Macroscopic quantum coherence in a repulsive Bose-Einstein condensate

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We consider a Bose-Einstein bicondensate of ^{87}Rb, trapped in two different internal levels, in a situation where the density undergoes a symmetry breaking in momentum space. This occurs for a suitable number of condensed atoms within a double-well dispersion curve, obtained by Raman coupling two internal states with two tilted and detuned light fields. Evidence of bistability results from the Gross-Pitaevskii equation. By second quantization, we evaluate the tunneling rate between the two asymmetric states; the effects of losses on coherence are negligible.

The transition from a superfluid to a Mott insulator has been recently demonstrated for a Bose-Einstein condensate (BEC), made of atoms with repulsive mutual interactions in a lattice potential [1]. Such a phenomenon goes beyond the mean-field approximation and its explanation requires to take into account the quantum fluctuations of the matter field. When the quantum tunneling between adjacent sites dominates the interaction energy, the probability distribution for the atomic occupation of a single site is Poissonian. In the opposite case, the minimum energy is obtained by reducing the quantum fluctuation of the local occupation number.

In a previous paper, we have studied a similar problem, but with only two wells and an attractive interaction [2]; in this case, the opposite effect occurs, that is, raising the atom number, i.e., the interaction energy, the minimum-energy state is obtained by increasing the atomic fluctuation in each site. This is indirectly demonstrated by the numerical observation of the symmetry breaking at a critical number of atoms. Approaching the threshold value, the quantum fluctuations increase, whereas above that value they blow up and a new minimum quantum state appears. The associated probability distribution of the condensate barycenter displays two peaks, which can be considered as the dead and alive states of a Schrödinger cat (SC), whose coherent superposition is called *macroscopic quantum coherence* (MQC). This term was introduced to describe the coherent superposition of two macroscopically distinct quantum states that differ for the value of a collective variable [3]. The phenomenon is observable only for attractive interactions that tend to cluster the atoms in one of the two wells. By contrast, a repulsive interaction tends to reduce the quantum fluctuations and to distribute the same number of atoms in each well, as observed in Ref. [1]. MQC has been observed with trapped ions [4] and microwave fields in high-Q cavity [5].

In this work, we discuss the feasibility of MQC in a BEC of mutually repulsive atoms. A repulsive interaction acts in the momentum space as an attractive interaction, therefore we expect that for a double-well dispersion curve the symmetry breaking occurs in the reciprocal space. Such a dispersion curve can be obtained by two detuned and tilted light fields, which transfer a net momentum to atoms as they jump from an internal state to another. We study the problem by finding the stationary solutions of two coupled Gross-Pitaevskii (GP) equations discretized over a space lattice. The system undergoes symmetry breaking in momentum

space for a suitable number of atoms and two new stationary states are created. We then introduce a quantum two-mode model, with the two modes chosen in such a way as to reproduce the stationary solutions of GP, and evaluate the quantum fluctuation-mediated tunneling rate between the two asymmetrical states. If the coupling with the environment is negligible, MQC occurs between these states. A Raman scheme for creating a superposition state with two Rb condensates in different internal quantum levels has already been discussed in Refs. [6,7]; however, both proposals have applicability problems, as discussed in Ref. [2]. We evaluate the decoherence rate due to the atomic losses and show that it is negligible, at variance with the previous schemes. Louis *et al.* [8] include a model of dissipation for the system studied in Ref. [7]. They claim that the main contribution to dissipation is due to the thermal cloud of noncondensed atoms. Instead, the dominant source of decoherence is the three-body loss rate, which is very large for atoms in different hyperfine levels [10]. In their case, it is necessary to use different hyperfine levels since the mutual scattering length has to be much larger than the single-species scattering length [7].

Here we refer to ^{87}Rb atoms, but our numerical results apply also to ^{23}Na, if some parameters are rescaled. In a previous work [9], we considered atoms in two different hyperfine levels ($F=1$, $m_F=-1$ and $F=2$, $m_F=1$), however, the associated depletion rate [10] is too high for our purposes, as it occurs in Refs. [7,8]. We consider condensate atoms that are optically trapped in the two Zeeman levels $F=1$, $m_F=-1$ and $F=1$, $m_F=1$. An all optical condensation has been reported in Ref. [11], alternatively, the condensate can be created with a magnetic confinement and transferred into an optical trap [12]. A homogeneous magnetic field has to be applied to remove the energy degeneracy. These levels are quasiresonantly coupled by means of two Raman fields L and R. We call ψ_0 and ψ_1 the fields associated with the $m_F=-1$ and $m_F=1$ levels, respectively. Furthermore, we call ψ_2 the upper state of the D_1 transition.

