Universality of decoherence for macroscopic quantum superpositions

Walter T. Strunz
Theoretische Quantendynamik, Fakultät für Physik, Universität Freiburg, Hermann-Herder-Strasse 3, 79104 Freiburg, Germany

Fritz Haake
Fachbereich Physik, Universität Essen, 45117 Essen, Germany

Daniel Braun
c/o IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 17 May 2002; published 6 February 2003)

We consider environment induced decoherence of quantum superpositions to mixtures in the limit in which that process is much faster than any competing one generated by the Hamiltonian H_{sys} of the isolated system. This interaction-dominated decoherence limit is of importance for the emergence of classical behavior in the macroscopic domain, since it will always be the relevant regime for large enough separations between the superposed wave packets. The usual golden-rule treatment then does not apply, but we can employ a short-time expansion for the free motion while keeping the interaction H_{int} in full. We thus reveal decoherence as a universal short-time phenomenon largely independent of the character of the system as well as the bath and of the basis the superimposed states are taken from. Simple analytical expressions for the decoherence time scales are obtained in the limit in which decoherence is even faster than any time scale emerging from the reservoir Hamiltonian H_{res}.

DOI: 10.1103/PhysRevA.67.022101 PACS number(s): 03.65.Yz

I. INTRODUCTION

A. Environment induced decoherence

Interferences from quantum superpositions of wave packets representing, say, the translational motion of a body, become more and more difficult to observe as the body becomes more massive and the superposed states are made more distinct. Eventually, when the separation of wave packets is increased towards macroscopic magnitudes (for which latter case we shall speak of ‘‘macroscopic superpositions’’), classical behavior, i.e., loss of the ability to interfere, emerges. Somehow, along the way from microscopic to macroscopic superpositions, the quantum capability of a particle to show up ‘‘here’’ and ‘‘there’’ simultaneously escapes detectability.

Two reasons are known for the elusiveness of macroscopic superpositions. One of these even has a classical wave analog. To explain it, let us imagine a plane wave with (de Broglie or classical) wavelength λ traversing a spatial structure of linear dimension d which splits the wave into partial ones. The parameter λ/d then determines the resolvability of interference effects. For instance, in a double-slit experiment an incoming plane wave gives rise to an outgoing interference pattern of angular aperture λ/d. The latter angle becomes exceedingly small when λ is the de Broglie wavelength of a macroscopic body.

The second reason for the notorious absence of quantum superpositions from the macroscopic domain, called environment induced decoherence [1,2], is of dissipative origin and is the one of concern to us here. Decoherence is, for microscopic bodies, just a facet of dissipation caused by interactions with many-freedom surroundings. However, if two sufficiently distinct wave packets $|\varphi_1\rangle,|\varphi_2\rangle$ are brought to an initial superposition $|\rangle=c_1|\varphi_1\rangle+c_2|\varphi_2\rangle$, the density operator $\rho(t)$ starts out as the projector $\rho(0)=|\rangle\langle|$ and then, for suitable coupling to the environment (see below), decoheres to the mixture $|c_1|^2|\varphi_1\rangle\langle\varphi_1|+|c_2|^2|\varphi_2\rangle\langle\varphi_2|$, with the weights $|c_i|^2$ still as in the initial superposition, on a time scale τ_{dec} while the subsequent relaxation of that mixture has a much longer characteristic time τ_{diss}. The smallness of the decoherence time τ_{dec} is manifest in its proportionality to a power of Planck's constant and inverse proportionality to a power of the ‘‘distance’’ d between the superposed packets,

$$\tau_{\mathrm{dec}}\propto\frac{\hbar^{\mu}}{d^{\nu}}\quad\text{with}\quad\mu,\nu>0. \tag{1.1}$$

We may interpret that power law as assigning a quantum scale of reference $\propto\hbar^{\mu/\nu}$ to the distance d such that the decoherence time τ_{dec} becomes vanishingly small when d assumes classical magnitude. On the other hand, the characteristic times for temporal changes of probabilities or other observables capable of a well defined classical limit remain finite in the formal limit $\hbar\rightarrow 0$. As a consequence, a given environment may have so weak an influence that probability relaxation is hard to follow because of τ_{diss} being very large, while giving rise to unresolvably small lifetimes τ_{dec} to coherences between sufficiently far apart wave packets.

A variety of experimental studies of decoherence have been undertaken [3–7], all of them involving weakly coupled environments (‘‘reservoirs’’ or ‘‘heat baths’’) and wave-packet separations of but modest magnitudes: the acceleration of decoherence over dissipation was not at all extreme, the time scale ratio $\tau_{\mathrm{dec}}/\tau_{\mathrm{diss}}$ not even down to 10^{-2} yet. Moreover, dissipation was sufficiently weak in all these experiments for the decoherence time to exceed the time scales τ_{sys} characteristic of the free motion of the system isolated from the environment. In that limit, a lot of free motion takes place during decoherence, and therefore the latter process becomes rather system specific in its characteristics. A unified treatment can, however, be based on the very fact that the environmental influence is weak and thus allows for perturbative treatment by the golden rule.

To illustrate decoherence in the golden-rule limit $\tau_{\rm sys}<\tau_{\rm dec}<\tau_{\rm diss}$, one often considers a harmonic oscillator of mass M and frequency Ω and a bath in thermal equilibrium. If the interaction Hamiltonian is the product of two coupling agents, one for the system (Q) and the other for the bath (B), i.e., $H_{\rm int}=QB$, and if two superposed wave packets are distinguished by the coupling agent Q in terms of the distance $d=|q_1-q_2|=|\langle\varphi_1|Q|\varphi_1\rangle-\langle\varphi_2|Q|\varphi_2\rangle|$, the golden rule is easily seen to yield the decoherence and dissipation times,

$$\frac{1}{\tau_{\rm dec}^{\rm GR}}=\frac{(q_1-q_2)^2}{\hbar^2}\int_0^\infty dt\left\langle\frac{1}{2}\{\tilde{B}(t),B\}\right\rangle\cos\Omega t,$$

$$\frac{1}{\tau_{\rm diss}^{\rm GR}}=\frac{1}{M\Omega}\int_0^\infty dt\left\langle\frac{i}{\hbar}[\tilde{B}(t),B]\right\rangle\sin\Omega t, \tag{1.2}$$

where $\tilde{B}(t)=e^{iH_{\rm res}t/\hbar}Be^{-iH_{\rm res}t/\hbar}$ refers to free time evolution of the bath; note that the dissipation time involves the response function $\langle(i/\hbar)[\tilde{B}(t),B]\rangle$ and the decoherence time the equilibrium correlation function $\langle\frac{1}{2}\{\tilde{B}(t),B\}\rangle$, with $\{\cdot,\cdot\}$ and $[\cdot,\cdot]$ denoting anticommutator and commutator, respectively, and $\langle\cdots\rangle$ thermal equilibrium average. Interestingly, the golden-rule decoherence time obeys the power law (1.1), while the dissipation time is independent of Planck's constant and of the distance d.

Our principal goal in the present paper is to contrast the golden-rule limit $\tau_{\rm sys}<\tau_{\rm dec}<\tau_{\rm diss}$ with the opposite case in which decoherence is the fastest process by far,

$$\tau_{\rm dec}\ll\tau_{\rm sys},\tau_{\rm diss}, \tag{1.3}$$

irrespective of the relative size of $\tau_{\rm sys}$ and $\tau_{\rm diss}$. That *interaction-dominated limit* prevails for sufficiently far apart wave packets and, in particular, for the decoherence of truly macroscopic superpositions; it may, therefore, be seen as relevant for the emergence of classical behavior in the macroscopic world and for the difficulties in experimentally pushing quantum coherent dynamics into the macroscopic domain. Moreover, the limit (1.3) must assign much more universal properties to decoherence since it allows no or "very little" free motion during times of the order $\tau_{\rm dec}$. We shall, in fact, see that the interaction-dominated limit (1.3) yields decoherence times independent of the force $F(Q)$ that may act on the isolated body. The decoherence times to be met with will involve different exponents μ,ν in expression (1.1) than the golden-rule one, $\tau_{\rm dec}^{\rm GR}$ of Eq. (1.2).

For a major part of the paper we not only base our analysis on limit (1.3), but furthermore assume

$$\tau_{\rm dec}\ll\tau_{\rm res}, \tag{1.4}$$

i.e., decoherence is fast even on environmental time scales. In that case, simple expressions of universal character, independent of the details of environmental dynamics, are obtained.

As soon as we drop limit (1.4) yet retain limit (1.3), we find more complicated decoherence dynamics, the temporal decay now being governed by the details of the time evolution of environmental correlations.

It would be highly desirable to experimentally observe the crossover from the golden-rule limit to the interaction-dominated limit (1.3), and further to the extreme limit where both limits (1.3) and (1.4) are satisfied. As already mentioned above, the experiments done thus far pertain to the golden-rule limit where the separation exponent ν takes on the value 2. We shall present some discussion of the crossover condition in Paper II of this series [8]. A quantitative treatment of that crossover itself will have (i) to be nonperturbative (like ours and in contrast to the golden rule) and (ii) have to avoid even the short-time approximation with respect to free motion whose simplicity we will take profit of in the present paper. In Paper II of this series [8], we treat the crossover in question for an exactly solvable model where both the system and the bath consist of harmonic oscillators.

The above remark about interaction-dominated decoherence showing greater universality than its golden-rule counterpart deserves some qualification. If both limits (1.3) and (1.4) are satisfied, of the three parts of the Hamiltonian of the composite system, $H=H_{\rm sys}+H_{\rm res}+H_{\rm int}$, the generators of free motion, $H_{\rm sys}$ and $H_{\rm res}$, play but a minor role in comparison to the interaction part $H_{\rm int}$. As a consequence, it is not of much importance whether the "body" under study is an oscillator, a large angular momentum or some other few-freedoms system. All we require is the possibility of superposing wave packets with large separations d, large in relation to microscopic quantum scales. Similarly, it does not matter whether the reservoir is composed of harmonic oscillators (such as modes of electromagnetic or elastic waves), atoms or other entities; what counts is that the reservoir has many degrees of freedom effective in $H_{\rm int}$, i.e., for $H_{\rm int}=QB$ in its coupling agent B; we shall assume $B=\sum_{i=1}^N B_i$ with N, the number of reservoir freedoms, large.

