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Dissipation-assisted quantum gates with cold trapped ions

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It is shown that a two-qubit phase gate and SWAP operation between ground states of cold trapped ions can be realized in one step by simultaneously applying two laser fields. Cooling *during* gate operations is possible without perturbing the computation. On the contrary, the cooling lasers even stabilize the desired time evolution of the system. This affords gate operation times of nearly the same order of magnitude as the inverse coupling constant of the ions to a common vibrational mode.

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For many years now ion trapping technology has been one of the standard techniques for investigating quantum phenomena with single particles. Paul traps or more complex structures have been used to create chains of several hundred ions [1]. Many schemes for implementing quantum gates using trapped ions have been proposed. Some of them require cooling into the vibrational ground state [2–5]. The feasibility of this has already been experimentally demonstrated [6]. Other schemes can even be implemented with "hot" ions [7] and have been applied to entangle up to four ions [8] and to observe violation of Bell's inequality with single atoms [9].

This paper proposes an alternative scheme for realizing a phase gate and SWAP operation with cold trapped ions in one step by simultaneously applying two laser fields. Compared with other schemes [2,4], the experimental effort for quantum computing can thus be greatly reduced. Ideally, the system remains during the whole gate operation in the ground state of a common vibrational mode. Thus cooling of this mode does not perturb the computation. On the contrary, it is shown that cooling can even improve the gate fidelity significantly. This affords gate durations of nearly the same order of magnitude as the inverse coupling constant of the ions to the common vibrational mode and there is no need to include a different ion species for sympathetic cooling [10] in the trap. In addition, the scheme only requires good control over one Rabi frequency. The size of the coupling strength of the ions to the vibrational mode does not enter the effective time evolution of the qubits.

Each qubit is obtained from the atomic ground states $|0\rangle$ and $|1\rangle$ of one ion, while the common vibrational mode is cooled down to its ground state $|0_{\rm vib}\rangle$. To establish coupling between qubits, we use a metastable state $|2\rangle$ and a strong laser field with Rabi frequency Ω_2 detuned by the phonon frequency ν, as shown in Fig. 1. The phase gate that adds a minus sign to the amplitude of the qubit state $|01\rangle$ then requires, in addition, only a weak laser pulse with the Rabi frequency Ω coupling resonantly to the 0-2 transition of ion 1. Let us denote the creation operator of a single phonon in the mode as $b^\dagger$ and introduce the coupling constant of the ions to the common vibrational mode as $g_2=\frac{1}{2}\eta\Omega_2$. (Here η is the Lamb-Dicke parameter characterizing the steepness of the trap.) The Hamiltonian of the system within the dipole and the rotating wave approximation and in the interaction picture with respect to the free Hamiltonian is then given by

$$H=\sum_{i=1}^{2} i\hbar g_2|1\rangle_i\langle 2|b^\dagger+\frac{1}{2}\hbar\Omega|0\rangle_1\langle 2|+\text{H.c.} \tag{1}$$

Here the Lamb-Dicke regime and the condition $g_2^2\ll\nu^2$ have been assumed, as in Ref. [2].

Let us first assume that the coupling constant g_2 is a few orders of magnitude larger than the Rabi frequency Ω. Then there are two different time scales in the system and the time evolution can be calculated to a very good approximation by adiabatic elimination. To do so, the amplitude of the state with n phonons in the vibrational mode and the ions in $|ij\rangle$ is denoted as c_{nij}. Only the coefficients of the qubit states, c_{000}, c_{001}, c_{010}, and c_{011}, and of the entangled state $|0a\rangle$ with $|a\rangle\equiv(|12\rangle-|21\rangle)/\sqrt{2}$ change slowly in time. Defining $|s\rangle\equiv(|12\rangle+|21\rangle)/\sqrt{2}$, their time evolution is given by

$$\dot{c}_{000}=-\frac{i}{2}\Omega\, c_{020},$$

$$\dot{c}_{001}=\frac{i}{2\sqrt{2}}\Omega\,(c_{0a}-c_{0s}),$$

$$\dot{c}_{0a}=\frac{i}{2\sqrt{2}}\Omega\, c_{001},$$

$$\dot{c}_{010}=\dot{c}_{011}=0. \tag{2}$$

Setting the derivatives of all other coefficients equal to zero yields $c_{020}=c_{0s}=0$. For $\Omega\ll g_2$, the time evolution of the system (2) can thus be summarized in the effective Hamiltonian