The starting equations are

$$i\hbar\dot{\psi}_{0,1} = (H_{0,1} - \hbar\omega_{0,1})\psi_{0,1} + \hbar E_{L,R}(t)e^{-i(\vec{k}_{L,R}\cdot\vec{x} - \omega_{L,R}t)}\psi_2,$$
(1)

$$i\hbar\dot{\psi}_2 = \hbar\omega_2\psi_2 + [\hbar E_L^*(t)e^{i(\vec{k}_L\cdot\vec{x} - \omega_L t)}\psi_0 + (L\leftrightarrow R)\psi_1],$$
(2)

where $H_0 = H_l + g_{00}|\psi_0|^2 + g_{01}|\psi_1|^2$, $H_1 = H_l + g_{11}|\psi_1|^2 + g_{10}|\psi_0|^2$, and $g_{ij} = 4\pi\hbar^2 a_{ij}/m$. $a_{ij} \sim 5.5$ nm [13] are the s-wave scattering lengths between atoms in i and j levels. We have called $H_l = -(\hbar^2/2m)\nabla^2 + V$ the one-atom term of the Hamiltonian, where V is the trapping potential. The field amplitudes $E_{L,R}$ are rescaled in order to be expressed in frequency units. They are thus the Rabi frequencies of the one-photon transition. $\hbar\omega_2$ is the energy of the upper state of the one-photon transition; $\hbar\omega_{0,1}$ are the energies of the $m_F = -1,1$ levels, respectively. We set $\omega_0 = 0$. In the adiabatic approximation, ψ_2 can be expressed in terms of ψ_1 and ψ_0 as $\psi_2 = -[E_L^* e^{i(\vec{k}_L \cdot \vec{x} - \omega_L t)}\psi_0 + E_R^* e^{i(\vec{k}_R \cdot \vec{x} - \omega_R t)}\psi_1]/\Delta$, where $\Delta = \omega_2 - \omega_L$. Thus, we have two closed equations for ψ_0 and ψ_1.

We introduce the gauge transformation

$$\tilde{\psi}_{0,1} = \exp\left(-i\int \frac{|E_L|^2 + |E_R|^2}{\Delta}dt\right)$$
$$\times \exp\left[\pm i\left(\frac{\vec{k}_d}{2}\cdot\vec{x} - \frac{\omega_d}{2}t \pm \frac{\hbar \vec{k}_d^2}{8m}t\right)\right]\psi_{0,1}, \quad (3)$$

where $k_d = \vec{k}_L - \vec{k}_R$ and $\omega_d = \omega_L - \omega_R$. As a result, the equations of motions become

$$i\hbar\dot{\tilde{\psi}}_{0,1} = \left(H_{0,1} \mp \frac{\hbar\delta}{2}\right)\tilde{\psi}_{0,1} - \hbar\Omega\tilde{\psi}_{1,0} \pm \frac{i\hbar^2 \vec{k}_d \cdot \vec{\nabla}}{2m}\tilde{\psi}_{0,1}. \quad (4)$$

Here, $\Omega \equiv E_L E_R^*/\Delta$ is the two-photon Rabi frequency, taken for simplicity as time independent and real, and the frequency δ is given by $\delta = \omega_1 - \omega_d + (|E_L|^2 - |E_R|^2)/\Delta$. We assume $|E_L|^2 = |E_R|^2$ and $\omega_d = \omega_1$, hence $\delta = 0$.

If the number of atoms is sufficiently small, we can neglect the nonlinear terms. Furthermore, let us initially consider a spatially homogeneous condensate (no trap potential). As a consequence, Eqs. (4) reduce to two linear equations with constant coefficients, and the eigenvalue problem in the reciprocal space is ruled by two linear algebraic equations for the transformed fields $\phi_0(\vec{k})$ and $\phi_1(\vec{k})$. The momenta of the atoms in the two levels $m_F = -1$ and $m_F = 1$, are respectively, $\hbar(\vec{k} - \vec{k}_d/2)$ and $\hbar(\vec{k} + \vec{k}_d/2)$.