It is appropriate to admit that in one other respect the interaction-dominated limit is no more but rather even a bit less universal than the golden-rule one. Obviously, the system coupling agent Q in $H_{\rm int}=QB$ is distinguished over other system observables not showing up in the interaction. As we shall see, the coupling agent Q is most effective in decohering superpositions of wave packets with large separations $|\langle\varphi_1|Q|\varphi_1\rangle-\langle\varphi_2|Q|\varphi_2\rangle|$ and considerably less effective if the distinction of the packets is one with respect to some other observable P not commuting with the coupling agent Q. In fact, packets far apart in Q will turn out to decohere with a Gaussian decay of suitable indicators, like $\exp[-(t/\tau_{\rm dec}^Q)^2]$, and that decay is captured already in zeroth order in $H_{\rm sys}+H_{\rm res}$; in that zeroth order, however, wave packets distinguished by P but not by Q would appear as retaining their relative coherence. Such latter packets are in fact also decohered by $H_{\rm int}=QB$, but in general in a non-Gaussian manner, like $\exp[-(t/\tau_{\rm dec})^n]$ with $n>2$ and a decoherence time τ_{dec} which differs from τ_{dec}^{Q} in the exponents μ,ν but still is of quantum character due to $\mu>0$; to capture that latter decoherence, a ''little bit'' of free motion must be accounted for in a systematic manner, as will be explained in Sec. III below. On the other hand, decoherence fully symmetric in Q and P would result from an interaction involving both of these observables as coupling agents towards different reservoirs, $H_{\mathrm{int}}=QB_1+PB_2$, as described in Sec. V and previously pointed out in a first report on this project [9].

The decay in terms of exponentials of powers t^n just mentioned arises when decoherence outruns both dissipation and the decay of bath correlations, i.e., when both limits (1.3) and (1.4) are satisfied. When only dissipation is cut short but bath correlation decay remains effective, such that limit (1.3) is respected but not limit (1.4), the qualitative picture of the above discussion does not change, yet the precise temporal course of decoherence involves the full time dependence of the bath correlation function and can no longer be written as an exponential of a power of time [9]; Sec. VI of the present paper is devoted to that case.

In Sec. VII, we treat decoherence of superpositions of angular-momentum coherent states, with one component, J_x, of an angular momentum $\vec{J}$ acting as coupling agent. In analogy to our findings for superpositions of states distinguished by P and Q, we shall be led to most rapid decoherence for pairs of states with differing mean values of the coupling agent and slowest decoherence for pairs of states with coinciding mean values for all system observables coupled to the coupling agent by the evolution generated by H_{sys}; ''rapid'' and ''slow'' will again be quantified by the exponents μ,ν in the power law (1.1).

B. Related literature

Some words about related literature are in order. Interaction dominance is often invoked in studies of the measurement process (see, for instance, von Neumann [10]), where the coupling of the system to a pointer degree of freedom is the most relevant part for the dynamics, and H_{sys} is simply neglected. The short-time limit of decoherence has been investigated very early by Joos and Zeh who expand an entanglement measure based on a Schmidt decomposition in powers of the elapsed time [11] (see also the earlier work by Kübler and Zeh [12]). Moreover, there are a number of articles on decoherence in the exactly solvable model of a harmonic oscillator coupled to a reservoir itself consisting of harmonic oscillators [13–17]. Decoherence dynamics may then be studied on all time scales, as in Paper II of this series [8], including very early times [15–17]. In none of these earlier works based on the oscillator model particular attention was paid to the investigation of decoherence in the limit of a more and more macroscopic initial separation of the superposed states, on which we focus here. As we will show, this limit is of astonishing simplicity, allowing us to reveal decoherence to be largely independent of the detailed nature of the isolated system and bath dynamics and to obtain simple analytical expressions for the decoherence time scale.

In a discussion of the quantum measurement process where an entangled state of a microscopic quantum system and a macroscopic pointer involves superpositions of macroscopically distinct pointer states, Haake and Żukowski [18] have employed the oscillator model and its exact solution to argue that the superposition in question decoheres in the limit $\tau_{\mathrm{dec}}\ll\tau_{\mathrm{sys}}$. More recently, discussing decoherence during a measurement process, the importance of that early-time limit has also been realized by Mozyrsky and Privman; in a later preprint Privman applies their approach to spin decoherence [19].

We emphasize that our results apply to decoherence of a fully coherent initial superposition of two macroscopically distinct states, and thus to an initial product of a pure system and arbitrary environment state. The only restriction on the latter is the applicability of the central limit theorem for an additive quantity comprising many degrees of freedom of the environment (see Sec. III for details).

The influence of initial correlations between system and environment on decoherence was investigated by Romero and Paz [16] using preparation function techniques and the harmonic-oscillator model. Their results indicate that such correlations may lead to a different temporal decay of the then less pronounced initial coherence. The second part of their work is devoted to a preparation resulting from a projection onto a pure superposition of distinct system states. That choice leads to a factorized state of system and environment, as in our work. Romero and Paz choose, however, not to investigate the corresponding early-time regime which becomes relevant for more and more macroscopic initial distance between the superposed states. The simplicity of this early-time regime is exploited here to liberate the discussion from oscillator models and thus demonstrate a large degree of universality of decoherence.

In a series of papers, Ford, Lewis, and O'Connell (FLO) [17,20] also emphasize that the presence of initial correlations and deviations from a fully coherent initial system state may lead to a different time scale governing the open-system dynamics. In Ref. [20], they argue that two wave packets of widths σ and separation d in Q space experience the time scale $\tau_{\mathrm{FLO}}=\sigma^2/dv$, where $v=\sqrt{k_BT/m}$ is a thermal velocity with m a typical mass; they point out that the latter time may be short compared to the golden-rule prediction for the decoherence time and thus also question the golden-rule approach to decoherence for more and more macroscopic superpositions. In contrast to Ref. [20], and in line with recent decoherence experiments [3–5], however, we do not assume that the initial system state had time to equilibrate at the environmental temperature T. We assume that the coherent initial state is created quasiinstantaneously, possibly achieved by a projection onto a pure system state, as investigated in the second part of Ref. [16]. The success of a matter interferometric experiment as in Vienna [3], strongly depends on that possibility to carefully velocity select the beam. No interference fringes could be observed if one were to use a thermal beam of buckyballs. Thus, we do not encounter the time scale τ_{FLO} in our analysis, yet nevertheless, we confirm that the speed of decoherence invalidates the golden-rule approach rather easily (see also Paper II of this series [8]). Interestingly, τ_{FLO} is independent of Planck's constant as well as of the strength of any interaction H_{int}. In fact, as Ford, Lewis, and O'Connell show in Ref. [20], the "attenuation factor" introduced by them as a measure of the fate of coherence may decay on the time scale $\tau_{\rm FLO}$ under the unitary evolution of the isolated system. This is provided the initial system density operator has already experienced a bath at temperature *T*, such that in position representation it has significant entries only along a diagonal band with a width of the order of a thermal de Broglie wavelength. Thus, the temporal behavior of the attenuation factor of Ford, Lewis, and O'Connell is markedly different from the evolution of standard measures employed to study *environment induced* decoherence, based on entropy or purity of the system state [1,2]: these latter quantities remain constant under unitary time evolution, as does our measure for coherence, the Hilbert-Schmidt norm of the off-diagonal part of the density operator (see Sec. II), which is closely related to purity. This qualitatively different behavior of the attenuation factor of Ford, Lewis and O'Connell and our standard measure for environment induced decoherence also explains why there is no agreement between these results even in those limiting cases where it might be naively expected.

Finally, in a very recent preprint, Lutz [21] takes up initial system-environment correlations as well. In particular, Lutz allows for a composite density matrix differing from the one for overall thermal equilibrium by a factor corresponding to a superposition of two wave packets for the system. As a result of such correlations, different early-time courses of decoherence when the system is a free and a harmonically bound particle are found in the Ford-Lewis-O'Connell attenuation factor; at the same time, it is stated clearly that such a "nonuniversal" decay only concerns thermally allowed coherences, which do not extend beyond the thermal de Broglie wavelength.

II. SUPERPOSITIONS OF DISTINCT WAVE PACKETS

We consider a single-freedom system for which the coordinate *Q* and momentum *P* obey the canonical commutation rule

$$[P,Q]=\frac{\hbar}{i}. \tag{2.1}$$

The initial states we shall have to deal with are pure states of the form of superpositions of two separate wave packets,

$$|\rangle=c_1|\varphi_1\rangle+c_2|\varphi_2\rangle,\quad |c_1|^2+|c_2|^2=1. \tag{2.2}$$

We may specify the individual packets in either the position or momentum representation and choose, for the sake of convenience, the Gaussians

$$\langle q|\varphi_i\rangle=\varphi_i(q)=\frac{1}{(2\pi\sigma)^{1/4}}e^{ip_i(q-q_i)/\hbar}e^{-(q-q_i)^2/4\sigma},$$
$$\langle p|\varphi_i\rangle=\tilde{\varphi}_i(p)=\frac{(2\pi\sigma)^{1/4}}{(\pi\hbar)^{1/2}}e^{-ipq_i/\hbar}e^{-\sigma(p-p_i)^2/\hbar^2}, \tag{2.3}$$

