Scissors mode of an expanding Bose-Einstein condensate

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We study the scissors mode of a free-expanding Bose-Einstein condensate. We find that the irrotational character of the superfluid flow within the condensate has dramatic consequences on the evolution of the scissors oscillation. After a short expansion the oscillation is amplified and distorted, while asymptotically it recovers its initial sinusoidal behavior, with an enhanced amplitude. We investigate this phenomenon by using a technique to excite the scissors mode in a condensate held in a magnetostatic potential.

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The close connection between Bose-Einstein condensation in dilute atomic gases and superfluidity has been shown by recent research. The most striking signatures of superfluidity observed are quantized vortices [1], reduction of dissipative phenomena [2,3], scissor modes [4,5], and irrotational flow [6,7].

In particular, the observation in a Bose-Einstein condensate (BEC) of the scissors mode, which is a shape-preserving angle oscillation [4], has provided the connection between superfluidity in atomic gases and in nuclei [8]. The phenomenon was studied theoretically for a BEC confined in a magnetic potential [4], while the experiment reported no changes of the oscillation during the ballistic expansion [5]. On the other hand, recent studies of the expansion of a BEC initially confined in a rotating trap [6,7], have shown that a rotating condensate expands in a distinctively different way with respect to a nonrotating one, due to the quenched moment of inertia of the superfluid.

In this paper we show that these two phenomena are actually connected, since a scissors oscillation is a time-dependent rotation of the condensate. In our analysis we find that superfluidity affects in a peculiar way the evolution of the scissors mode of a free-expanding BEC. Because of the changing moment of inertia, a scissors mode initially excited in the trap is significantly perturbed during the first phase of the expansion, but it remarkably recovers the sinusoidal behavior at longer expansion times. Our predictions are confirmed by an experiment with an elongated BEC held in a magnetostatic potential. For this experiment we develop a technique, which also allows us to investigate the expansion of a rotating superfluid in a regime where the implications of the irrotationality are the most dramatic.

The wave function of a condensate at zero temperature can be conveniently written in terms of the density ρ and the phase S,

$$\psi(\mathbf{x},t)=\sqrt{\rho(\mathbf{x},t)}e^{iS(\mathbf{x},t)/\hbar}. \tag{1}$$

In the Thomas-Fermi approximation the Gross-Pitaevskii equation for ψ can be transformed in two coupled hydrodynamic equations for the density ρ and the velocity $\mathbf{v}=\boldsymbol{\nabla}S/m$ [9,10],

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}(\rho\mathbf{v})=0, \tag{2}$$

$$m\frac{\partial\mathbf{v}}{\partial t}+\boldsymbol{\nabla}\left(\frac{\mathbf{v}^2}{2m}+U(\mathbf{x},t)+g\rho\right)=\mathbf{0}, \tag{3}$$

where U is the trapping potential, $g=4\pi\hbar^2 a/m$ is the coupling strength, m is the atomic mass, and a is the interatomic scattering length.

Here we are interested in the solution of Eqs. (2) and (3) which corresponds to a scissors mode. We consider a harmonic potential of the form

$$U(\mathbf{x},t)\equiv\frac{1}{2}m\omega_{ho}^2\sum_{ij=1}^{3}x_iW_{ij}(t)x_j, \tag{4}$$

where $W=W^T$ for symmetry reasons, with initial conditions $W_{ij}(t<0)=\omega_i^2\delta_{ij}/\omega_{ho}^2$ and $\omega_{ho}\equiv(\omega_x\omega_y\omega_z)^{1/3}$. By tilting the trap potential by a small angle θ_{0i} around the z axis at $t=0$, one can excite a scissors mode in the x-y plane, that oscillates about the rotated trap eigenaxes by the angle

$$\theta_0(t)=\theta_{0i}\cos(\omega_{sc}t), \tag{5}$$

with frequency $\omega_{sc}=(\omega_x^2+\omega_y^2)^{1/2}$ [4]. In this case Eqs. (2) and (3) can be solved exactly with a quadratic ansatz for the condensate density and phase [10,11]

$$\rho(\mathbf{x},t)=\frac{m\omega_{ho}^2}{g}\left(\rho_0(t)-\frac{1}{2}\sum_{i,j=1}^{3}x_iA_{ij}(t)x_j\right), \tag{6}$$

$$S(\mathbf{x},t)=m\omega_{ho}\left(s_0(t)+\frac{1}{2}\sum_{i,j=1}^{3}x_iB_{ij}(t)x_j\right). \tag{7}$$

