Entangling two Bose-Einstein condensates by stimulated Bragg scattering

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(Received 7 October 2002; published 10 February 2003)

We propose an experiment for entangling two spatially separated Bose-Einstein condensates by Bragg scattering of light. When Bragg scattering in two condensates is stimulated by a common probe, the resulting quasiparticles or particles in the two condensates get entangled due to quantum communication between the condensates via the probe beam. The entanglement is shown to be significant and occurs in both number and quadrature phase variables and depends strongly on the relative detuning of the two pumps and the relative atom-field coupling strengths of the two condensates. We present two methods of detecting the generated entanglement.

DOI: 10.1103/PhysRevA.67.023603 PACS number(s): 03.65.Ud, 42.50.Dv

Apparently puzzling, yet most profound, first formulated as a paradox [1], quantum entanglement lies at the very heart of quantum information processing and many issues in the foundations of quantum mechanics. Generation and manipulation of entanglement is, therefore, of prime interest. Bose-Einstein condensates (BEC) [2] of weakly interacting atomic gases seem to be suitable macroscopic objects for producing many-particle entanglement [3]. A BEC has intrinsic entanglement character due to reduced quantum fluctuations in momentum space. For instance, in the condensate ground state, a pair of mutually opposite momentum modes is maximally entangled in atomic number variables [4].

Stimulated resonant Bragg scattering of light by a condensate generates quasiparticles [5], predominantly in two momentum side modes $\mathbf{q}$ and $-\mathbf{q}$, where $\mathbf{q}$ is the momentum transferred from light fields to the atoms. Bragg spectroscopy [6] with coherent or classical light produces coherent states of the quasiparticles in a BEC. When these quasiparticles are projected into a particle domain, that is, into the Bogoliubov-transformed momentum modes, they form two-mode squeezed as well as an entangled state [4]. Bragg spectroscopy can also generate tripartite entanglement [4]. Rayleigh scattering of light by a condensate under certian conditions can produce super-radiance [7,8] and entangled atom-photon pairs [8]. Spin degrees of freedom of a spinor BEC [9] are also useful in describing entanglement in spin variables. Apart from BECs, multiatom entanglement in other macroscopic systems has been realized [10] on the basis of collective-spin squeezing [11,12]. Furthermore, the entanglement in collective-spin variables of two ensembles of gaseous Cs atoms has been experimentally demonstrated [13]. Continuous variables such as the quadratures of a field mode (which are analogous to position and momentum) have also been employed [14] in entanglement studies.

We here propose a scheme for producing quantum entanglement between two spatially separated BECs of a weakly interacting atomic gas. The entanglement we consider is in momentum modes of BECs. The schematics of the proposed experiment is shown in Fig. 1. The condensates A and B are illuminated by pump lasers $L1$ and $L2$, respectively. A single stimulating probe field acts on both the condensates. This common probe can be a single-mode field of a ring cavity. All these three fields are detuned far off the resonance of an electronic excited state of the atoms. The frequencies and the directions of propagation of these fields are so chosen such that Bragg-resonance (phase matching) conditions of scattering in both the condensates are fulfilled. The Hamiltonian of the system is $H=H_A+H_B+H_F+H_{AF}+H_{BF}$. Retaining the dominant momentum side modes $\mathbf{q}$ and $-\mathbf{q}$ only under Bragg-resonance condition, in the Bogoliubov approximation, $H_A=\hbar\omega^B_{q_1}(\hat{\alpha}^\dagger_{\mathbf{q}_1}\hat{\alpha}_{\mathbf{q}_1}+\hat{\alpha}^\dagger_{-\mathbf{q}_1}\hat{\alpha}_{-\mathbf{q}_1})$, where $\hat{\alpha}_{\mathbf{q}_1}$ represents quasiparticle with momentum $\mathbf{q}_1$, and $\omega^B_{q_1}=[(\omega_{q_1}+\mu/\hbar)^2-(\mu/\hbar)^2]^{1/2}$ is the frequency of Bogoliubov's quasiparticle. Here, $\omega_{q_1}=\hbar^2 q_1^2/2m$, and $\mu=\hbar^2\xi^{-2}/2m$ is the chemical potential with $\xi=(8\pi n_0 a_s)^{-1/2}$ being the healing length. Similarly, $H_B=\hbar\omega^B_{q_2}(\hat{\beta}^\dagger_{\mathbf{q}_2}\hat{\beta}_{\mathbf{q}_2}+\hat{\beta}^\dagger_{-\mathbf{q}_2}\hat{\beta}_{-\mathbf{q}_2})$, with $\hat{\beta}_{\mathbf{q}_2}$ being the quasiparticle operator of the condensate B. The pumps are treated classically. Let $\hat{c}$ represent the common probe field mode, then the field Hamiltonian $H_F=\hbar\omega_3\hat{c}^\dagger\hat{c}$, where ω_3 is the frequency of the probe. The quasiparticle operators $\hat{\alpha}(\hat{\beta})$ are related to the particle operators $\hat{a}(\hat{b})$ by Bogoliubov transformation : $\hat{a}_{\mathbf{q}}=u_q\hat{\alpha}_{\mathbf{q}}-v_q\hat{\alpha}^\dagger_{-\mathbf{q}}$, where $v_q=(u_q^2-1)^{1/2}=[\frac{1}{2}((\omega_q+\mu/\hbar)/\omega^B_q-1)]^{1/2}$. The atom-field interaction Hamiltonian for condensate A is