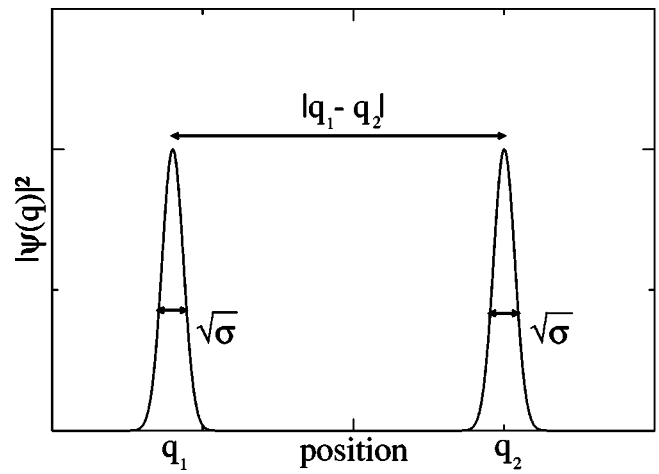

FIG. 1. Position-space density $|\psi(q)|^2$ for a coherent superposition of two Gaussian wave packets as envisaged in this paper. The distance between the packets is assumed much larger than their individual spread.

with $i=1,2$. Needless to say, $\varphi_i(q)$ and $\tilde{\varphi}_i(p)$ are Fourier transforms of one another. These packets are located in position space at q_i with (rms) uncertainty $\Delta q=\sqrt{\sigma}$ and in momentum space at p_i with uncertainty $\Delta p=\hbar/2\sqrt{\sigma}$; the uncertainty product $\Delta q\Delta p=\hbar/2$ is the minimum one allowed by the uncertainty principle; were we to choose σ as a classical quantity independent of Planck's constant, we would confront two extremely squeezed states with the momentum much more sharply defined than the position; we will actually envisage the symmetric situation $\sigma\propto\hbar$, where both Δq and Δp are $\propto\sqrt{\hbar}$, like for coherent states [22]. To ensure good separation, we stipulate that either $\Delta q\ll|q_1-q_2|$ or $\Delta p\ll|p_1-p_2|$ or both (see Fig. 1). Actually, inasmuch as we are interested in "macroscopic superpositions," we may assume at least one of the two distances $|q_1-q_2|,|p_1-p_2|$ of classical magnitude, i.e., independent of $\hbar$.

Our choice of Gaussian packets is a matter of convenience; it will allow us to evaluate all subsequently encountered integrals analytically. The universal decoherence laws to be established rest on sufficient separation of the two packets, however, rather than on their specific form or their minimum-uncertainty property.

The initial density operator corresponding to the state (2.2) is a sum of four terms,

$$\rho_{\rm sys}(0)=\sum_{i,j=1}^{2}c_ic_j^*|\varphi_i\rangle\langle\varphi_j|=\sum_{i,j}c_ic_j^*\rho_{\rm sys}^{ij}(0), \tag{2.4}$$

two "diagonal" ones weighted by probabilities $|c_i|^2$ and two off-diagonal "interference terms" $\rho_{\rm sys}^{12}(0)=|\varphi_1\rangle\langle\varphi_2|=\rho_{\rm sys}^{21}(0)^\dagger$ weighted by the "coherences" $c_1c_2^*$ and $c_1^*c_2$.

Inasmuch as quantum-mechanical time evolution is represented by linear operators each of the four terms in Eq. (2.4) has its own temporal successor $c_ic_j^*\rho_{\rm sys}^{ij}(t)$. To show that interaction with an environment tends to destroy the interference terms before the diagonal terms change noticeably, we shall employ the norms

$$N_{ij}(t)=\mathrm{Tr}_{\mathrm{sys}}\rho_{\mathrm{sys}}^{ij}(t)\rho_{\mathrm{sys}}^{ij}(t)^{\dagger}. \tag{2.5}$$

Clearly, if the system in question were isolated, these norms would all remain time independent, $N_{ij}(t)=1$, since the unitary time evolution operators $U_{\mathrm{sys}}(t)=e^{-iH_{\mathrm{sys}}t/\hbar}$ would cancel under the trace operation. The time scale separation we are after arises only due to the interaction with an environment, and then only if the initial wave packets φ_i are sufficiently distinct.

III. INTERACTION-DOMINATED DECOHERENCE

To allow for dissipative motion of Q and P, we introduce a reservoir with many degrees of freedom and deal with a Hamiltonian of the structure

$$H=H_{\mathrm{sys}}+H_{\mathrm{res}}+H_{\mathrm{int}}. \tag{3.1}$$

We need not specify the Hamiltonian H_{res} governing the free motion of the environment; the Hamiltonian of the isolated single-freedom system is taken as the usual sum of a kinetic and a potential term, $H_{\mathrm{sys}}=P^2/2M+V(Q)$; for the interaction Hamiltonian, however, we do assume a slightly restrictive form involving only one of the two system observables, say Q, as a coupling agent,

$$H_{\mathrm{int}}=QB, \tag{3.2}$$

with B some reservoir coupling agent which should involve all degrees of freedom of the reservoir in a way to be commented on below.

The simplest initial state to deal with has our single-freedom system prepared so as to be statistically independent from the reservoir; the initial joint density operator then takes the form of a product

$$\rho(0)=\rho_{\mathrm{sys}}(0)\rho_{\mathrm{res}}(0), \tag{3.3}$$

with ρ_{sys} representing the superposition of two distinct wave packets as described in the preceding section. The initial density operator of the reservoir could but need not be the thermal equilibrium state with respect to H_{res}; our precise demand on $\rho_{\mathrm{res}}(0)$ will be given presently.

The reduced system density operator originating from any one of the four terms $\rho_{\mathrm{sys}}^{ij}(0)$ can now be written as

$$\rho_{\mathrm{sys}}^{ij}(t)=\mathrm{Tr}_{\mathrm{res}}e^{-iHt/\hbar}\rho_{\mathrm{sys}}^{ij}(0)\rho_{\mathrm{res}}(0)e^{iHt/\hbar}. \tag{3.4}$$

In view of our intention to evaluate the norms $N_{ij}(t)$ defined in Eq. (2.5), it is advantageous to pass to the interaction picture and write the time evolution operator as

$$\begin{gathered}
e^{-iHt/\hbar}=e^{-i(H_{\mathrm{sys}}+H_{\mathrm{res}})t/\hbar}\widetilde{U}(t)=U_0(t)\widetilde{U}(t),\\
\widetilde{U}(t)=\left(\exp\left[-i\int_0^t dt'\,\widetilde{H}_{\mathrm{int}}(t')/\hbar\right]\right)_+,\\
\widetilde{H}_{\mathrm{int}}(t)=U_0^{\dagger}(t)H_{\mathrm{int}}U_0(t)=\widetilde{Q}(t)\widetilde{B}(t),
\end{gathered} \tag{3.5}$$

where $(\cdots)_+$ demands time ordering of the operator product $(\cdots)$. The norms in question thus read

$$\begin{gathered}
N_{ij}(t)=\mathrm{Tr}_{\mathrm{sys}}\widetilde{\rho}_{\mathrm{sys}}^{ij}(t)\widetilde{\rho}_{\mathrm{sys}}^{ji}(t),\\
\widetilde{\rho}_{\mathrm{sys}}^{ij}(t)=\mathrm{Tr}_{\mathrm{res}}\widetilde{U}(t)\rho_{\mathrm{sys}}^{ij}(0)\rho_{\mathrm{res}}(0)\widetilde{U}^{\dagger}(t).
\end{gathered} \tag{3.6}$$

We are interested in the limiting case where decoherence, i.e., the decay of $N_{12}(t)$ is faster than any process arising in the absence of the coupling H_{int}. Our results will self-consistently confirm this limit as relevant for large enough separations (with respect to position, or momentum, or both) between the wave packets. The short-time behavior thus aimed at allows to approximate the interaction-picture evolution operator $\widetilde{U}(t)$ by expanding its logarithm as a power series in the time t. To find that expansion we start with the interaction-picture Hamiltonian

$$\begin{aligned}
\widetilde{Q}(t)\widetilde{B}(t)=&(Q+M^{-1}Pt-V'(Q)t^2/2M+\cdots)\\
&\times(B+\dot{B}t+\ddot{B}t^2/2+\cdots),
\end{aligned} \tag{3.7}$$

where $\dot{B}=(i/\hbar)[H_{\mathrm{res}},B],\ddot{B}=(i/\hbar)[H_{\mathrm{res}},\dot{B}]$. Notice that this short-time expansion is meaningful only if both conditions (1.3) and (1.4) are satisfied. We shall drop the latter condition in Sec. VI where we keep the full time dependence of $\widetilde{B}(t)$.

A simple sequence of unitary transformations, described in the Appendix, brings about the desired expansion as

$$\widetilde{U}(t)=e^{-(i/\hbar)\{Q(Bt+\dot{B}t^2/2)+PBt^2/2M+\cdots\}}, \tag{3.8}$$

where the dots refer to cubic and higher-order terms in t; in particular, the force $-V'(Q)$ enters $\ln\widetilde{U}(t)$ only in order t^3.

We intend to evaluate the trace $\mathrm{Tr}_{\mathrm{sys}}$ in the Q representation, where $Q|q\rangle=q|q\rangle$ and, with an arbitrary state vector $|\psi\rangle$, $\langle\psi|P|q\rangle=i\hbar\,\partial/\partial q\langle\psi|q\rangle$, $\langle q|P|\psi\rangle=-i\hbar\,\partial/\partial q\langle q|\psi\rangle$. We thus have

$$\langle q|\rho_{\mathrm{sys}}^{ij}(t)|q'\rangle=D_Q(t)\langle q|\rho_{\mathrm{sys}}^{ij}(0)|q'\rangle \tag{3.9}$$

with the decoherence factor

$$D_Q(t)=\langle e^{-(i/\hbar)(q-q')(Bt+\dot{B}t^2/2)-(\partial/\partial q+\partial/\partial q')Bt^2/2M}\rangle; \tag{3.10}$$

the angular brackets in the last member of the foregoing equation denote an average with respect to the initial state of the reservoir.

