_2\rangle|e_3\rangle], \tag{7}$$

where we have discarded the common phase factor. After the atoms 2 and 3 emit from the cavity *A*, these two atoms and the atom 1, which carries the information, will simultaneously be sent to another cavity *B*. But before, a classical field is applied to rotate atom 1 and atom 2 along the z axis by the angles θ_1 and θ_2, respectively. The corresponding transformation is given by

$$U_j=\exp[-i\theta_j(|e_j\rangle\langle e_j|-|g_j\rangle\langle g_j|)]. \tag{8}$$

Thus, the state $|\Psi_{in}\rangle$ becomes

$$|\Psi_{in}'\rangle=\alpha e^{i\theta_1}|g_1\rangle+\beta e^{-i\theta_1}|e_1\rangle, \tag{9}$$

where θ_1 will be determined later. If we choose the parameter $\theta_2=\pi/4$, the two-atom quantum state $|\Psi_{23}\rangle$ becomes

$$|\Psi_+\rangle=\frac{1}{\sqrt{2}}[|e_2\rangle|g_3\rangle+|g_2\rangle|e_3\rangle]. \tag{10}$$

The state of the total system $|\Psi_{in}'\rangle\otimes|\Psi_+\rangle$ can directly be used to implement the $1\rightarrow2$ universal quantum cloning machine. Now we send three atoms into the cavity *B*. These three atoms interact with each other via cavity-assisted atomic collision processes according to the effective Hamiltonian (3):

$$H=\lambda\Bigg(\sum_{j=1}^{3}|e_j\rangle\langle e_j|+\sigma_+^1\sigma_-^2+\sigma_-^1\sigma_+^2+\sigma_+^1\sigma_-^3+\sigma_-^1\sigma_+^3+\sigma_+^2\sigma_-^3+\sigma_-^2\sigma_+^3\Bigg). \tag{11}$$

If the atoms are in the state $|g_1\rangle|\Psi_+\rangle$ or $|e_1\rangle|\Psi_+\rangle$, the quantum state evolves as follows:

$$\begin{aligned}|g_1\rangle|\Psi_+\rangle\rightarrow e^{-i3\lambda t/2}\Bigg\{&\left[\cos\left(\frac{3}{2}\lambda t\right)-\frac{i}{3}\sin\left(\frac{3}{2}\lambda t\right)\right]|g_1\rangle|\Psi_+\rangle\\&-\frac{i2\sqrt{2}}{3}\sin\left(\frac{3}{2}\lambda t\right)|e_1\rangle|g_2\rangle|g_3\rangle\Bigg\},\\|e_1\rangle|\Psi_+\rangle\rightarrow e^{-i5\lambda t/2}\Bigg\{&\left[\cos\left(\frac{3}{2}\lambda t\right)-\frac{i}{3}\sin\left(\frac{3}{2}\lambda t\right)\right]|e_1\rangle|\Psi_+\rangle\\&-\frac{i2\sqrt{2}}{3}\sin\left(\frac{3}{2}\lambda t\right)|g_1\rangle|e_2\rangle|e_3\rangle\Bigg\}.\end{aligned} \tag{12}$$

If we choose the interaction time to satisfy $\lambda t=2\pi/9$, the state of the total system $|\Psi_c'\rangle\otimes|\Psi_+\rangle$ evolves into

$$\begin{aligned}|\Psi_m\rangle=&\alpha e^{i\theta_1+i\pi/6}\left[\sqrt{\frac{2}{3}}|e_1\rangle|g_2\rangle|g_3\rangle+\sqrt{\frac{1}{3}}e^{i\pi/3}|g_1\rangle|\Psi_+\rangle\right]\\&+\beta e^{-i\theta_1+i\pi/18}\left[\sqrt{\frac{2}{3}}|g_1\rangle|e_2\rangle|e_3\rangle\right.\\&\left.+\sqrt{\frac{1}{3}}e^{i\pi/3}|e_1\rangle|\Psi_+\rangle\right].\end{aligned} \tag{13}$$