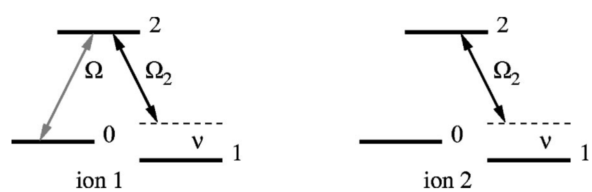

FIG. 1. Level scheme for implementation of a phase gate. A strong laser field with detuning ν establishes coupling between the ions via a common vibrational mode. In addition, a laser pulse individually addressing the 0-2 transition of ion 1 is required.

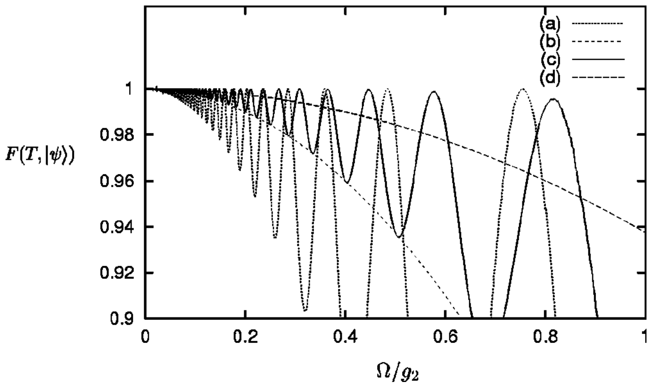

FIG. 2. The fidelity of a single phase gate as a function of Ω/g_2 for the initial qubit states $|00\rangle$ (a), (b) and $|01\rangle$ (c), (d). The curves (a), (c) result from an exact solution of the time evolution (1) while the curves (b), (d) result from Eq. (5).

$$H_{\rm eff}=-\frac{1}{2\sqrt{2}}\hbar\Omega\,[|001\rangle\langle 0a|+{\rm H.c.}]. \tag{3}$$

If the duration of the laser pulse equals $T=2\sqrt{2}\pi/\Omega$, then the resulting evolution is the desired phase gate. As the effective Hamiltonian $H_{\rm eff}$ and the gate operation time T are independent of the coupling constant g_2, the proposed scheme is widely protected against fluctuations of this system parameter.

Deviations from the time evolution (3) arise because population accumulates unintentionally in the states $|110\rangle$ and $|111\rangle$. In first order in Ω/g_2 one has

$$c_{110}=-\frac{i\Omega}{2g_2}c_{000},\quad c_{111}=-\frac{i\Omega}{4g_2}c_{001} \tag{4}$$

and the fidelity of the proposed phase gate equals

$$F(T,|\psi\rangle)=1-\frac{\Omega^2}{4g_2^2}\left[|c_{000}|^2+\frac{1}{4}|c_{001}|^2\right]. \tag{5}$$

Figure 2 compares this fidelity with the exact solution resulting from numerical integration of the time evolution (1) [11]. Good agreement is only found for $\Omega\ll g_2$. For somewhat larger Rabi frequencies, in general, fidelities worse than the result predicted by Eq. (5) are obtained due to nonadiabaticity. Thus the fidelity is above 99% for all initial states if $\Omega<0.1\,g_2$ is chosen. This corresponds to gate operation times $T>90/g_2$. In the following we aim at enlarging the parameter regime for which the fidelity is at least 99% [12].

A dominating error source in the scheme is heating. To avoid this, the gate should be performed fast. Therefore we assume in the following that Ω is of nearly the same size as the coupling constant g_2. As can be seen from above, increasing Ω leads to the population of unwanted states. To reduce the error rate of the scheme one could therefore measure the population of states with $n>0$ at the end of each gate. Under the condition that no population is found in these states, the system gets projected back onto the subspace with $n=0$ and the fidelity of the prepared state increases. In the case of detection of an error, the gate failed and the whole computation has to be repeated.