At this point we need to specify the previously announced structure of the reservoir coupling agent as *additively* comprising a large number N of degrees of freedom,

$$B=\sum_{i=1}^{N} B_i . \tag{3.11}$$

Moreover, we require the reservoir initial state ρ_{res} to involve those many degrees of freedom with sufficiently weak correlations for the central limit theorem to hold for the statistical behavior of B as well as its time derivative $\dot{B}$. To avoid unnecessarily voluminous expressions in the sequel we also stipulate vanishing initial means of these observables, $\langle B\rangle =\langle\dot{B}\rangle=0$. The exponent $\mathcal{B}\equiv-(i/\hbar)(q-q')(Bt+\dot{B}t^2/2) -(\partial/\partial q+\partial/\partial q')Bt^2/2M$ in the reservoir expectation value in Eq. (3.10) is thus assigned Gaussian statistics according to $\langle e^{\mathcal{B}}\rangle=e^{(1/2)\langle\mathcal{B}^2\rangle}$. The decoherence factor (3.10) then takes the form

$$\begin{aligned}D_Q(t)&=e^{-(q-q')^2\langle(Bt+\dot{B}t^2/2)^2\rangle/2\hbar^2}\\&\quad\times e^{i(q-q')(\partial/\partial q+\partial/\partial q')\langle(Bt+\dot{B}t^2/2)Bt^2\rangle/2M\hbar}\\&\quad\times e^{(\partial/\partial q+\partial/\partial q')^2\langle B^2\rangle t^4/8M^2};\end{aligned} \tag{3.12}$$

it may be worth noting that we could write three separate exponentials, since the relative displacement and the center-of-mass momentum commute, $[q-q',\partial/\partial q+\partial/\partial q']=0$. We shall save a lot of space and gain better transparency by retaining, in each of the three exponentials in $D_Q(t)$, only the respective leading-order terms in the time t; it will become clear further below that nothing of relevance for the final result is thus lost. A similar calculation in the P basis yields

$$\langle p|\rho_{\mathrm{sys}}^{ij}(t)|p'\rangle=D_P(t)\langle p|\rho_{\mathrm{sys}}^{ij}(0)|p'\rangle, \tag{3.13}$$

$$\begin{aligned}D_P(t)&=e^{(\partial/\partial p+\partial/\partial p')^2\langle B^2\rangle t^2/2}e^{i(p-p')(\partial/\partial p+\partial/\partial p')\langle B^2\rangle t^3/2M\hbar}\\&\quad\times e^{-(p-p')^2\langle B^2\rangle t^4/8M^2\hbar^2}.\end{aligned}$$

Note that now we have retained only the $O(t^2)$ term in the first exponential and the $O(t^3)$ term in the second exponential.

The asymmetry between Q and P in the matrix elements (3.9), (3.12), and (3.13) arises from the distinction of the coordinate Q as the system coupling agent in the interaction (3.2). By their asymmetric appearance these matrix elements already suggest different temporal courses of decoherence for superpositions of wave packets macroscopically distinguished in Q and in P; that difference will become yet easier to discern once we have evaluated the norms (3.6). Upon there inserting the matrix elements (3.9),(3.12), integrating by parts, and changing integration variables to relative and center-of-mass coordinate as $k=q-q',\overline{q}=\frac{1}{2}(q+q')$, and $\partial/\partial\overline{q}=\partial/\partial q+\partial/\partial q'\equiv\partial$ we get

$$\begin{aligned}N_{ij}&=\int d\overline{q}dk\varphi_i^*(\overline{q}+k/2)\varphi_j(\overline{q}-k/2)D_Q^2(t)\\&\quad\times\varphi_i(\overline{q}+k/2)\varphi_j^*(\overline{q}-k/2),\\D_Q^2(t)&=e^{-k^2\langle B^2\rangle t^2/\hbar^2}e^{ik\partial\langle B^2\rangle t^3/M\hbar}e^{\partial^2\langle B^2\rangle t^4/4M^2}.\end{aligned} \tag{3.14}$$

The second and third exponentials in the foregoing quantity $D_Q^2(t)$ are integral operators acting on the subsequent functions of the center-of-mass variable $\overline{q}$ as, respectively, $e^{\Delta\partial}f(\overline{q})=f(\overline{q}+\Delta)$ and $e^{\tau\partial^2}f(\overline{q}) =\int dx(4\pi\tau)^{-1/2}e^{(\overline{q}-x)^2/4\tau}f(x)$, i.e., like shift and diffusion. Of course, apart from the change of variables just indicated $D_Q^2(t)$ is nothing but the square of $D_Q(t)$ given in Eq. (3.12). After inserting the initial states (2.3) and doing the three Gaussian integrals over $\overline{q},k,x$ we finally obtain the "coherence norm" $N_{12}(t)$ in its dependence on the time t and the separations q_1-q_2 and p_1-p_2 of the two wave packets in Q space and P space,

$$\begin{aligned}N_{12}(t)&=\{1+4\sigma\langle B^2\rangle t^2/\hbar^2+O(t^4)\}^{-1/2}\\&\quad\times\exp\{-(q_1-q_2)^2\langle B^2\rangle t^2/\hbar^2\}\\&\quad\times\exp\{-(q_1-q_2)(p_1-p_2)\langle B^2\rangle t^3/M\hbar^2\}\\&\quad\times\exp\{-(p_1-p_2)^2\langle B^2\rangle t^4/4M^2\hbar^2\}\\&\equiv\mathcal{P}(t)\mathcal{E}^Q(t)\mathcal{E}^{QP}(t)\mathcal{E}^P(t).\end{aligned} \tag{3.15}$$

For typographical reasons we have not indicated the corrections $\propto t^{n+1}$ to the leading-order terms t^n in the three exponentials; they are independent of the separations q_1-q_2 and p_1-p_2; neither do these separations enter the prefactor $\mathcal{P}(t)$.

We have thus established one of the central results of the present paper and proceed to a critical appreciation.

IV. DISCUSSION OF INTERACTION-DOMINATED DECOHERENCE

A. Decoherence time scales

If the two wave packets in our superposition differ both in their center positions and momenta, the three exponentials in the coherence norm $N_{12}(t)$ have the decay times

$$\begin{aligned}\tau_{\mathrm{dec}}^Q&=\frac{\hbar}{|q_1-q_2|\sqrt{\langle B^2\rangle}},\\\tau_{\mathrm{dec}}^{QP}&=\left(\frac{M\hbar^2}{|(q_1-q_2)(p_1-p_2)|\langle B^2\rangle}\right)^{1/3},\\\tau_{\mathrm{dec}}^P&=\left(\frac{4M^2\hbar^2}{(p_1-p_2)^2\langle B^2\rangle}\right)^{1/4}.\end{aligned} \tag{4.1}$$

All three of these decoherence times are quantum in character and tend to vanish in the formal classical limit $\hbar\rightarrow 0$. They may be considered ordered in their magnitudes by the respective powers of Planck's constant [23] $\tau_{\mathrm{dec}}^Q\propto\hbar,\tau_{\mathrm{dec}}^{QP}\propto\hbar^{2/3},\tau_{\mathrm{dec}}^P\propto\hbar^{1/2}$. The distances $|q_1-q_2|$ and $|p_1-p_2|$ between the two wave packets appear as referred to quantum scales and in those units tend to take on huge values if mesoscopic or even macroscopic. At any rate, it is the smallness of the decoherence times for which macroscopic superpositions would have little chance to be detectable even if preparable.

Which of the three exponentials wins out in governing the decoherence of macroscopically distinct packets depends on the distances $|q_1-q_2|$ and $|p_1-p_2|$; obviously, different cases arise, and these will be dealt with individually below.

It may be worth noting that the powers of Planck's constant as well as those of the distances differ from the ones more familiar from the golden-rule result (1.2).

B. Universality and limits of validity

Inasmuch as our result for the decay of coherence between the superposed wave packets is based on a short-time expansion of the (logarithm of) the time evolution operator $\tilde{U}(t)$, [cf. Eqs. (3.5) and (3.8)], we have to emphasize its limit of validity. To appreciate that limit we must realize that it is the free motion of the single-freedom system and the reservoir which was treated as nearly ineffective during the decoherence, while the interaction $H_{\rm int}$ was kept in full; in particular, the first exponential $\mathcal{E}^Q(t)$ in the coherence norm (3.15) can immediately be checked to arise from entirely neglecting $H_{\rm sys}+H_{\rm res}$ in Eq. (3.5) and thus taking $\tilde{U}(t)=e^{-iH_{\rm int}t/\hbar}=e^{-iQBt/\hbar}$. Shouldering the burden of the $O(t^2)$ terms in Eq. (3.8), which bring in $\dot{Q}=(i/\hbar)[H_{\rm sys},Q]=P/M,\dot{B}=(i/\hbar)[H_{\rm res},B]$ is necessary only in the case $q_1=q_2$; note again that the potential energy $V(Q)$ is barred from entering at all, to the order in t accepted. It follows that our result (3.15) for the coherence norm is valid only in the limit when the decoherence times (4.1) are much smaller than any of the time scales characteristic of the free motions of the single-freedom system as well as the reservoir,

$$\tau_{\rm dec} \ll \tau_{\rm sys}\,,\ \tau_{\rm res}\,. \tag{4.2}$$

Just for the sake of illustration, if the single-freedom system were an oscillator, the relevant system time scale would be the basic period of oscillation, while for the reservoir the shortest time scale is either the inverse of the highest frequency provided by the environmental degrees of freedom or the thermal time $\hbar/kT$. In Sec. VI, we discuss the more general regime where no assumption is made about the relative size of $\tau_{\rm dec}$ and $\tau_{\rm res}$.

The self-consistency condition (4.2) is fulfilled for sufficiently large distances between the superposed wave packets; it is hard, probably impossible, to violate for truly macroscopic superpositions and that fact may be seen as the reason for the absence of quantum interferences from the macroscopic world. We hasten to add that the condition is not fulfilled for the present-day experiments on decoherence, which all still operate in the situation $\tau_{\rm res}\ll\tau_{\rm sys}\ll\tau_{\rm dec}$ where the golden rule applies. In order to distinguish the decoherence phenomena taking place in our short-time limit from the golden-rule-type decoherence processes thus far observed we speak of interaction-dominated decoherence in the limit (4.2).