$$H_{AF}=\exp(-i\omega_1 t)\hbar\eta_A\hat{c}^\dagger(\hat{\alpha}^\dagger_{\mathbf{q}_1}+\hat{\alpha}_{-\mathbf{q}_1})+\text{H.c.},\qquad(1)$$

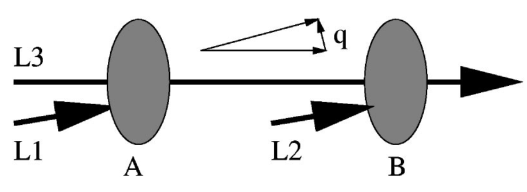

FIG. 1. The scheme for creation of entanglement. A and B are two condensates, $L1$ and $L2$ are pump lasers, and $L3$ is a common entangling probe field. The wave vectors of L1 and L2 are $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively; probe's wave vector is $\mathbf{k}_3$. The fields are in the Bragg resonance with the momentum mode $\mathbf{q}_{1(2)}=\pm(\mathbf{k}_{1(2)}-\mathbf{k}_3)$ of the condensate $A(B)$ for blue(+) and red(-) detuning of the respective pump from the probe.

where ω_1 is the frequency of the $L1$ pump and $\eta_A = \sqrt{N_A}\Omega_A \int d^3\mathbf{r} \exp(i\mathbf{q}_1\cdot\mathbf{r})|\Psi_0^A(\mathbf{r})|^2 f_{q_1}$ is the effective atom-field coupling constant. Here, N_A is the number of atoms in condensate A, Ω_A is the two-photon Rabi frequency of an atom in A, $\Psi_0^A(\mathbf{r})$ is the ground state of the condensate A, and $f_{q_1}=u_{q_1}-v_{q_1}$. H_{BF} is given by the similar expression as H_{AF} with α replaced by β and the subscripts A and 1 replaced by B and 2, respectively. $\eta_B = \exp(ik_3X_{\rm AB})\sqrt{N_B}\Omega_B \int d^3\mathbf{r} \exp(i\mathbf{q}_2\cdot\mathbf{r})|\Psi_0^B(\mathbf{r})|^2 f_{q_2}$, where $X_{\rm AB}$ is the separation between the two condensates. Here, the dependency of the effective coupling constants on the total number of atoms $N_{A(B)}$ can be noted. This holds good if the zero-momentum state (condensate) remains macroscopically occupied during the dynamical evolution meaning that the dynamical depletion of this state is negligible. This is physically more appropriate if we work below the super-radiance threshold [7]. The Heisenberg equations of motion are

$$\dot{\hat{\alpha}}_{\mathbf{q}_1} = -i\omega^B_{q_1}\hat{\alpha}_{\mathbf{q}_1} - i\eta_A \hat{c}^\dagger, \tag{2}$$

$$\dot{\hat{\alpha}}^\dagger_{-\mathbf{q}_1} = i\omega^B_{q_1}\hat{\alpha}^\dagger_{-\mathbf{q}_1} + i\eta_A \hat{c}^\dagger; \tag{3}$$