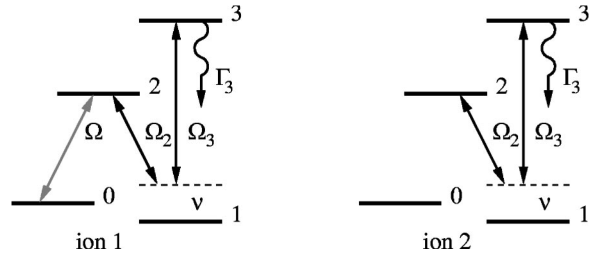

FIG. 3. Level scheme of a dissipation-assisted phase gate with cold trapped ions. In addition to the basic setup shown in Fig. 1, a strong laser field couples the state $|1\rangle$ with detuning ν to an auxiliary atomic state $|3\rangle$. Populating this level leads to the emission of photons with a rate Γ_3.

To realize such an error detection measurement one could use a laser field that couples the atomic ground state $|1\rangle$ with detuning ν to an auxiliary state $|3\rangle$ and another laser that excites the atomic 1-2 transition with the same detuning. To assure that populating level 3 leads to the emission of many photons, an even stronger laser should couple $|3\rangle$ to a rapidly decaying fourth level that decays into $|3\rangle$ with a high spontaneous decay rate. Gate failure leads thus to an effect that is known as a "macroscopic light period" [13], i.e., fast Rabi oscillations between level 3 and 4 accompanied by photon emission at a high rate. This can easily be detected and the whole computation can be repeated, if necessary.

However, such an error detection measurement would take much longer than the inverse of the coupling constants of the ions to the vibrational mode and much longer than T. Hence, error detection does not help to decrease the gate operation time for a given minimum fidelity F. To shorten the gate duration without reducing the fidelity more than predicted by Eq. (5), we propose that the desired time evolution (3) be stabilized during the gate performance by using dissipation. This is achieved by continuously applying the laser fields, proposed for the implementation of error detection, as shown in Fig. 3. For simplicity, the strong laser coupling to the fourth level and spontaneous decay from this level have been combined into a single decay rate assigned to the metastable state $|3\rangle$. Let us denote this spontaneous decay rate as Γ_3, while $g_3=\frac{1}{2}\eta\Omega_3$ is the coupling strength of the atomic 1-3 transition to the vibrational mode. In the following, $g_3\sim g_2$ and $10\,g_2<\Gamma_3<100\,g_2$ is assumed.

More generally, any process that indicates whether the phonon mode is excited or not can serve as an error detection measurement. This applies to ground-state cooling because populating the vibrational mode leads to the emission of photons at a high rate while no emission takes place if the ions are in the vibrational ground state [14]. Thus level 3 and the additional strong laser field in Fig. 3 could as well be replaced by the cooling laser setup. Indeed, continuous cooling can improve the fidelity of the performed gate operation. However, for simplicity, the continuous read out of the phonon mode is in the following modeled as shown in Fig. 3.

The basic mechanism of the improved scheme is that observing for emitted photons implements a (conditional) no-photon time evolution, thus resulting in continuous damping of the population in unwanted states. As long as the amplitudes c_{020} and c_{0s} are negligible, the time evolution of the other states resembles the desired phase gate, as can be seen from Eq. (2). In addition, we show that the population that now accumulates in the states $|110\rangle$ and $|111\rangle$ is about the same as predicted for an adiabatic process and the fidelity of the phase gate coincides with $F(T,|\psi\rangle)$ given in Eq. (5) to a very good approximation. The price one has to pay for this improvement of the precision of the gate is that photon emission might occur with a small probability. Then the gate operation would have failed.