Within its range of applicability, our short-time result has a certain universal character. Inasmuch as the free-motion Hamiltonian $H_{\rm sys}+H_{\rm res}$ is not operative through the full cycle of any free oscillation in either the single-freedom system or the reservoir, the character of such oscillations remains irrelevant for decoherence as a short-time phenomenon. It does not matter whether the single-freedom system is a harmonic or anharmonic oscillator since, as already stressed before, the force $-V'(Q)$ gets no chance to act; likewise, whether the bath consists of oscillators (such as lattice vibrations, electromagnetic or gravitational waves), two-level atoms, or other elementary units is immaterial.

The insensitivity of interaction-dominated decoherence to the character of the oscillations generated by $H_{\rm sys}+H_{\rm res}$ also implies ignorance of whether the reservoir will, at larger times, impose underdamped or overdamped motion to the single-freedom system.

C. Wave packets distinguished by the coupling agent Q

If the center positions q_1,q_2 of the superposed wave packets are classically distinct the decay of the interference term $\rho_{\rm sys}^{12}(t)$ is governed by the first of the three exponentials, $\mathcal{E}^Q(t)=\exp[-(t/\tau_{\rm dec}^Q)^2]$, in the norm $N_{12}(t)$. We then encounter a Gaussian falloff on the time scale $\tau_{\rm dec}^Q$. The second and third exponentials are trivially ineffective for $p_1=p_2$; but even for $|p_1-p_2|\neq 0$ and independent of $\hbar$ they may be considered as practically constant in time since their lifetimes $\tau_{\rm dec}^{QP}\propto\hbar^{2/3},\tau_{\rm dec}^{P}\propto\hbar^{1/2}$ are much larger than $\tau_{\rm dec}^{Q}\propto\hbar$. Equally ineffective are the corrections of third and higher order in t within that first exponential, in our limit of large distances $|q_1-q_2|$. This is because the whole exponent in $\mathcal{E}^Q(t)$ depends on the distance only through the common factor $(q_1-q_2)^2$; the leading t^2 term thus defines a scaling variable $\tau=t|q_1-q_2|$ such that higher-order corrections involve $t^n=\tau^n/|q_1-q_2|^{n-2}$. For sufficiently large distances $|q_1-q_2|$, the higher-order corrections would come into effect only for times t at which the leading Gaussian has already suppressed the coherence norm to rather uninterestingly small values. For the same reason the prefactor $\mathcal{P}(t)$, which arises from the Gaussian integrals, cannot noticeably deviate from its initial value unity during the lifetime of the leading exponential.

D. Wave packets distinguished by the conjugate momentum P

An interesting situation arises when $q_1=q_2$ and $|p_1-p_2|$ is of classical magnitude, i.e., independent of Planck's constant, since then the first and second exponentials in the coherence norm remain equal to unity at all times. Interaction-dominated decoherence is thus described by the third exponential, $N_{12}(t)=\mathcal{E}^P(t)=e^{-(t/\tau_{\rm dec}^P)^4}$. Due to the different power of Planck's constant in $\tau_{\rm dec}^P$, i.e., $\tau_{\rm dec}^P/\tau_{\rm dec}^Q\propto\hbar^{1/2}$, we may say that under the influence of the position Q as a coupling agent, momentum-space superpositions decohere more slowly than position-space superpositions. It has in fact been known for quite some time that an interaction $H_{\rm int}=QB$ decoheres superpositions of wave packets most rapidly if these packets are distinct in the eigenrepresentation of Q; Zurek [2] speaks of the ''distinction of the pointer basis;'' we here see that distinction carrying over to the short-time limit of decoherence. Part of the importance of our result (3.15) lies in bringing to light the rapid decay of superpositions of packets not at all distinguished by the coupling agent Q. As already mentioned before, the capability of $H_{\rm int}=QB$ to decohere momentum-space superpositions would be overlooked if the action of the free-motion Hamiltonian $H_{\rm sys}+H_{\rm res}$ were dropped entirely, with overzealous appeal to the limit (4.2) of interaction predominance. Clearly, our short-time expansion of the (logarithm of the) interaction-picture propagator $\tilde{U}(t)$ accounts, in the next-to-leading order in the time t, for just that much free motion as necessary to let a pure-momentum superposition acquire a bit of a Q component and thus to become visible and fall prey to $H_{\rm int}=QB$.

E. Transition between position space and momentum-space superpositions

The borderline between position-space and momentum-space distinction is worth a moment of special attention. When both $|q_1-q_2|$ and $|p_1-p_2|$ are nonzero and of classical magnitude (independent of $\hbar$), the first exponential $\mathcal{E}^Q(t)$ with its Gaussian decay dominates the decoherence process, as already emphasized above. Now imagine the momentum distinction fixed and the distance $|q_1-q_2|$ decreased; eventually, the lifetime $\tau_{\rm dec}^Q$ of the first exponential will have grown to the magnitude of its competitors $\tau_{\rm dec}^{QP},\tau_{\rm dec}^P$, and then the dominance of the first exponential is lost. The emancipation of the competing exponentials takes place when, respectively, $\tau_{\rm dec}^Q/\tau_{\rm dec}^{QP}=O(\hbar^0)\approx 1$ and $\tau_{\rm dec}^Q/\tau_{\rm dec}^P=O(\hbar^0)\approx 1$; inserting the various decoherence times according to Eq. (4.1), we see that both transitions concur at

$$|q_1-q_2|^2/|p_1-p_2|=O(\hbar), \tag{4.3}$$

i.e., for classical magnitude of the momentum distinction at $|q_1-q_2|\propto\sqrt{\hbar}$. Interestingly, then, the transition in question requires keeping all three exponentials in the coherence norm (3.15) for a proper description. Actually, to obtain good quantitative reliability it would be advisable to include the order-t^2 term in the prefactor $\mathcal{P}(t)$ as well, $\mathcal{P}(t)=[1+4\sigma\langle B^2\rangle t^2/\hbar^2+O(t^4)]^{-1/2}\approx\exp(-2\sigma\langle B^2\rangle t^2/\hbar^2)$, since the position-space width of each of the superposed wave packets was assumed as $\sqrt{\sigma}\propto\sqrt{\hbar}$, i.e., as of the same order in $\hbar$ as the transitional distance $|q_1-q_2|$.

V. SEVERAL RESERVOIRS AND COUPLING AGENTS

A single-freedom system may be coupled to two many-freedom reservoirs with both the position Q and the momentum P serving as system coupling agents, according to the interaction Hamiltonian [9]

$$H_{\rm int}=QB_Q+PB_P\,. \tag{5.1}$$

The two separate reservoirs enter with the respective coupling agents B_Q,B_P; for these we assume the structure (3.11), i.e., $B_Q=\Sigma_i B_{Qi},B_P=\Sigma_i B_{Pi}$, and vanishing means with respect to the initial state of the reservoirs. The "Q reservoir" and the "P reservoir" are independent and have their own free-motion Hamiltonians such that $H_{\rm res}=H_Q+H_P$.

To describe the decoherence of an initial superposition, like Eqs. (2.2) and (2.3) in the limit (4.2), we may again employ the short-time expansion of the (logarithm of the) interaction-picture propagator. In analogy to Eqs. (3.5) and (3.8) we have

$$\begin{aligned}\tilde{U}(t)&=\left(\exp\left[-i\int_0^t dt'\tilde{H}_{\rm int}(t')/\hbar\right]\right)_+\\&=e^{-i\{(QB_Q+PB_P)t+O(t^2)\}/\hbar}.\end{aligned} \tag{5.2}$$

Note that we need not go to higher than first order in t, since the presence of both reservoirs entails the appearance of both the position *and* the momentum in first order; no bit of free motion must be invoked here to assist any underprivileged distinction of the superposed wave packets. The central limit theorem then yields Gaussian decay of the coherence norm,

$$\begin{aligned}N_{12}(t)&=\exp\{-(t/\tau_{\rm dec}^Q)^2\}\exp\{-(t/\tau_{\rm dec}^P)^2\},\\\tau_{\rm dec}^Q&=\hbar/|q_1-q_2|\sqrt{\langle B_Q^2\rangle},\\\tau_{\rm dec}^P&=\hbar/|p_1-p_2|\sqrt{\langle B_P^2\rangle}.\end{aligned} \tag{5.3}$$

The remarks about limits of validity and universality of the preceding section apply again, except for the simplification that the presence of both reservoirs makes for symmetry between the pair of observables. In particular, higher-order corrections in t are irrelevant if at least one of the two distances $|q_1-q_2|$, $|p_1-p_2|$ is of classical magnitude.

VI. COMPETITION OF DECOHERENCE AND BATH CORRELATION DECAY

Thus far we have assumed that decoherence is by far the fastest process, shorter even in duration than environmental time scales such that the two conditions (1.3) and (1.4) could be exploited. Of greater experimental relevance, however, is the case in which limit (1.3) is satisfied, while the bath correlation time scale $\tau_{\rm res}$ may be comparable with or even shorter than the decoherence time. To that important case we shall now generalize our above discussions.

The analysis of Sec. III goes through unchanged up to the short-time expansion (3.7), except that this very expansion must now be confined to the free time evolution of the system coupling agent, $\tilde{Q}(t)=Q+M^{-1}Pt+\cdots$, while the time dependence of the bath coupling agent $\tilde{B}(t)$ generated by the free bath Hamiltonian $H_{\rm res}$ must be kept in full,

$$\tilde{H}_{\rm int}(t)=\tilde{Q}(t)\tilde{B}(t)=(Q+\cdots)\tilde{B}(t). \tag{6.1}$$

Actually, we shall simplify even further by retaining only the lowest-order term of the expansion of the system coupling agent, $\tilde{Q}(t)\approx Q$, thus confining ourselves to treating the decoherence of wave packets with different locations in Q space.