$$\dot{\hat{c}}^\dagger = -i\delta_1 \hat{c}^\dagger + i[\eta_A^*(\hat{\alpha}_{\mathbf{q}_1} + \hat{\alpha}^\dagger_{-\mathbf{q}_1}) + \eta_B^*(\hat{\beta}_{\mathbf{q}_2} + \hat{\beta}^\dagger_{-\mathbf{q}_2})]; \tag{4}$$

$$\dot{\hat{\beta}}_{\mathbf{q}_2} = -i(\omega^B_{q_2} + \delta_1 - \delta_2)\hat{\beta}_{\mathbf{q}_2} - i\eta_B \hat{c}^\dagger, \tag{5}$$

$$\dot{\hat{\beta}}^\dagger_{-\mathbf{q}_2} = i(\omega^B_{q_2} - \delta_1 + \delta_2)\hat{\beta}^\dagger_{-\mathbf{q}_2} + i\eta_B \hat{c}^\dagger; \tag{6}$$

where $\delta_{1(2)} = \omega_{1(2)} - \omega_3$. The operators have been redefined to eliminate fast time dependence. The above five equations can be written in a matrix form $\dot{\mathbf{X}} = -i\mathbf{M}\mathbf{X}$. In the absence of any dissipation or for a time short enough compared to all damping times, the dynamics of the system would be mainly controlled by the eigenvalues of the $\mathbf{M}$ matrix. Let Γ_2 denote the two-photon resonance linewidth and Γ_c the cavity linewidth. We choose $\omega^B_{q_{1,2}} \gg \Gamma_2$ and $\eta_{A,B} \gg \Gamma_c$ so that we can work in a reversible dynamical regime.

We next discuss how to quantify entanglement between two BECs. If the entanglement occurs in number operators of the quasiparticle modes 1 and 2, then it can be quantified by the parameter [11,4]

$$\xi_n(1,2) = \langle[\Delta(\hat{n}_1 - \hat{n}_2)]^2\rangle/(\langle\hat{n}_1\rangle + \langle\hat{n}_2\rangle). \tag{7}$$

If $\xi_n < 1$, then the two modes are entangled. If the entanglement is described by two noncommuting Gaussian operators $\hat{X}$ and $\hat{P}$ which are analogous to position and momentum variables, then the entanglement parameter is defined by [15]

$$\xi_p(1,2) = \tfrac{1}{2}[\langle[\Delta(X_1 + X_2)]^2\rangle + \langle[\Delta(P_1 - P_2)]^2\rangle]. \tag{8}$$

The two modes are entangled in quadrature phase, when $\xi_p < 1$.

For numerical illustration, we consider two homogeneous identical Na condensates. We consider two cases; case (I) both the pumps are blue detuned from the common probe. In this case, pump photons are predominantly scattered into the probe mode at both the condensate sites. In case (II), first pump ($L1$) is blue detuned and the other ($L2$) is red detuned from the probe. In this case, at site A, pump photons are predominantly scattered into the probe mode while at the other site B, probe photons are scattered into the pump mode. We here enlist the important results: (1) In case (I), if the effective coupling (η) of $B(A)$ is stronger than that of $A(B)$, then entanglement arises between $\mathbf{q}_1(-\mathbf{q}_1)$ of A and $-\mathbf{q}_2(\mathbf{q}_2)$ of B only, other pairs of modes are immune to any entanglement. (2) In case II, if $\eta_B > \eta_A$, the modes $\mathbf{q}_1$ of A and $-\mathbf{q}_2$ of B are involved in entanglement. In Fig. 2, we display entanglement parameters between these two chosen modes as a function of time for case (I) [Fig. 2(a)] and case (II) [Fig. 2(b)]. A comparison between Fig. 2(a) and 2(b) shows that the entanglement in case (II) can be very large compared to that in case (I), particularly in free-particle regime ($q \gg \xi^{-1}$). Figure 2(a) exhibits that the entanglement parameter at long time diverges hyperbolically, while it oscillates all the time in Fig. 2(b). The reasons for this contrast are that in the case of Fig. 2(a), two of the five eigenvalues of $\mathbf{M}$ matrix are complex, while in the case of Fig. 2(b) all the eigenvalues are real. Far below the free-particle limit, we