Solving the eigenvalue problem, we find the two dispersion curves $\hbar\omega(\vec{k}) = \hbar^2 k^2/(2m) \pm \{[\hbar^2 \vec{k}_d \cdot \vec{k}/(2m) + \hbar\delta/2]^2 + \hbar^2|\Omega|^2\}^{1/2}$, plotted in Fig. 1(a) for $\Omega = 200$ s^{-1} and $|\vec{k}_d| = 2\pi \times 4.5 \times 10^5$ m^{-1}. As shown in the figure, the electromagnetic coupling modifies the parabolic dispersion curves associated with the two hyperfine levels, lifting the degeneracy at their intersection point. The energy gap for $\delta = 0$ is $2\hbar|\Omega|$. By varying δ, one can rise or lower the energy separation between the two minima. For $\delta = 0$, the two minima have the same energy. Introducing the harmonic trap potential, the ground state has no longer a definite momentum. Furthermore, the two wells of the dispersion curve are equally populated by quantum tunneling [Fig. 1(b), solid line]. In this state, the atoms in the Zeeman levels have an opposite nonzero average momentum. When the number of atoms is sufficiently high, a symmetry breaking occurs be-
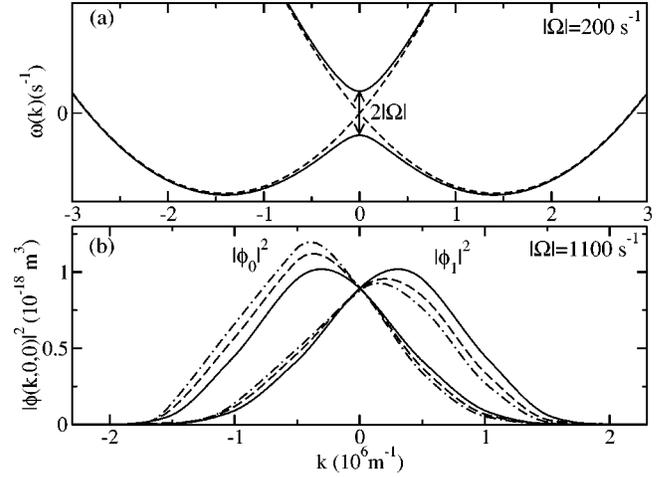

FIG. 1. (a) Dispersion curves of a free atom for $|\vec{k}_d| = 2\pi \times 4.5 \times 10^5$ m^{-1} and $\Omega = 200$ s^{-1}. (b) Density distributions $|\phi_{0,1}|^2$ for $k_d = 2\pi \times 4.5 \times 10^5$ m^{-1}, $\Omega = 1100$ s^{-1}, $\omega_\parallel = \omega_\perp = 2\pi \times 100$ s^{-1} and three different boson numbers: $N = 1390$ (solid), 1420 (dashed), 1460 (dot dashed).

cause of the atomic interactions and the population of the two levels becomes unbalanced. We report in Fig. 1(b) the distributions $|\phi_0|^2$ and $|\phi_1|^2$ for $\delta = 0$ and for three different values of the number of atoms N. $\phi_{0,1}$ are the Fourier transforms of the ground-state solution ($\tilde{\psi}_0, \tilde{\psi}_1$) of Eq. (4) for a spherical trap potential V corresponding to equal longitudinal ($\omega_\perp$) and radial ($\omega_\parallel$) trap frequencies. As it results, this interaction clusters the majority of atoms within a single well, thus contrasting the quantum tunneling across the barrier. Due to the geometry of the problem, and taking into account that the scattering lengths a_{ij} are practically equal, there is another ground state which is obtained from that of Fig. 1(b) by inverting the horizontal axis and interchanging ϕ_0 and ϕ_1. Thus we have two stable stationary states with equal energy. The numerical evidence of Fig. 1(b) is also supported by a synthetic variational argument, already exploited in Ref. [2] for Li, and based upon a suitable two-mode approximation.