The propagator (3.8) is now replaced by

$$\tilde{U}(t)=\left(\exp\left[-(i/\hbar)\left\{Q\int_0^t ds\tilde{B}(s)+\cdots\right\}\right]\right)_+ . \quad (6.2)$$

Proceeding in the very same fashion as in Sec. III we find the variant of Eq. (3.10),

$$\langle q|\rho_{\text{sys}}^{ij}(t)|q'\rangle=D_Q(t)\langle q|\rho_{\text{sys}}^{ij}(0)|q'\rangle,$$

$$D_Q(t)=\left\langle\left(\exp(i/\hbar)\left[\left\{q'\int_0^t ds\tilde{B}(s)\right\}\right]\right)_- \times\left(\exp\left[-(i/\hbar)\left\{q\int_0^t ds\tilde{B}(s)\right\}\right]\right)_+\right\rangle . \quad (6.3)$$

Again, the large angular brackets denote an average with respect to the initial state of the reservoir, and $(\cdots)_-$ refers to antitime ordering, opposite in sense to $(\cdots)_+$.

As in Sec. III we now take advantage of the multicomponent structure of the bath coupling agent B which allows to regard $\tilde{B}(t)$ as (an operator process) of Gaussian statistics. The reservoir average in Eq. (6.3) may then be evaluated analytically (most straightforwardly by expanding all exponentials),

$$\left\langle\left(\exp\left[(i/\hbar)q'\int_0^t ds\tilde{B}(s)\right]\right)_- \times\left(\exp\left[-(i/\hbar)q\int_0^t ds\tilde{B}(s)\right]\right)_+\right\rangle$$
$$=\exp\left\{-(1/\hbar^2)(q-q')\int_0^t ds\int_0^s ds'[q\langle\tilde{B}(s)\tilde{B}(s')\rangle - q'\langle\tilde{B}(s')\tilde{B}(s)\rangle]\right\}. \quad (6.4)$$

For the superposition of wave packets with different positions studied here, this result is a generalization of Eq. (3.12), valid also for times long compared to environmental correlation times. A short-time expansion of Eq. (6.4) recovers Eq. (3.12). We had previously [9] derived Eq. (6.4) for the special case of a reservoir of harmonic oscillators; here, we can rejoice in the validity for general baths with effectively Gaussian coupling agents B.

All that remains to be done is to determine the coherence norm $N_{12}(t)$ as in Sec. III. That task, now simplified inasmuch as the momentum P is barred, yields

$$N_{12}(t)=\exp\left(-\frac{(q_1-q_2)^2}{\hbar^2}\int_0^t ds\int_0^s ds'\langle\{\tilde{B}(s),\tilde{B}(s')\}\rangle\right), \quad (6.5)$$

with no restriction on the validity beyond $\tau_{\text{dec}}\ll\tau_{\text{sys}}$. Clearly, the foregoing result generalizes the first factor $\mathcal{E}^Q(t)$ in the coherence norm (3.15) so as to allow for competition of decoherence and bath correlation decay.

While we still observe the quadratic dependence of the exponential suppression of coherence on the distance $|q-q'|$, the precise time evolution of decoherence is governed by the symmetric part of the bath correlation function. No system time scale is involved here, in contrast to the analogous expression (1.2) for exponential golden-rule decay. In fact, Eq. (6.5) describes nonexponential decay for $t\to\infty$, unless the Fourier transform of $\langle\{\tilde{B}(t),\tilde{B}(0)\}\rangle$ (the spectral density), differs from zero at zero frequency. This is seen by writing $\int_0^t ds\int_0^s ds'\langle\{\tilde{B}(s),\tilde{B}(s')\}\rangle=\int_0^t ds(t-s)\times\langle\{\tilde{B}(s),\tilde{B}(0)\}\rangle$, taking advantage of the stationarity of the Gaussian process. As $t\to\infty$, no rate of decay can be defined, unless $\int_0^\infty ds\langle\{\tilde{B}(s),\tilde{B}(0)\}\rangle$ remains finite. Examples of such decay will be presented for the exactly solvable harmonic-oscillator model in Paper II of this series [8].

VII. ANGULAR-MOMENTUM DECOHERENCE

To emphasize the universality of interaction-dominated decoherence $\tau_{\text{dec}}\ll\tau_{\text{sys}},\tau_{\text{res}}$, we here consider an angular-momentum vector $\vec{J}$ whose three components obey the commutation relations $[J_x,J_y]=i\hbar J_z$, etc., coupled to a reservoir. As the Hamiltonian we take

$$H_{\text{sys}}=\Omega J_z,\quad H_{\text{int}}=J_xB. \quad (7.1)$$

The squared angular momentum is thus conserved, $J^2=j(j+1)$, with the quantum number j capable of taking on integer or half integer values; large values of j enable the angular momentum to near classical behavior.

Suitable wave packets are provided by coherent states [24], which specify a direction for (the expectation value of) $\vec{J}$ in terms of two angles, θ and ϕ, with the minimal uncertainty allowed by the commutation relations. We shall denote those states by $|j,\theta,\phi\rangle\equiv|\alpha\rangle$, the latter shorthand dropping the quantum number j and introducing the complex amplitude $\alpha=e^{i\phi}\tan(\theta/2)$. The whole complex plane is visited by α as the "polar" angle ranges in $0\leq\theta\leq\pi$ and the "azimuthal" angle in $0\leq\phi<2\pi$. (We may speak of the mapping of the surface of the unit sphere onto the complex plane; the sphere $\lim_{j\to\infty}J^2/(\hbar j)^2=1$ is the classical phase space.) The coherent-state mean of $\vec{J}$ reads

$$\langle\alpha|J_x|\alpha\rangle=\hbar j\frac{\alpha+\alpha^*}{1+\alpha\alpha^*}=\hbar j\cos\phi\sin\theta,$$

$$\langle\alpha|J_y|\alpha\rangle=\hbar j\frac{i(\alpha^*-\alpha)}{1+\alpha\alpha^*}=\hbar j\sin\phi\sin\theta, \quad (7.2)$$

$$\langle\alpha|J_z|\alpha\rangle=\hbar j\frac{1-\alpha\alpha^*}{1+\alpha\alpha^*}=\hbar j\cos\theta.$$

The coherent state $|\alpha\rangle$ can be expressed in terms of eigenstates $|j,m\rangle$ of J^2 and J_z [eigenvalues $j(j+1)$ and m, respectively] as

$$
\begin{aligned}
|\alpha\rangle &= (1+\alpha\alpha^*)^{-j} e^{\alpha J_-/\hbar}|j,j\rangle \\
&= (1+\alpha\alpha^*)^{-j} \sum_{n=0}^{2j} \sqrt{\binom{2j}{n}}\, \alpha^n |j,j-n\rangle \\
&\equiv (1+\alpha\alpha^*)^{-j} ||\alpha\rangle, \qquad (7.3)
\end{aligned}
$$

where $J_- = J_x - iJ_y$. It will in fact be convenient to work with the non-normalized Dirac ket $||\alpha\rangle$ which is holomorphic in α and the corresponding antiholomorphic bra $\langle\alpha||$. The coherent state $|\alpha\rangle$ itself is normalized as $\langle\alpha|\alpha\rangle = 1$.

We now turn to a superposition of two coherent states, $|\rangle = c_\alpha|\alpha\rangle + c_\beta|\beta\rangle$, which in the limit $j \gg 1$ is a macroscopic superposition, and inquire about the temporal fate of the coherence norm $N_{\alpha\beta}(t) = \mathrm{Tr}_{\mathrm{sys}}\rho_{\mathrm{sys}}^{\alpha\beta}(t)\rho_{\mathrm{sys}}^{\beta\alpha}(t)$ with

$$
\begin{aligned}
\rho_{\mathrm{sys}}^{\alpha\beta}(t) &= [(1+\alpha\alpha^*)(1+\beta\beta^*)]^{-j} \\
&\quad\times \mathrm{Tr}_{\mathrm{res}}\, e^{-iHt/\hbar} ||\alpha\rangle\langle\beta|| \rho_{\mathrm{res}}(0) e^{iHt/\hbar} \qquad (7.4)
\end{aligned}
$$

the temporal successor of $\rho_{\alpha\beta}(0) = |\alpha\rangle\langle\beta|$. Like in Sec. III, we go to the interaction picture, where

$$
\begin{aligned}
\tilde{H}_{\mathrm{int}} &= \tilde{B}(t)(J_x \cos\Omega t - J_y \sin\Omega t) \\
&= J_x B + (J_x\dot{B} - \Omega J_y B)t + (-\Omega^2 J_x B - 2\Omega J_y \dot{B} + J_x \ddot{B})t^2/2 \\
&\quad + \cdots \qquad (7.5)
\end{aligned}
$$

gives rise to the propagator

$$
\begin{aligned}
\tilde{U}(t) = \exp\Bigg\{ -\frac{i}{\hbar}\Bigg[& BJ_x t - (\dot{B}J_x - B\Omega J_y)t^2/2 + \Bigg(-\Omega^2 J_x B \\
& - 2\Omega J_y \dot{B} + J_x\ddot{B} + \frac{i}{2\hbar}[B,\dot{B}] + \tfrac{1}{2}\Omega J_z B^2 \Bigg) t^3/6 + \cdots \Bigg]\Bigg\}; \qquad (7.6)
\end{aligned}
$$

see the Appendix for the derivation of the foregoing short-time expansion. Note that we have here included the third-order term of the expansion, for a reason that will become clear presently. When the propagator $\tilde{U}(t)$ acts on the holomorphic $||\alpha\rangle$ we may use the identities

$$
J_x||\alpha\rangle = \frac{\hbar}{2}\left(2j\alpha - (\alpha^2-1)\frac{\partial}{\partial\alpha}\right)||\alpha\rangle \equiv \hat{X}_\alpha||\alpha\rangle,
$$

$$
J_y||\alpha\rangle = \frac{\hbar}{2i}\left(2j\alpha - (\alpha^2+1)\frac{\partial}{\partial\alpha}\right)||\alpha\rangle \equiv \hat{Y}_\alpha||\alpha\rangle, \qquad (7.7)
$$

$$
J_z||\alpha\rangle = \hbar\left(j - \alpha\frac{\partial}{\partial\alpha}\right)||\alpha\rangle \equiv \hat{Z}_\alpha||\alpha\rangle
$$

and their adjoints $\langle\beta||J_x = \hat{X}_{\beta^*}\langle\beta||$, etc., such that $U(t)||\alpha\rangle = U(t;\alpha)||\alpha\rangle$ with $U(t;\alpha)$ differing from $U(t)$ only by the replacements (7.7); similarly, $\langle\beta||U^\dagger(t) = U^\dagger(t,\beta^*)\langle\beta||$ with $U^\dagger(t,\beta^*)$ obtained from $U^\dagger(t)$ by $J_x \to X_{\beta^*}$, etc. We thus get

$$
\begin{aligned}
&\tilde{\rho}_{\mathrm{sys}}^{\alpha\beta}(t)[(1+\alpha\alpha^*)(1+\beta\beta^*)]^j \\
&= \mathrm{Tr}_{\mathrm{res}} U(t)||\alpha\rangle\langle\beta||\rho_{\mathrm{res}}(0)U^\dagger(t) \\
&= \mathrm{Tr}_{\mathrm{res}} U^\dagger(t,\beta^*)U(t,\alpha||\alpha\rangle\langle\beta||\rho_{\mathrm{res}}(0) \\
&= \langle U^\dagger(t,\beta^*)U(t,\alpha)\rangle||\alpha\rangle\langle\beta||. \qquad (7.8)
\end{aligned}
$$