Therefore, in general the system can be described by ten time-dependent dimensionless parameters: ρ_0, S_0, A_{ij}, and B_{ij} [A and B are 3×3 symmetric matrices, reflecting the symmetry property of U, with $A_{k3}=0=B_{k3}$ due to the vanishing of $W_{k3}(t)$, $k=1,2$]. These parameters obey the following set of first-order differential equations [11]

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$$\frac{d\rho_0}{d\tau} = \rho_0 \mathrm{Tr}B, \quad \frac{dS_0}{d\tau} = \rho_0, \tag{8}$$

$$\frac{dA}{d\tau} = 2A\mathrm{Tr}B + \{A,B\}, \tag{9}$$

$$\frac{dB}{d\tau} = W - A - B^2, \tag{10}$$

with boundary conditions determined by the equilibrium distribution of the condensate at $\tau \equiv \omega_{ho} t = 0$. Notice that these equations depend on the number of atoms N and on the scattering length a only through the initial value of $\rho_0(0) = 0.5 a_{ho}^2 (15Na/a_{ho})^{2/5}$, $a_{ho} = \sqrt{\hbar/m\omega_{ho}}$ being the harmonic oscillator characteristic length.

The subsequent angular motion is evaluated in terms of the rotation which diagonalizes the quadratic part of the density $\rho(\mathbf{x},t)$, (that is the matrix A_{ij}). The angle of rotation is fixed by the relation [6]

$$\tan(2\theta_0) = \frac{2A_{12}}{A_{11} - A_{22}}. \tag{11}$$

The same Eqs. (8)–(10) can be used to study the expansion of the condensate after the release from the trap ($W \equiv 0$), with the appropriate initial conditions.

Experimentally we study the scissors mode of a BEC of ^{87}Rb atoms in an elongated potential created by a QUIC magnetostatic trap [12]. The potential has a cylindrical symmetry around the x axis, with frequencies $\omega_x = 2\pi \times 16.3$ Hz and $\omega_y = \omega_z = 2\pi \times 200$ Hz, with the y axis aligned along the direction of gravity. The BEC is typically composed by 10^5 atoms. In all the measurements we report no thermal component was detectable.

Since the trap is static, the scissors mode cannot be excited with the technique demonstrated for time-orbiting-potential traps [5]. However, we can take advantage of the fact that gravity breaks the symmetry of the magnetic potential resulting in anharmonicity even at the trap minimum. Actually, we found that the eigenaxes of our magnetic trap are rotated in the x-y plane in a localized region around the minimum. The rapid motion of the condensate through such deformation is therefore equivalent to a sudden rotation of the trap, like that considered in Eq. (5). In fact, when we excite a dipolar oscillation along the weak x axis by a sudden displacement of the trap minimum, a scissors mode appears as soon as the BEC travels for the first time through the minimum. Since for an elongated BEC the scissors mode proceeds essentially at $\omega_y \gg \omega_x$, it is possible to study several scissor oscillations before the BEC travels again through the center of the trap. By choosing an appropriately small dipolar amplitude, we observe a "pure" scissors mode with amplitude $\theta_{0i} \simeq 15$ mrad, without any apparent shape deformation.

To study such mode, we need first to recall the peculiar behavior of a rotating BEC during the expansion after release from the trap [6]. In fact, due to the irrotational nature of a superfluid ($\nabla \times \mathbf{v} = 0$) the moment of inertia Θ of a condensate is quenched with respect to the rigid body value Θ_{rig} [13],

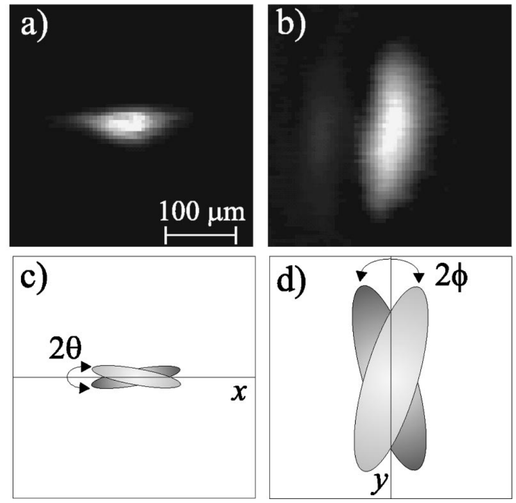

FIG. 1. Absorption images of a rotating, elongated Rb condensate during the ballistic expansion. In (a) the BEC has just been released from the trap ($t_{exp} = 2$ ms), and in (b) it has expanded for 23 ms. The other two pictures, not to scale, show the corresponding evolution of a scissors mode. The initial oscillation about the horizontal x axis (c) is transformed in an oscillation about the vertical y axis for long expansion times (d).