The matter field fluctuations allow the passage from one stae to the other. If the decoherence is negligible, coherent oscillations between such states can be observed, demonstrating MQC. To evaluate the oscillation frequency, we quantize the two-mode system, as done in Ref. [2]. First of all, we write the classical Hamiltonian corresponding to the equations of motions (4), taking $g_{00} = g_{11} = g_{01} \equiv g$,

$$\mathcal{H} = \int \left[\psi_0^* H_l \psi_0 + \psi_1^* H_l \psi_1 + \frac{g}{2}(|\psi_0|^2 + |\psi_1|^2)^2 \right.$$
$$\left. + \frac{i\hbar^2 \vec{k}_d}{2m}(\psi_0^* \vec{\nabla}\psi_0 - \psi_1^* \vec{\nabla}\psi_1) - \hbar\Omega(\psi_1^*\psi_0 + \psi_0^*\psi_1) \right] d^3x$$
$$(5)$$

[from now on we omit the tilde on the ψ's, even though we are always in the gauge Eq. (3)]. We then introduce the spinorial ground states
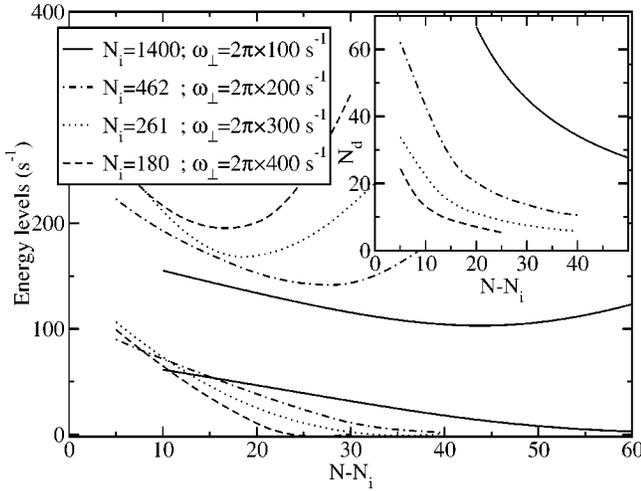
FIG. 2. First and second excited energy levels of the rubidium BEC versus $N - N_i$ (the excess of atoms above the breakup value N_i) for $\omega_\parallel = 2\pi \times 100$ s^{-1} and some values of $\omega_\perp$. In the inset, we plot N_d for the same values of the trap frequencies.

$$\vec{\psi}_{g,1}(\vec{x}) \equiv \begin{pmatrix} \psi_{0g}(\vec{x}) \\ \psi_{1g}(\vec{x}) \end{pmatrix}, \quad \vec{\psi}_{g,2}(\vec{x}) \equiv \begin{pmatrix} \psi_{1g}(-\vec{x}) \\ \psi_{0g}(-\vec{x}) \end{pmatrix}, \quad (6)$$

where ψ_{0g} and ψ_{1g} are the ground-state wave functions associated with the two internal states. $\vec{\psi}_{g,2}$ is obtained from $\vec{\psi}_{g,1}$ by interchanging the spinorial components and inverting the axes.