To within a further correction of order t^4 we can merge the two exponentials in the last member of the foregoing equation by simply adding the exponents. We proceed to the coherence norm

$$
\begin{aligned}
N_{\alpha\beta}(t) = [(1+\alpha\alpha^*)(1+\beta\beta^*)]^{-2j}\Bigg\langle \exp\Bigg(-i\Bigg\{ & (\hat{X}_\alpha - \hat{X}_{\beta^*}) \\
& \times[Bt + \dot{B}t^2/2 + (\ddot{B} - \Omega^2 B)t^3/6] - (\hat{Y}_\alpha - \hat{Y}_{\beta^*}) \\
& \times(Bt^2/2 + \dot{B}t^3/3) + (\hat{Z}_\alpha - \hat{Z}_{\beta^*})B^2t^3/12 + (\hat{X}_\alpha^2 \\
& - \hat{X}_{\beta^*}^2)\frac{i}{\hbar}[B,\dot{B}]t^3/12 \Bigg\}\Bigg)\Bigg\rangle \langle\exp(-i\{\text{same with } \alpha \\
& \to \beta, \beta^* \to \alpha^*\})\rangle[(1+\alpha\alpha^*)(1+\beta\beta^*)]^{2j}, \qquad (7.9)
\end{aligned}
$$

where we have encountered $\mathrm{Tr}_{\mathrm{sys}}||\alpha\rangle\langle\beta||\beta\rangle\langle\alpha|| = [(1+\alpha\alpha^*)(1+\beta\beta^*)]^{2j}$; it is on this latter function of $\alpha,\alpha^*,\beta,\beta^*$ that the various differential operators like $\partial/\partial\alpha$ in the exponentials in Eq. (7.9) act. To leading order in j these differentiations act as $\partial/\partial\alpha \to 2j\alpha^*/(1+\alpha\alpha^*)$, etc., whereupon the differential operators $\hat{X}_\alpha, \hat{Y}_\alpha, \hat{Z}_\alpha$ become replaced by real c numbers, in fact the coherent-state expectation values of J_x, J_y, J_z given in Eq. (7.2), $\hat{X}_\alpha \to \langle\alpha|J_x|\alpha\rangle/\hbar$, $\hat{X}_{\beta^*} \to \langle\beta|J_x|\beta\rangle/\hbar$, etc. The two reservoir means $\langle\exp(\cdots)\rangle$ in the foregoing expression for the coherence norm thus become mutual complex conjugates and are controlled by the three distances

$$
d_i = \langle\alpha|J_i|\alpha\rangle - \langle\beta|J_i|\beta\rangle, \quad i = x,y,z, \qquad (7.10)
$$

and even by the differences of the mean values of J_x^2 with respect to the coherent states $|\alpha\rangle, |\beta\rangle$. Confining ourselves to the leading order in j we have

$$
\begin{aligned}
N_{\alpha\beta}(t) = \Bigg|\Bigg\langle \exp\Bigg(-\frac{i}{\hbar}\Bigg\{ & d_x[Bt + \dot{B}t^2/2 + (\ddot{B} - \Omega^2 B)t^3/6] \\
& - d_y(Bt^2/2 + \dot{B}t^3/3) + d_z B^2 t^3/12 + (\langle\alpha|J_x|\alpha\rangle^2 \\
& - \langle\beta|J_x|\beta\rangle^2)\frac{i}{\hbar}[B,\dot{B}]t^3/12 \Bigg\}\Bigg)\Bigg\rangle\Bigg|^2, \qquad (7.11)
\end{aligned}
$$

and this can now be seen to imply a greater wealth of decoherence courses than previously encountered for a canonical pair of observables.

The system coupling agent, J_x in the interaction (7.1), again plays a distinguished role; it is most efficient in decohering a superposition $c_\alpha|\alpha\rangle + c_\beta|\beta\rangle$ if it has macroscopically distinct means in the two superposed coherent states, macroscopic now meaning $j \gg 1$. In that situation only the single term linear in the time t needs to be kept in the exponent of the coherence norm (7.11). The Gaussian average for the bath coupling agent B then yields $N_{\alpha\beta}(t) = |\langle e^{-id_x Bt/\hbar}\rangle|^2 = e^{-(t/\tau_{\rm dec}^x)^2}$ with the decoherence time

$$\tau_{\rm dec}^x = d_x\sqrt{\langle B^2\rangle}/\hbar = [\langle B^2\rangle j^2(\cos\phi_\alpha\sin\theta_\alpha - \cos\phi_\beta\sin\theta_\beta)^2]^{-1/2} \quad (7.12)$$

in analogy with $\tau_{\rm dec}^Q$ of Eq. (4.1).

The competing terms in the coherence norm can become effective only when the distance d_x vanishes (or is of subclassical magnitude). This happens in four distinct cases, three of which come with $\cos\phi_\alpha = \cos\phi_\beta, \sin\theta_\alpha = \sin\theta_\beta$: (i) $\beta = 1/\alpha^* \Leftrightarrow \{\phi_\alpha = \phi_\beta, \theta_\alpha = \pi - \theta_\beta\}$ such that the two points in the spherical phase space distinguished by α and β are reflections of one another in the equatorial plane $\theta = \pi/2$; (ii) $\beta = \alpha^* \Leftrightarrow \{\phi_\alpha = 2\pi - \phi_\beta, \theta_\alpha = \theta_\beta\}$ whereupon the two points are mutually opposite on the circular section of the spherical phase space with the plane $\theta = \theta_\alpha = \theta_\beta$; (iii) $\beta = 1/\alpha \Leftrightarrow \{\phi_\alpha = 2\pi - \phi_\beta, \theta_\alpha = \pi - \theta_\beta\}$ and then the two points are mutual antipodes. A fourth case, (iv), arises from $\cos\phi_\alpha = \sin\theta_\beta, \cos\phi_\beta = \sin\phi_\alpha$. At any rate, if $d_x = 0$ but d_y is of classical magnitude we may drop all terms of order t^3 in the coherence norm and get $N_{\alpha\beta}(t) = |\langle e^{-id_y B\Omega t^2/2\hbar}\rangle|^2 = e^{-(t/\tau_{\rm dec}^y)^4}$ with a decoherence time much larger than $\tau_{\rm dec}^x$,

$$\tau_{\rm dec}^y = (d_y^2\Omega^2\langle B^2\rangle/4\hbar^2)^{-1/4} = [\tfrac{1}{4}j^2\Omega^2\langle B^2\rangle(\sin\phi_\alpha\sin\theta_\alpha - \sin\phi_\beta\sin\theta_\beta)^2]^{-1/4}, \quad (7.13)$$

in analogy with $\tau_{\rm dec}^P$ of Eq. (4.1). Such "protection of coherence by symmetry" has been discussed previously in Ref. [25], in the context of golden-rule-type decoherence.

Specific to the angular-momentum algebra is the possibility that both d_x and d_y vanish but $d_z \neq 0$; this actually happens in case (i) above as well as in the subcase $\cos(\phi_\alpha \pm \theta_\alpha) = 0$ of case (iv). We then get the coherence norm, after doing a slightly different Gaussian integral, as $N_{\alpha\beta}(t) = |\langle e^{(-id_z B^2 t^3/12\hbar)}\rangle|^2 = e^{-(t/\tau_{\rm dec}^z)^6}$; the pertinent time scale is

$$\tau_{\rm dec}^z = (d_z^2\Omega^2\langle B^2\rangle^2/36\hbar^2)^{-1/6} = [\tfrac{1}{36}j^2\Omega^2\langle B^2\rangle^2(\cos\theta_\alpha - \cos\theta_\beta)^2]^{-1/6}. \quad (7.14)$$

We refrain from a detailed discussion of the various transitional regimes that may arise when d_x and d_y are not strictly zero but of subclassical magnitude, a discussion that would proceed much in analogy to the one in Sec. IV.

We would like to emphasize that the decoherence times $\tau_{\rm dec}^{x,y,z}$ all obey the power law (1.1), with $1/j$ as a dimensionless representative of Planck's constant; the exponents tend to order the decoherence times in magnitude as $\tau_{\rm dec}^x \ll \tau_{\rm dec}^y \ll \tau_{\rm dec}^z$; that ordering expresses decreasing power of the coupling agent J_x in decohering the respective superpositions. As usual, the coupling agent is most effective with respect to superpositions of states it "sees" as distinct in terms of its respective mean values; next come superpositions of states distinct by the mean values of J_y since the free motion generated by $H_{\rm sys} = \Omega J_z$ rotates J_x into J_y; finally, superpositions of states distinguished only by J_z undergo slowest decoherence since J_z enters the short-time expansion of the propagator $(\exp[-i\int_0^t dt' H_{\rm int}(\tilde{t}')/\hbar])_+$ only in the third-order term t^3, due to the commutator $[J_x, J_y] = i\hbar J_z$.