$$\Theta = \frac{\langle x^2 - y^2 \rangle^2}{\langle x^2 + y^2 \rangle^2} \Theta_{rig}. \tag{12}$$

Therefore, due to conservation of energy and angular momentum, a rotating condensate cannot reach a symmetric configuration during the expansion, and undergoes a rapid rotation causing the inversion of the "long" and "short" axes. In particular, if at release the condensate is rotating in the x-y plane, with its long axis forming a small angle θ_0 with the x axis, as shown in Fig. 1, initially it will start expanding in the short direction. Then, when the aspect ratio approaches unity, after a fast rotation, the condensate will continue to expand in the other direction (the "long" one) [4], see Fig. 2.

We have studied the evolution of the rotation angle θ of the condensate as a function of the expansion time t_{exp}, as shown in Fig. 2(a). The measurements are performed by releasing the condensate after a fixed evolution time t of the scissor oscillations in the trap. For comparison we also plot the angle evolution of the thermal cloud just above condensation, which has been put under rotation with the same technique described above. Note that the rotation of the thermal cloud can be experimentally investigated only until the cloud becomes spherical at $t_{exp} \simeq 1/\omega_x$. Initially, the expansion of the BEC and the thermal cloud is almost indistinguishable, being characterized by the angular velocity Ω_0 at release from the trap. This is due to the fact that the moment of inertia of an elongated condensate is close to the rigid body value [see Eq. (12)]. The quenching becomes important as soon as the aspect ratio approaches unity, where the nonclas-

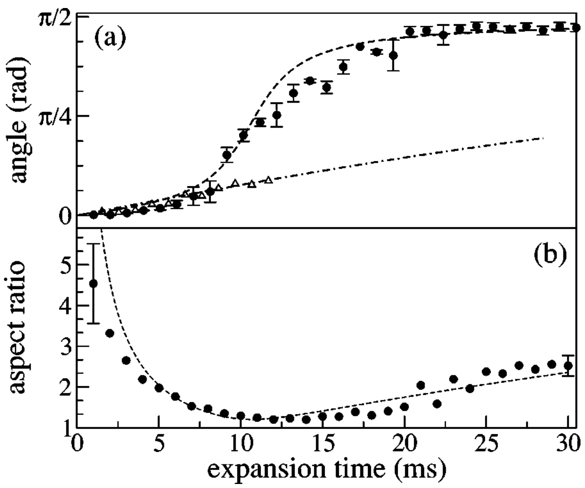

FIG. 2. (a) Evolution of the rotation angle for a condensate (circles) and a thermal cloud (triangles) released with small angular velocity from an elongated trap. (b) Evolution of the aspect ratio of the rotating condensate. When the aspect ratio gets close to unity the angular velocity shows a rapid increase. The lines represent the theoretical predictions.

sical behavior of the condensate is evident, as predicted in Ref. [6]. Indeed, its rotation undergoes a fast acceleration [Fig. 2(b)], followed by a slow evolution towards an asymptotic angle, close but smaller than $\pi/2$. The agreement of experiment and theory is quite good. In contrast, the rotation angle of the thermal cloud continues to follow the predicted behavior [6]

$$\theta(t;t_{exp}) = \theta_0(t) + \arctan[\Omega_0(t)t_{exp}], \tag{13}$$

as shown in Fig. 2(a). This figure shows that the use of very elongated condensates allows for the investigation of novel regimes where the superfluid expansion is dramatically different from the classical one.