After the three atoms have crossed through the cavity B, we send the atoms 2 and 3 again into the third cavity C. The effective interaction between the atoms can be described by Eq. (5). After the interaction time τ, the quantum state (13) evolves into

$$\begin{aligned}|\Psi_n\rangle = \alpha e^{i\theta_1+i\pi/6}\Bigg[\sqrt{\frac{2}{3}}\,|e_1\rangle|g_2\rangle|g_3\rangle \\ +\sqrt{\frac{1}{3}}\,e^{i\pi/3-2i\lambda\tau}|g_1\rangle|\Psi_+\rangle\Bigg] \\ +\beta e^{-i\theta_1+i\pi/18}\Bigg[\sqrt{\frac{2}{3}}\,e^{-2i\lambda\tau}|g_1\rangle|e_2\rangle|e_3\rangle \\ +\sqrt{\frac{1}{3}}\,e^{i\pi/3-2i\lambda\tau}|e_1\rangle|\Psi_+\rangle\Bigg]. \quad (14)\end{aligned}$$

We will determine the interaction time τ later. After the atom 2 and the atom 3 have crossed through the cavity C, these two atoms are rotated along the z axis by the angles θ_3 and θ_4. Therefore a classical microwave pulse is used. The corresponding transformation is described by Eq. (8). If we choose the condition $\theta_3=\theta_4$, the quantum state (14) becomes

$$\begin{aligned}|\Psi_o\rangle = \alpha e^{i\theta_1+i\pi/6}\Bigg[\sqrt{\frac{2}{3}}\,e^{-2i\theta_3}|e_1\rangle|g_2\rangle|g_3\rangle \\ +\sqrt{\frac{1}{3}}\,e^{i\pi/3-2i\lambda\tau}|g_1\rangle|\Psi_+\rangle\Bigg] \\ +\beta e^{-i\theta_1+i\pi/18}\Bigg[\sqrt{\frac{2}{3}}\,e^{-2i\lambda\tau+2i\theta_3}|g_1\rangle|e_2\rangle|e_3\rangle \\ +\sqrt{\frac{1}{3}}\,e^{i\pi/3-2i\lambda\tau}|e_1\rangle|\Psi_+\rangle\Bigg]. \quad (15)\end{aligned}$$

We choose the parameters θ_1, θ_3, and $\lambda\tau$ to satisfy $\theta_1=-\pi/18$, $\theta_3=\pi/6$, and $\lambda\tau=\pi/3$. In this case the state (15) reduces to

$$\begin{aligned}|\Psi_f\rangle = \alpha\Bigg[\sqrt{\frac{2}{3}}\,|e_1\rangle|g_2\rangle|g_3\rangle+\sqrt{\frac{1}{3}}\,|g_1\rangle|\Psi_+\rangle\Bigg] \\ +\beta\Bigg[\sqrt{\frac{2}{3}}\,|g_1\rangle|e_2\rangle|e_3\rangle+\sqrt{\frac{1}{3}}|e_1\rangle|\Psi_+\rangle\Bigg], \quad (16)\end{aligned}$$

if the common phase factor is discarded. This equation demonstrates, that the optimal $1\to2$ cloning process is implemented [2].

In summary, a scheme for the implementation of the optimal $1\to2$ cloning process is proposed. In contrast to the scheme [19], only simple two-level atoms are required, which interact with the cavity fields. This might simplify the experimental implementation of the scheme of Buzek and Hillery. In contrast to the scheme of Milman *et al.* [19], our scheme requires a cavity-assisted collisions between atoms [20]. This technique has been experimentally demonstrated [21]. In this scheme, the cavity field is only virtually excited. There is no transfer of quantum information between atoms and cavity fields. That's why the requirement on the quality factor of the cavity can be loosened. This scheme is essentially insensitive to cavity losses and to thermal cavity excitations. It should be pointed out that the presented scheme involves three cavities, which might make the experimental implementation of the present scheme more complicated.

Finally we give a brief discussion on the experimental feasibility of the presented scheme. To implement the scheme, we need to preserve the coherence of the cavity field before the atoms are flying out of the cavity. For the Rydberg atoms with principle quantum number 50 and 51, the radiative time is about $T_r=3\times10^{-2}$ sec. The coupling of the atoms and the cavity field is $g/2\pi=50$ kHz [21]. In order to control the entanglement in the cavity-assisted collision process, the detuning δ should be much greater than g. With the choice $\delta=10g$ the interaction time between the atom and the cavity field is of the order $\pi\delta/g^2=10^{-4}$ sec. At this scale the time, which is needed to rotate the single qubit, is negligible. Thus, the interaction time, which is needed to perform the total procedure is shorter than the time, which is needed by the scheme [19]. This time interval is much shorter than T_r and the photon lifetime 1 ms in the present cavity. Therefore, based on the cavity QED technique the presented scheme is realizable.

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