To describe the time evolution of the system under the condition of no photon emission, we use in the following the Schrödinger equation with the conditional Hamiltonian $H_{\rm cond}$. As predicted by the quantum jump approach [15], the norm of a vector developing with this non-Hermitian Hamiltonian decreases in general with time and

$$P_0(T,|\psi\rangle)=\|U_{\rm cond}(T,0)|\psi\rangle\|^2 \tag{6}$$

is the probability of no photon emission in $(0,T)$ if $|\psi\rangle$ is the initial state of the system. For the level configuration shown in Fig. 3 the conditional Hamiltonian equals

$$H_{\rm cond}=\sum_{i=1}^{2}\sum_{j=2}^{3} i\hbar g_j|1\rangle_i\langle j|b^\dagger+\frac{1}{2}\hbar\Omega|0\rangle_1\langle 2|+\text{H.c.} -\frac{i}{2}\hbar\Gamma_3|3\rangle_i\langle 3|. \tag{7}$$

Because of the different time scales of the scheme, the no-photon time evolution of the system can again be calculated by adiabatic elimination. This yields the same effective Hamiltonian as in Eq. (3). In first order in Ω/g_2, population accumulates unintentionally in the states $|020\rangle$, $|110\rangle$, $|030\rangle$, $|0s\rangle$ $|111\rangle$, $|013\rangle$, and $|031\rangle$ and it is

$$(c_{0s},c_{111},c_{013},c_{031})=-\frac{i\Omega}{2g_2}\left(\frac{\sqrt{2}g_3^2}{g_2\Gamma_3},\frac{1}{2},\frac{g_3}{\Gamma_3},\frac{g_3}{\Gamma_3}\right)c_{001},$$

$$(c_{020},c_{110},c_{030})=-\frac{i\Omega}{g_2}\left(\frac{g_3^2}{g_2\Gamma_3},\frac{1}{2},-\frac{g_3}{\Gamma_3}\right)c_{000}. \tag{8}$$

To optimize the fidelity, Γ_3 should be much larger than g_3 so that all coefficients proportional to g_3/Γ_3 become negligible. In this case, $F(T,|\psi\rangle)$ becomes the fidelity calculated in Eq. (5) to a very good approximation.

That this is indeed the case is shown in Fig. 4 which results from a numerical solution of the time evolution (7). As expected, the dissipation channel continuously introduced in the system stabilizes the desired time evolution and corrects for errors resulting from the nonadiabaticity of the scheme if Ω becomes of about the same order of magnitude as g_2. The gate fidelity (5) applies now to a much wider parameter regime. Fidelities above 99% are obtained if $\Omega<0.18\,g_2$ is chosen. However, for too big spontaneous decay rates Γ_3, the damping of unwanted amplitudes becomes ineffective which is why $\Gamma_3<100\,g_2$ has been assumed.

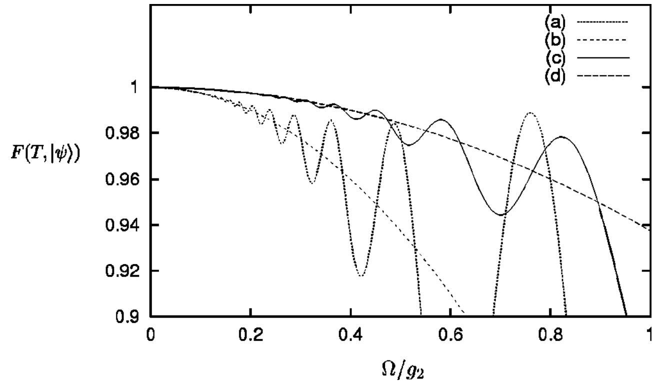

FIG. 4. Fidelity of a single phase gate under the condition of no photon emission as a function of Ω/g_2 for the initial qubit states $|\psi\rangle=|00\rangle$ (a), (b) and $|\psi\rangle=|01\rangle$ (c), (d). Deviations from the time evolution with $g_3=\Gamma_3=0$ are continuously damped away with $g_3=g_2$ and $\Gamma_3=20\,g_2$ (a), (c) and the fidelity is close to the theoretically predicted fidelity (5) assuming adiabaticity (b), (d).