By analyzing the expansion of the condensate for various conditions of initial angle and angular velocity at release from the trap, it is possible to reconstruct the evolution of a scissors oscillation for the expanding BEC. As an example, in Fig. 3 we show the calculated evolution of the angle θ during the time of flight, for conditions corresponding to very small angular velocity and maximal angular velocity of the trapped oscillations. The behavior of θ in all these regimes is qualitatively similar to that shown in Fig. 2. However, in the situations of small initial angular velocity the transient rotation is even faster than in other cases, while the asymptotic angle is closer to $\pi/2$. By studying the solutions of Eqs. (8)–(10), we can identify three different regimes for the expanding scissors mode, depending on the expansion time t_{exp}. (i) For $t_{exp} < 1/\omega_x$ the condensate expands in a way similar to a classical gas, and the sinusoidal behavior of the scissors mode is preserved, as is possible to see from Eq. (13). (ii) When $t_{exp} \simeq 1/\omega_x$ the rotation angle is always close to $\pm\pi/4$, depending on the sign of the angular velocity at release. The angle oscillation is therefore close to a square wave with the same phase of the angular velocity in the trap $\Omega_0(t)$. (iii) For $t_{exp} > 1/\omega_x$ the angle of the condensate approaches $\pi/2$, and it is more convenient to study the oscillations of the angle ϕ from vertical. We find that the original sinusoidal behavior of the oscillations is restored, with a time dependence which can be fitted by the form

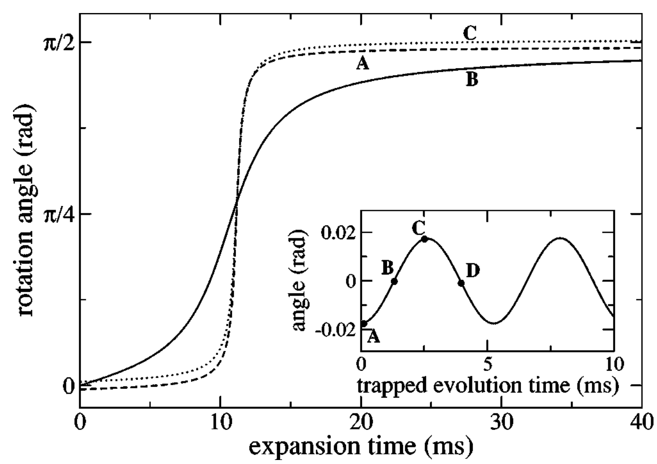

FIG. 3. Calculated rotation angle of the condensate during the expansion, released at different times during the scissors oscillation in the trap, as shown in the inset. Cases *A* and *C* correspond to small angular velocities; *B* and *D* to the maximum velocity. The evolution of *D* is the opposite of *B* (negative angles).

$$\begin{aligned}\phi(t;t_{exp}) &= \theta_0(t) + F(t_{exp})\Omega_0(t)\\ &= \theta_{0i}\sqrt{1+\omega_{sc}^2F^2(t_{exp})}\cos(\omega_{sc}t+\varphi),\end{aligned} \tag{14}$$

where F is a nontrivial function of the expansion time and of the trap geometry, and φ is a phase shift. This behavior is remarkable, since in passing over the "critical" time region at $t_{exp} \simeq 1/\omega_x$, the condensate does not lose memory of its initial angular velocity, and it starts to behave again as a classical gas.

We have verified these expectations in the experiment. Qualitatively, we observe that the scissors oscillation has a sinusoidal behavior just after the release of the condensate from the trap, it gets closer to square wave at $t_{exp} \approx 12$ ms, and then turns back to a sinusoid for longer expansion times. For a quantitative analysis, since the finite resolution of the imaging system prevents us from studying the condensate inside the trap, we probe the oscillation of the condensate after a minimum expansion $t_{exp} = 4$ ms, as shown in Fig. 4(a), and we reconstruct the scissors mode in the trap. In Fig. 4(b) we compare such a motion to the experimental scissors mode after a long expansion of 23 ms. The initial scissors mode is clearly amplified and phase shifted after the expansion. For the experimental parameters we calculate indeed that $\omega_{sc}F(t_{exp} = 23\ ms) \simeq 10.7$ is much larger than one, and therefore the time evolution of the scissors mode is substantially a replica of that of the angular velocity in the trap. We have observed a similar behavior also for collision-induced scissor oscillations of a binary BEC system [14].

To study the role played by the trap geometry in the amplification mechanism, we have calculated the ratio between the final amplitude and the trapped one as a function of the trap anisotropy ω_x/ω_y, for various expansion times. In Fig.