It is convenient to use the basis vectors $\vec{\psi}_a = \vec{\psi}_{g,1} + \vec{\psi}_{g,2}$ and $\vec{\psi}_b = \vec{\psi}_{g,1} - \vec{\psi}_{g,2}$. We write the quantized spinorial field of the bicondensate as $\vec{\psi}(\vec{x}) = \hat{a} \vec{\psi}_a(\vec{x}) + \hat{b} \vec{\psi}_b(\vec{x})$, and substitute this expression in the operator version of Eq. (5). $\hat{a}$ and $\hat{b}$ are annihilation operators. Thus, we obtain a reduced Hamiltonian $\mathcal{H}_{red}$ of the same form found in the attractive case of Ref. [2]. By $\mathcal{H}_{red}$, we evaluate the difference between the lowest eigenvalues, which provides the tunneling rate. In Fig. 2, we plot the first and second energy levels as a function of the number of atoms and for some values of $\omega_\perp$. The energy of the ground state is set to zero, so the energy of the first level gives the tunneling frequency. The entanglement between the condensate and the lost atoms induces a decoherence of the superposition. Using the approach of Ref. [2], we find that the coherence is given by $\tilde{C} = e^{-\epsilon M}$, where M is the number of lost atoms and $\epsilon = 1 - 2\int d^3 x \, \psi_0^*(\vec{x}) \psi_1(-\vec{x})/N$. The quantity $N_d = 1/\epsilon$ is the number of atoms which must be lost in order to reduce the coherence by $1/e$. The inset of Fig. 2 shows how N_d scales with N. The relevant loss processes are two-body inelastic and three-body collisional decays. In Fig. 3 we report the average three-body and two-body (inset) loss rates, the latter one refers to atoms in the $m = -1$ level. We have used the upper limit of 1.6×10^{-16} cm^3/s for the two-body loss rate coefficient and 5.8×10^{-30} cm^6/s for the three-body processes, both of them measured in Ref. [14] for the trapped Zeeman level $F = 1, m = -1$. Two-body decay can occur also by means of collisions between atoms in different Zeeman
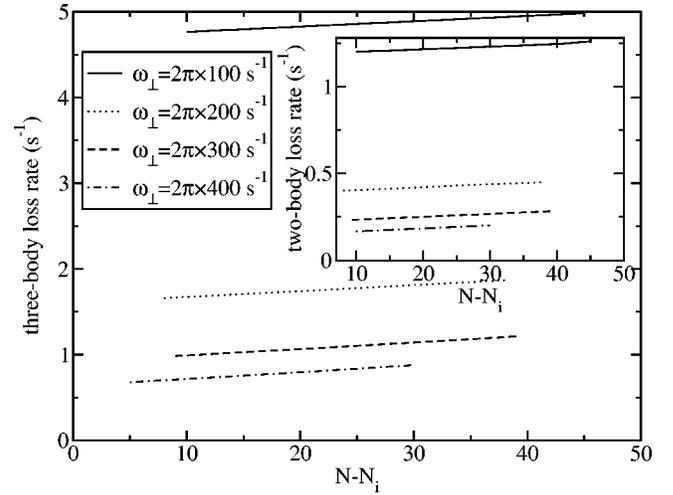

FIG. 3. Three-body and two-body (inset) loss rates. The latter one refers to atoms trapped in the $F = 1, m = -1$ level and represents an upper limit [14].

levels. We assume that the corresponding loss rate is of the same order of magnitude as the measured value. From Figs. 2 and 3 we find that the decoherence effects are negligible during a MQC oscillation period. For high $\omega_\perp$, the threshold N_i and the loss rates are lower. For $\omega_\perp = 2\pi \times 200$–$400$ s^{-1}, the overall loss rate is much smaller than the corresponding tunneling frequencies. Therefore, we can observe many oscillations before a single atom is lost. For $\omega_\perp = 2\pi \times 100$ s^{-1} the loss rate is > 5 s^{-1}. With a tunneling frequency of 8 Hz also in this case we can observe an oscillation before a single atom is lost. If the loss of atoms does not transfer energy to the trapped atoms, the escape of a few atoms does not reduce the superposition coherence (inset of Fig. 2), but modifies slightly the tunneling rate. In Ref. [15], it is shown that inelastic collisional processes induce local variations of the mean-field interparticle interaction and are accompanied by the creation or annihilation of elementary excitations. This phenomenon depends on the density and is completely negligible in our case. Notice that Eqs. (4) are invariant if N is varied by a factor α and the lengths and the energies are multiplied by α and α^{-2}, respectively. So for $\omega = \omega_\perp = 2\pi \times 70$ s^{-1} the threshold is $N_i = 1980$. By the two-mode model, we find that the tunneling frequencies are reduced by nearly a factor of 0.5. However, the two-body and three-body decay rates are reduced by the factors 0.25 and 0.125, respectively.