Using the coefficients c_{0s} and c_{020} given in Eq. (8) and the differential equations that govern the time evolution of the qubit states and the entangled state $|0a\rangle$ [the same as Eq. (2)], the unnormalized state of the system at the end of the gate operation under the condition of no photon emission can be calculated up to first order in Ω/g_2. Its norm squared equals the gate success rate (6) and

$$P_0(T,|\psi\rangle)=1-2\sqrt{2}\pi\frac{\Omega g_3^2}{g_2^2\Gamma_3}\left[\left|c_{000}\right|^2+\frac{1}{4}\left|c_{001}\right|^2\right]. \tag{9}$$

This is in good agreement with the numerical results shown in Fig. 5. For example, for $\Omega<0.1\,g_2$ and $\Gamma_3=20\,g_2$ one has $P_0(T,\psi)>95\%$, independent of the initial state of the system. The probability for photon emission during the gate operation is of the order of Ω/g_2 and for $\Omega g_3^2/(g_2^2\Gamma_3)\ll 1$ close to unity. Gate failure might be a bit more likely than for

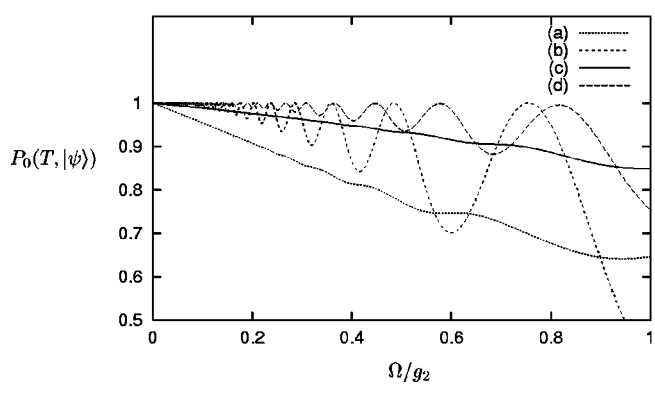

FIG. 5. Success rate of a single phase gate as a function of Ω/g_2 for the initial qubit states $|\psi\rangle=|00\rangle$ (a), (b) and $|\psi\rangle=|01\rangle$ (c), (d) for the case of continuous monitoring of the system (a), (c) and the case of only a single error detection measurement at the end of the gate (b), (d).

quantum error detection (see Fig. 5). However, no additional time is required which would increase the sensitivity of the scheme with respect to heating.

Another two-qubit gate that can easily be realized with the same experimental setup is SWAP operation. This gate exchanges the states of two qubits without the corresponding ions having to change their places physically. Compared with the above phase gate, its implementation does not require individual laser addressing. To realize SWAP operation, the laser field with Rabi frequency Ω should address both ions simultaneously. To improve the gate fidelity, the same ideas as described above can be used. By analogy with Eq. (7), the conditional Hamiltonian of the system is now given by

$$H_{\rm cond}=\sum_{i=1}^{2}\sum_{j=2}^{3} \mathrm{i}\hbar g_j|1\rangle_i\langle j|b^\dagger+\frac{1}{2}\hbar\Omega|0\rangle_i\langle 2|+\mathrm{H.c.} -\frac{i}{2}\hbar\Gamma_3|3\rangle_i\langle 3|. \tag{10}$$

Proceeding as above leads to the effective Hamiltonian

$$H_{\rm eff}=\frac{1}{2\sqrt{2}}\hbar\Omega[-|001\rangle\langle 0a|+|010\rangle\langle 0a|+\mathrm{H.c.}] \tag{11}$$

and if $T=2\pi/\Omega$, then the time evolution of the system exchanges the amplitudes of the qubit states $|01\rangle$ and $|10\rangle$ of the initial state. Deviations of the fidelity from unity are, for the same parameter regime as considered above, about the same size as for the proposed phase gate since implementation of the two gates is very similar. The same applies to the improvement of the gate precision that can be achieved with the help of dissipation and to the gate success rates.

Summarizing, we have shown that dissipation can be used to construct relatively simple and precise gates for quantum computing. As an example we discussed the implementation of a two-qubit phase gate and SWAP operation with cold trapped ions. The corresponding gate operation times are of nearly the same order of magnitude as the inverse coupling constant of the atoms to the vibrational mode. Here the parameter regime has been chosen such that auxiliary decay channels, resulting from continuous cooling of the ions, stabilize the desired adiabatic time evolution of the system and improve the fidelity of the gate operation significantly.

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