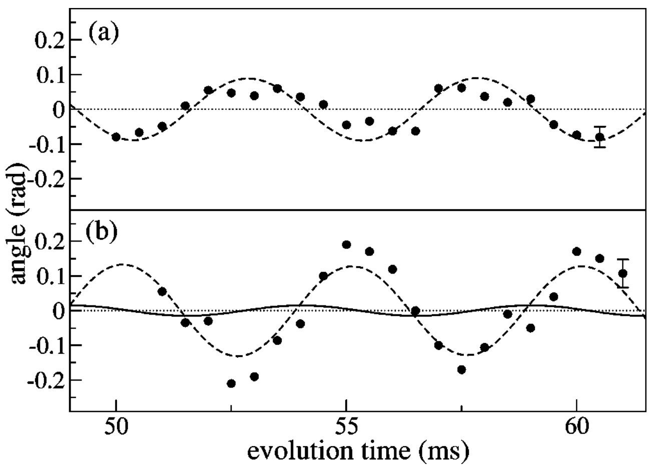

FIG. 4. Evolution of the scissors mode during the ballistic expansion. In (a) the angle $\theta(t)$ after a short expansion of 4 ms (circles) is compared to theory (dashed line) to extrapolate the oscillation in the trap. In (b) we compare the latter (continuous line) to the oscillation $\phi(t)$ after a longer expansion of 23 ms (circles). The data show an enhancement of the scissors mode, as expected from theory (dashed line).

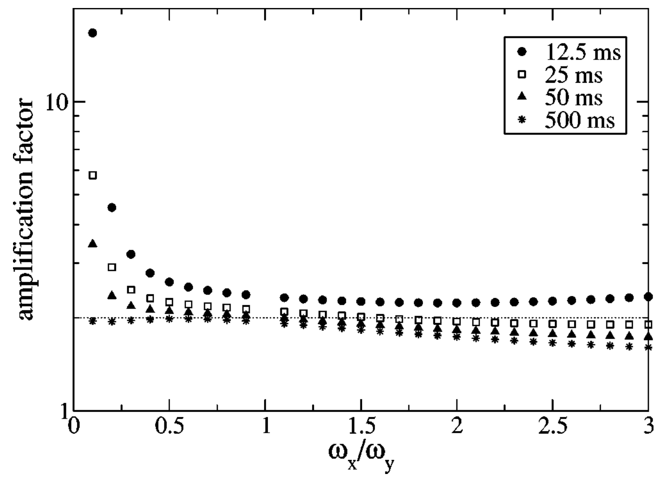

FIG. 5. Calculated amplification factor $\sqrt{1+\omega_{sc}^2 F^2}$ of a scissors mode as a function of ω_x/ω_y, for various expansion times: $t_{exp}=25,50,100,500$ ms. Here the scissor frequency is kept fixed to $\omega_{sc}=2\pi\times 200.7$ Hz and the radial and axial frequencies are varied accordingly.

5 we show the results, obtained by keeping fixed the scissor frequency to the value $2\pi\times 200.7$ Hz, and varying the radial and axial frequencies accordingly. The behavior shown is independent of the initial angle θ_{0i}, in the small-angle regime. This picture confirms that in cigar-shaped traps, for typical expansion times ($t_{exp}\simeq 10$–30 ms), the scissors mode amplitude is substantially amplified by the expansion. It also shows that the amplification factor asymptotically tends to a value ~ 2, regardless the trap geometry. It is worth noticing that this asymptotic behavior is similar to that of a 2D condensate in the limit $\omega_x/\omega_y=1$, where relation (14) can be demonstrated analytically [15]. Notice that in contrast, Eq. (13) would not imply any amplification of the asymptotic oscillations of the thermal cloud.

In general, the scissors modes are coupled to compressional modes, since a rotation of the condensate can induce shape deformation for large enough rotation angles [4]. In our trap, we can increase the angular velocity of the BEC by increasing the amplitude of the dipolar oscillation, accessing the nonlinear regime, where the scissors mode couples to quadrupole shape oscillations.

In conclusion, we have shown that after an intermediate phase, in which the oscillation is strongly perturbed, asymptotically the scissors mode of an expanding BEC recovers its initial sinusoidal behavior. The final amplitude is enhanced, with an amplification factor which depends on both the initial amplitude and the angular velocity. The demonstrated capability of exciting scissor oscillations in elongated condensates opens the possibility of studying rotating superfluids in a different regime of large moments of inertia and small angular velocities.

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