As we have shown in Ref. [2], the probability distribution $P(m)$ associated with the observable $\hat{M} = \hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger$ displays two peaks for the ground state, which are the alive and dead states of the Schrödinger cat. They have an unbalanced average number of atoms in the two levels. This observable is associated with the population measurement in one of the two Zeeman levels. To observe the coherence, we have to put the system in one of the two states, that corresponds to taking a superposition of the ground and first excited states of the condensate. This can be obtained by measuring with a nondemolition technique (e.g., a phase contrast technique) the population in a Zeeman level when only the ground state
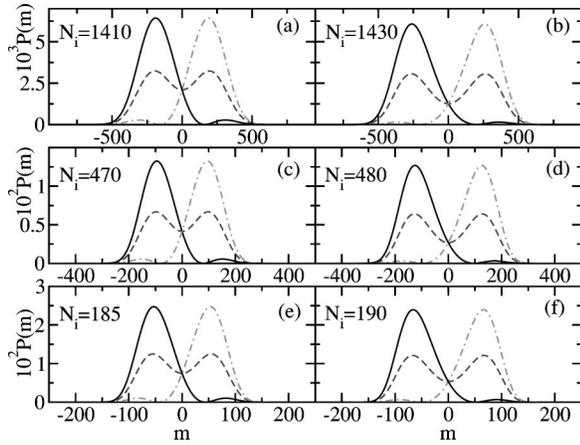
FIG. 4. Distribution $P(m)$ for some parameters. The solid, dashed, and dashed-dot lines correspond to $P(m)$ at the initial time, at a quarter of period and half a period, respectively. $\omega_\perp$ is (a), (b) $2\pi \times 100$ s^{-1}, (c), (d) $2\pi \times 200$ s^{-1}, and (e), (f) $2\pi \times 400$ s^{-1}.

is populated. This observation collapses the $P(m)$ distribution to one peak, as discussed in Ref. [2]. If the energy transfer is not too large, we expect that only the first two energy states are populated. With this initial preparation, the system begins to oscillate at the frequencies of Fig. 2 between the two Schrödinger cat states, as reported in Fig. 4. At the initial time, only one peak is present (solid line). At a quarter of period corresponding to the frequency separation between ground and first excited states, the $P(m)$ displays two peaks (dashed line). At half a period, the only peak is that absent at the initial time, thus there is a coherent oscillation between the two states. Detecting such an oscillation would provide evidence of a SC at an intermediate time when both peaks are present. It is important to point out that distributions with two oscillating peaks can be obtained also classically. For example, in a classical harmonic oscillator, a solution of the Liouville equation is the phase-space probability distribution $[\pi(1+a^2)]^{-1} e^{-(x^2+p^2)}[(x+a\cos\omega t)^2+(x+a\sin\omega t)^2]$. Integrating over p, this gives the distribution in x as $\pi^{-1/2}(1+a^2)^{-1}\exp(-x^2)(x^2+1/2+a^2+2ax\cos\omega t)$, which displays two oscillating peaks similar to the ones reported in of Fig. 4. However, in our case, the oscillation effect is more pronounced, since the distributions at the initial time and at half

a period display nearly a single peak, in contrast with the classical behavior. A sufficiently precise measurement of m and N and a sufficiently strong control of noise and temperature are necessary to extract distributions that produce evidence of quantum coherence. A precise measurement of N could be obtained by a phase contrast imaging technique. A method that may help to distinguish a quantum oscillation from a classical one consists in studying the effect of decoherence, since the quantum coherence is very sensitive to the environment coupling. Decoherence can be obtained by a phase contrast imaging of the condensate during its oscillations. The decoherence rate grows increasing the intensity of the probing beam.

In conclusion, we have shown that a double-well dispersion curve can be obtained by a suitable Raman coupling. In this situation, a symmetry breaking in momentum space is demonstrated solving two coupled Gross-Pitaevskii equations. The condensate can oscillate between the two emerging asymmetrical steady solutions (SC states) by means of the field quantum fluctuations (MQC). We have found that it is possible to obtain an oscillation frequency between the SC states around 50–100 s^{-1} and that the decoherence due to the atomic losses is negligible, at variance with the previous schemes. In order to neglect the thermal activation, the second excited level energy (E_2) has to be higher than the thermal energy. From Fig. 2 we can see that E_2 ranges between 100 and 300 s^{-1}, which correspond to a temperature of 0.7–2.3 nK. If the cooling is performed below the threshold, when the symmetry breaking does not occur, the required temperature can be ~ 5 nK. However, it may not be necessary to cool at very low temperatures the whole condensate, but just the involved degrees of freedom, provided that these latter ones are weakly coupled with the other modes, which act as a thermal bath. We remark that for low densities, the evaporative cooling allows to reach much lower temperatures, because of a smaller three-body decay rate. To be sure that no excitation is present, one could tailor the trapping potential in such a way that only the first two levels are bound. In this work, we have chosen the parameters for which the tunneling frequency is much larger than the decoherence rate, however, the symmetry breaking and the super-Poissonian atom fluctuations below threshold can be observed with a much higher number of atoms, thus these phenomena are observable with the present technology.

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