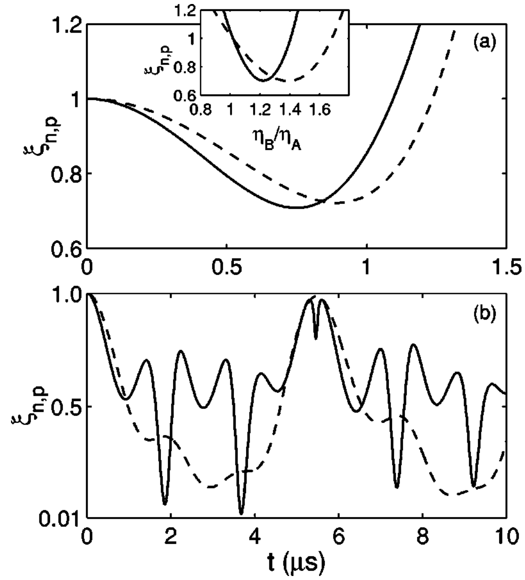

FIG. 2. (a) Entanglement parameters ξ_n (solid) and ξ_p (dashed) between **q** mode of A and **-q** mode of B as a function of time in microsecond when both the pumps are blue detuned by the same amount from the probe. For both the condensates, $q = 2\xi^{-1}$, $\omega_q^B = 0.21$ MHz, and $\delta_1 = \delta_2 = 0.17$ MHz. The coupling constants $\eta_A = 1.62$ MHz, $\eta_B = 1.25\eta_A$. Both the condensates are initially in the ground states and the common probe field is in coherent state with average number of photons $n_p = 10$. The inset to (a) shows entanglement parameters ξ_n (solid) and ξ_p (dashed) as a function of η_B/η_A for a fixed time $t = 0.75$ μs. The other parameters are the same as in (a). (b) Same as in (a) but entanglement is now between the particles rather than quasiparticles and with first pump detuned to the blue and the second one detuned to the red of the common probe, $q = 8.33\xi^{-1}$, $\omega_q^B = 2.96$ MHz, $\delta_1 = -\delta_2 = 2.92$ MHz, $\eta_A = 2.22$ MHz, $\eta_B = 1.25\eta_A$, and $n_p = 100$.

find that the entanglement in both the cases is not so large. In case (I), the entanglement diminishes in the free-particle regime. (3) For equal couplings, there is no entanglement between any pair of modes in any case. Inset to Fig. 2(a) shows the variation of entanglement parameters as a function of the ratio of the two coupling constants at a fixed time in case (I). The coupling parameters can be made different either by using pump lights of different intensities or by taking different atom numbers for the two, otherwise identical condensates. (4) We find entanglement both in quasiparticle ($\hat{\alpha}$, $\hat{\beta}$), and particle or atomic modes ($\hat{a}$, $\hat{b}$). It is worth mentioning that in a single condensate, as shown in Ref. [4], coherent light scattering can generate entanglement only in atomic modes, and not in quasiparticle modes.

The light scattering events occurring at A and B are not independent, since a quantum communication has been set between the generated quasiparticles in A and B via the common probe. When the second condensate interacts with the probe, the probe has back action on the first condensate. Had we treated the common *probe classically*, then the Hamiltonian [Eq. (1)] would have been linear in the atomic operators. A Hamiltonian linear in bosonic opeartors cannot generate nonclassical correlation. Therefore, the *probe* must be treated *quantum mechanically*. Light scattering by either condensate generates entanglement (reduced quantum fluctuation) between the scattered atom and the emitted (scattered) photon. This entanglement can only be imprinted into an atom in the other condensate when the photon (carrier of entanglement) emitted by the former condensate is absorbed by the latter condensate. In both the cases (I) and (II), a quasiparticle in $-\mathbf{q}$ is likely to be produced when the probe photon is absorbed and then emitted into the pump mode. In case (I), $-\mathbf{q}$ is the off-resonant mode in both the condensates while in case (II), it is resonant in one of the condensates. This explains why we find entanglement only between $\mathbf{q}$ and $-\mathbf{q}$ and also why it is much larger in case (II).