The partial immunity to decoherence of superpositions of angular-momentum coherent states expressed in the ordering just discussed may be broken by reducing the symmetry of the dynamics. One way of achieving that is to generalize the free motion as $H_{\rm sys} = \Omega_z J_z + \Omega_y J_y$; another is to allow for more reservoirs [9], e.g., according to $H_{\rm int} = J_x B_x + J_z B_z$.

Clearly, a larger set of observables like the generators of, say, SU(n) with $n = 3,4, \ldots$ would give rise to a yet richer decoherence scenario if $H_{\rm sys}$ and $H_{\rm int}$ both linearly involved different such generators.

VIII. CONCLUSIONS AND PERSPECTIVES

Quantum superpositions are fragile objects with respect to almost all environmental influences. In quantum mechanics, "openness" of a system is a more involved concept than in classical mechanics. Although good isolation from the environment may allow damping to be hardly noticeable for quantities with a classical limit, coherences in a quantum system may be subjected to rapid decay. The underlying time scale separation between $\tau_{\rm dec}$ and $\tau_{\rm diss}$ becomes ever more drastic as the distance between the superposed states grows. The decoherence time scale is shortened by a factor involving the distance, measured in units of a quantum reference "length" and thus enormously big when it comes to mesoscopic or even macroscopic scales. For more and more macroscopic superpositions, the decoherence time scale eventually becomes the smallest time scale involved. It follows that standard approaches to open-system dynamics, based on golden-rule-type assumptions fail to describe the rapid decay of such superpositions.

We have shown that a short-time expansion of the logarithm of the interaction propagator is the appropriate approach to decoherence in the limit of macroscopic superpositions. Remarkably, decoherence dynamics in this limit is largely independent of the nature of the system and the bath. No classical forces will have time to exert their influence on the very short decoherence time scale.

A remark about the use of a factorized initial condition is in order: Our results ignore the problem of how to actually create macroscopic superpositions. We assume they are given and determine the ensuing dynamics. Clearly, under laboratory conditions, it will take a certain time to prepare such an initial state, time enough for decoherence to possibly be effective. Initial system-environment correlations are thus an important ingredient for the discussion of the decay of macroscopic superpositions, a problem that will be addressed in future work.

How far the creation of superpositions can be stretched to the macroscopic is a question of central importance not only for quantum foundations but also for engineering in the fields of quantum information. Our results suggest that for these fascinating developments, environmental effects need to be described with new theoretical input. Well established methods of open-system dynamics, historically developed with an eye to near-equilibrium behavior become questionable for the nonequilibrium dynamics of coherent phenomena and may well turn out to be too limited to meet the quantum challenges of the future.

ACKNOWLEDGMENTS

We have enjoyed discussions with Wojciech Zurek, Dieter Forster, and Hans Mooij as well as the hospitality of the Institute for Theoretical Physics at the University of Santa Barbara during the workshop "Quantum Information: Entanglement, Decoherence, Chaos," where this project was completed. Support by the Sonderforschungsbereiche "Unordnung und Große Fluktuationen" and "Korrelierte Dynamik hochangeregter atomarer und molekularer Systeme" of the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

APPENDIX: SHORT-TIME EXPANSION

To derive the expansions (3.8) and (7.6) of the interaction-picture propagator we start with expanding the interaction Hamiltonian (3.7),

$$\tilde{H}_{\rm int}(t)=H_0+H_1t+H_2t^2/2+O(t^3). \tag{A1}$$

Separating the time-independent term H_0 we write the propagator

$$\tilde{U}(t)=\exp\left(-i\int_0^t dt'\,\tilde{H}_{\rm int}(t')/\hbar\right)=e^{-iH_0t/\hbar}U_1(t),$$

$$U_1(t)=\left(\exp\left[-i\int_0^t dt'\,H_1(t')/\hbar\right]\right)_+,$$

$$\begin{aligned}H_1(t)&=e^{iH_0t/\hbar}[H_1t+H_2t^2/2+O(t^3)]e^{-iH_0t/\hbar}\\&=H_1t+\frac{i}{\hbar}[H_0,H_1]t^2+H_2t^2/2+O(t^3).\end{aligned} \tag{A2}$$

Next, we split off the leading term H_1t in $H_1(t)$,

$$U_1(t)=e^{-iH_1t^2/2\hbar}U_2(t),$$

$$U_2(t)=\left(\exp\left[-i\int_0^t dt'\,H_2(t')/\hbar\right]\right)_+, \tag{A3}$$

$$\begin{aligned}H_2(t)&=e^{iH_1t^2/2\hbar}\left[\left(\frac{2i}{\hbar}[H_0,H_1]+H_2\right)t^2/2\right]e^{-iH_1t^2/2\hbar}\\&=\left(\frac{2i}{\hbar}[H_0,H_1]+H_2\right)t^2/2+O(t^3),\end{aligned}$$

$$\Rightarrow U_2(t)=e^{-i(2i[H_0,H_1]/\hbar+H_2)t^3/6+O(t^4)}.$$

When finally merging the three unitary factors $e^{-iH_0t/\hbar}U_1(t)U_2(t)$ into a single exponential we encounter a correction of the t^3 term due to

$$e^{-iH_0t/\hbar}e^{-iH_1t^2/2\hbar}=e^{(-(i/2\hbar)\{H_0t+H_1t^2-(i/4\hbar)[H_0,H_1]t^3+O(t^4)\})}, \tag{A4}$$

whereupon we get

$$\tilde{U}(t)=e^{(-(i/\hbar)\{H_0t+H_1t^2/2+(2H_2+(i/\hbar)[H_0,H_1])t^3/12+O(t^4)\})}. \tag{A5}$$

The foregoing general identity yields Eq. (3.8), since the interaction Hamiltonian (3.7) implies $H_0=QB$ and $H_1=M^{-1}PB+Q\dot{B}$. For the angular-momentum case we needed the third-order term in $\ln\tilde{U}(t)$ to reveal the quantum acceleration of decoherence for the most obstinate superpositions of coherent states.

[1] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I. O. Stamatescu, and H. D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996).

[2] W.H. Zurek, Phys. Today **44** (10), 36 (1991); W.H. Zurek, S. Habib, and J.P. Paz, Phys. Rev. Lett. **70**, 1187 (1993); J.P. Paz and W.H. Zurek, *ibid.* **82**, 5181 (1999).

[3] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, Nature (London) **401**, 680 (1999).

[4] M. Brune, S. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996); J.M. Raimond, M. Brune, and S. Haroche, *ibid.* **79**, 1964 (1997).

[5] C.J. Myatt, B.E. King, Q.A. Turchette, C.A. Sackett, D. Kielpinski, W.M. Itano, C. Monroe, and D.J. Wineland, Nature (London) **403**, 269 (2000); C. Monroe, D.M. Meekhof, B.E. King, and D.J. Wineland, Science **272**, 1131 (1996).

[6] C. van der Wal, A. ter Haar, F. Wilhelm, R. Schouten, T. Orlando, Seth Loyd, and J. Mooij, Science **290**, 773 (2000).

[7] J.R. Friedman, V. Patel, W. Chen, S.K. Tolpygo, and J.E. Lukens, Nature (London) **406**, 43 (2000).

[8] W.T. Strunz and F. Haake, following paper Phys. Rev. A **67**, 022102 (2003).

[9] D. Braun, F. Haake, and W.T. Strunz, Phys. Rev. Lett. **86**, 2913 (2001).

[10] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).

[11] E. Joos and H.D. Zeh, Z. Phys. B: Condens. Matter **59**, 223 (1984).

[12] O. Kübler and H.D. Zeh, Ann. Phys. (N.Y.) **76**, 405 (1973).

[13] P. Ullersma, Physica (Amsterdam) **32**, 27 (1966); **32**, 56 (1966); **32**, 90 (1966).

[14] F. Haake and R. Reibold, Phys. Rev. A **32**, 2462 (1985).

[15] W.G. Unruh and W.H. Zurek, Phys. Rev. D **40**, 1071 (1989); B.L. Hu, J.P. Paz, and Y. Zhang, *ibid.* **45**, 2843 (1992); J.P. Paz, S. Habib, and W.H. Zurek, *ibid.* **47**, 488 (1993).

[16] L.D. Romero and J.P. Paz, Phys. Rev. A **55**, 4070 (1997).

[17] G.W. Ford and R.F. O'Connell, Phys. Rev. D **64**, 105020 (2001).

[18] F. Haake and M. Żukowski, Phys. Rev. A **47**, 2506 (1993).

[19] D. Mozyrsky and V. Privman, Mod. Phys. Lett. B **14**, 303 (2000); V. Privman, e-print cond-mat/0203039.

[20] G.W. Ford, J.T. Lewis, and R.F. O'Connell, Phys. Rev. A **64**, 032101 (2001); G.W. Ford and R.F. O'Connell, Phys. Lett. A **286**, 87 (2001).

[21] E. Lutz, e-print cond-mat/0208503.

[22] R.J. Glauber, Phys. Rev. **131**, 2766 (1963); C.G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).

[23] Our power counting assumes $\langle B^2\rangle$ independent of $\hbar$; if $\langle B^2\rangle \propto \hbar^\gamma$, with $\gamma<2$, as, e.g., in the zero-temperature harmonic-oscillator case, we still retain $\tau_{\rm dec}\to 0$ for $\hbar\to 0$, see Ref. [8].

[24] F.T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972); R.J. Glauber and F. Haake, *ibid.* **13**, 357 (1976).

[25] D. Braun, P.A. Braun, and F. Haake, Opt. Commun. **179**, 195 (2000); in *Decoherence: Theoretical, Experimental, and Conceptual Problems*, edited by P. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.O. Stamatescu (Springer, Berlin, 2000).