To explain the results further, we here resort to an approximate analysis. Let us consider $\delta_1=\delta_2=\delta$ and $\mathbf{q}_1=\mathbf{q}_2=\mathbf{q}$. Suppose, $\omega_{q_{1(2)}}\gg\eta_{A(B)}$ and $\delta_{1(2)}\gg\eta_{A(B)}$, then we can neglect in the Hamiltonian the diagonal terms proportional to $\omega_{q_{1(2)}}$ and $\delta_{1(2)}$. This approximation enables us to obtain solutions of the Heisenberg equations in an analytical form. Let us write the quadratures $X_A(\mathbf{q})=1/\sqrt{2}(\hat{\alpha}_{\mathbf{q}}+\hat{\alpha}_{\mathbf{q}}^{\dagger})$, $P_A(\mathbf{q})=-i1/\sqrt{2}(\hat{\alpha}_{\mathbf{q}}-\hat{\alpha}_{\mathbf{q}}^{\dagger})$, and similarly for $X_B(-\mathbf{q})$ and $P_B(-\mathbf{q})$. Using the analytical solutions, we calculate $\xi_p=1-\eta_A\eta_B t^2(1-\eta_A/\eta_B)$, which is less than unity (the two modes are entangled in quadrature variables) if $\eta_A\eta_B t^2(1-\eta_A/\eta_B)>0$, which is only possible if $\eta_A\neq\eta_B$ and $\eta_A<\eta_B$. Similarly, we can prove that for $-\mathbf{q}$ (off resonant) of A and $\mathbf{q}$ (resonant) of B, $\xi_p=1+(\eta_B t)^2(1-\eta_A/\eta_B)$, which is always greater than unity for $\eta_A<\eta_B$ and less than unity if $\eta_A>\eta_B$. For the same resonant $\mathbf{q}$ mode of A and B, $\xi_p=1+\eta_B t^2/2+(\eta_A^2+\eta_B^2)^2 t^4/4$, which is always greater than unity. In the same way, we can show that for the remaining mode pair $(-\mathbf{q},-\mathbf{q})$, ξ_p is also greater than unity. Next, we prove that to generate entanglement in number variables (ξ_n), the two coupling parameters should also be different.

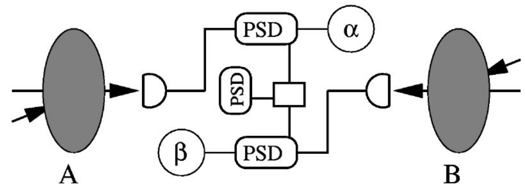

FIG. 3. The scheme for verification of the entanglement. Apart from pump lasers, two extremely weak verifying probes of the same frequency as that of entangling probe—one each for each condensate—are switched on. The probes acting on A and B have momentum $\mathbf{k}_2$ and $-\mathbf{k}_2$, respectively. The momentum of pump laser for A is $\mathbf{k}_1$, while that for B is $-\mathbf{k}_1$.

Substituting $\hat{n}_1=\hat{\alpha}_{\mathbf{q}}^{\dagger}\hat{\alpha}_{\mathbf{q}}$ and $\hat{n}_2=\hat{\beta}_{-\mathbf{q}}^{\dagger}\hat{\beta}_{-\mathbf{q}}$ in Eq. (7) we can express $\xi_n=1-R/(\langle\hat{n}_1\rangle+\langle\hat{n}_2\rangle)$, where

$$R=8\,\eta_A^2 t^4[\,\eta_B^2-2\,\eta_A^2+4n_p(\,\eta_B^2-\eta_A^2)] \tag{9}$$

and n_p is the initial number of photons in the coherent probe beam. Now, $\xi_n<1$ implies that $R>0$ which amounts to $(\eta_B/\eta_A)^2>1+1/(1+4n_p)$, that is, $\eta_B>\eta_A$. On the other hand, if $\eta_B\leqslant\eta_A$, then $\xi_n>1$. We also carry out an alternative analysis to check whether the two resonant modes $\mathbf{q}$ of A and B exhibit any entanglement in other parameter regimes. By neglecting the off-resonant mode $-\mathbf{q}$ in both the condensates and keeping only the resonant mode, it can be analytically proved that $\xi_p(\mathbf{q},\mathbf{q})=1+\sinh^2(\eta t)$ and $\xi_n(\mathbf{q},\mathbf{q})=1+[1+n_p/n_p+1](\eta_A^2-\eta_B^2)^2/(\eta_A^2+\eta_B^2)^2\sinh^2(\eta t)$, that is, both the parameters ξ_p and ξ_n are always greater than unity. Here, $\eta=\sqrt{\eta_A^2+\eta_B^2}$.

We next show how a setup as shown in Fig. 3 can be utilized to verify the generated entanglement after switching off the lasers $L1$, $L2$ and detuning the probe (cavity) further faroff the atomic resonance so that cavity cannot exert any influence on the atoms. Two different pairs of verifying pump-probe Bragg pulses are then applied to the condensates. The modes $\mathbf{q}$ of A and $-\mathbf{q}$ of B are in Bragg resonance with the respective Bragg pulses. The atom-field coupling in both the condensates is very small compared to the Bogoliubov frequency ω_q^B. Let $\hat{c}_{probe,A}$ and $\hat{c}_{probe,B}$ denote the verifying probe-field modes for the condensates A and B, respectively. By neglecting the off-resonant terms $\hat{\alpha}_{2\mathbf{q}}^{\dagger}\hat{\alpha}_{\mathbf{q}}$ and $\hat{\alpha}_{-2\mathbf{q}}^{\dagger}\hat{\alpha}_{-\mathbf{q}}$ in the Hamiltonian, the time evolution of the output probe modes, in a frame rotating with pump-probe detuning δ, can be written as

$$\hat{c}_{probe,A}^{(out)}\simeq\hat{c}_{probe,A}^{(in)}+\frac{\eta_A}{\delta-\omega_q^B}(\exp[i(\delta-\omega_q^B)t]-1)\hat{\alpha}_{\mathbf{q}}^{\dagger}$$
$$+\frac{\eta_1}{\delta+\omega_q^B}(\exp[i(\delta+\omega_q^B)t]-1)\hat{\alpha}_{-\mathbf{q}}. \tag{10}$$

$\hat{c}_{probe,B}^{(out)}$ is given by a similar expression as above with α replaced by β; and the subscripts A and $\mathbf{q}$ replaced B and $-\mathbf{q}$, respectively. The output probes have oscillating parts at frequency $\delta-\omega_q^B$ proportional to the quasiparticle amplitudes $\alpha_{\mathbf{q}}$ and $\beta_{-\mathbf{q}}$, and at frequency $\delta+\omega_q^B$ proportional to the amplitudes $\alpha_{-\mathbf{q}}$ and $\beta_{\mathbf{q}}$. Therefore, phase-sensitive measurements of the spectral components of the output probe beams corresponding to these frequencies would provide measures of the quasiparticle operators which can be employed to calculate the entanglement parameter in number operators, i.e., ξ_n. For calculating entanglement parameters in quadrature phase variables, both the output probe beams coming from *A* and *B*, can be mixed via a beam splitter to form the superposition operators Σ which can be measured by a similar phase-sensitive detection scheme (not shown in Fig. 3).

Following the recent experiment of J. M. Vogels *et al.* [16], we also suggest that the quasiparticles can be detected by imparting a large momentum to them with additional Bragg pulses. Alternatively, in the large-momentum regime $(q \gg \xi^{-1})$, the Bragg-scattered atoms which essentially behave as free particles $(\omega_q^B \propto q^2)$, can be outcoupled by switching off the trap. Since entanglement is between two opposite momentum states, by proper geometric arrangement, the two moving entangled atomic ensembles can be made to collide and interfere. From the interference pattern obtainable via absorption imaging, the atomic number fluctuations can be deduced using the theoretical model used in Ref. [17], and thus entanglement parameter in number variables can be calculated.

In conclusion, we have theoretically demonstrated how light scattering leads to quantum entanglement between two Bose-Einstein condensates. The generated entanglement may be useful in quantum communication using coherent light [18]. We have particularly focused on the conditions under which the entanglement can be obtained. We have also suggested how the generated entanglement could be verified by using Bragg scattering of far-off-resonant fields.

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