Spin-based quantum-information processing with semiconductor quantum dots and cavity QED

Mang Feng,[1,2] Irene D'Amico,[1,3] Paolo Zanardi,[1,3] and Fausto Rossi [1,3,4]
[1]*Institute for Scientific Interchange (ISI) Foundation, Villa Gualino, Viale Settimio Severo 65, I-10133 Torino, Italy*
[2]*Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China*
[3]*Istituto Nazionale per la Fisica della Materia (INFM), Torino, Italy*
[4]*Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi, 24-10129 Torino, Italy*
(Received 10 September 2002; published 30 January 2003)

A quantum-information-processing scheme is proposed with semiconductor quantum dots located in a high-Q single-mode QED cavity. The spin degrees of freedom of one excess conduction electron of the quantum dots are employed as qubits. Excitonic states, which can be produced ultrafast with optical operation, are used as auxiliary states in the realization of quantum gates. We show how properly tailored ultrafast laser pulses and Pauli-blocking effects can be used to achieve a universal encoded quantum computing.

DOI: 10.1103/PhysRevA.67.014306 PACS number(s): 03.67.−a, 32.80.Lg, 42.50.−p

Quantum computing [1] has drawn much attention over the past few years due to the speedup it promises in the treatment of classically hard computational problems, such as factoring [2] and database search [3]. Experiments have been done so far in systems of trapped ions, cavity atoms, and nuclear magnetic resonance, which demonstrated the feasibility of small-scale quantum computing [1]. However, it is generally believed that, in order to boost the current techniques to a large scale, e.g., thousands of qubits, quantum computer architecture should be based on solid-state hardware exploiting present nanotechnology.

The ideas we will discuss in this paper are within the framework of semiconductor quantum dot (QD) quantum-information processing (QIP), which has been intensively studied by envisaging two different kinds of qubit [4–11] based either on spin or on orbital degrees of freedom. In the latter approach, by using the electron-hole pair states, i.e., excitonic states, as qubits, one can have an ultrafast implementation of quantum computing with optical operations. The physical coupling between two (neighboring) qubits is provided by dipole-dipole interaction. Decoherence due to phonons is the main obstacle to the implementation of this QIP scheme [4,5]. In the former kind of proposals [6,7], the spin states of the only excess conduction electron of each QD are employed as qubits. The two-qubit gate is performed on two adjacent QDs exploiting the exchange interaction. This scheme benefits from a much longer decoherence time [8], but the implementation of quantum gates on spin states is slower than that on excitonic states. A common problem for the two schemes cited above is that only the nearest-neighbor qubits are coupled. So significant overhead is necessary for coupling two distant qubits. On the other hand, recent developments in semiconductor nanotechnology have shown that quantum dots located in the high-Q cavity provide alternative two-level systems in which the coupling between two distant QDs is mediated by the cavity mode [9,10]. So QIP can in principle be implemented in this kind of system.

In the present work, we will try to perform quantum computing with an array of GaAs-based QDs confined in a high-Q single-mode cavity, by *merging* the methods of spintronics (i.e., spin-based electronics), optoelectronics, and cavity QED. There is only one excess conduction electron in each QD. As the cavity mode acts as the "bus" qubit, two distant qubits can interact directly, which would simplify greatly the quantum computing manipulation. Our scheme is inspired by the idea proposed in Ref. [9]. In that paper, the spin states $m_x=\frac{1}{2}$ and $-\frac{1}{2}$ of the only excess conduction electron are employed as qubit states by applying an additional magnetic field along the x axis, and an effective long-range interaction is present between two distant quantum dot spins, mediated by the vacuum field of the cavity mode. In our scheme, instead, the magnetic field is applied along the z axis. By means of the auxiliary electron-hole pair states, i.e., excitonic states, we employ the spin states $m_z=\frac{1}{2}$ and $-\frac{1}{2}$ of the only conduction electron as qubit states $|1\rangle$ and $|0\rangle$, respectively. Since excitonic states are introduced as auxiliary states in our scheme, the quantum gates must be performed quickly because the decoherence time of the exciton is much shorter than that of spin states. Moreover, we should also pay attention to the cavity mode, whose decoherence time is of the same order as the excitonic one. Fortunately, as we will show below, both the exciton and the cavity mode are only virtually excited in our two-qubit gating. Therefore, we can achieve universal quantum computing based on a recently proposed model of encoded quantum computing (EQC), in which no single-qubit operation is needed [12]. The experimental feasibility of our scheme will also be discussed.

We assume that, besides being radiated by the cavity light, the QDs can be individually addressed by lasers. Due to the Pauli exclusion principle, the radiation of a σ^- polarized light with suitable energy on the QD will produce an exciton with state $|m_J^e=-\frac{1}{2},m_J^h=-\frac{1}{2}\rangle$ in the s shell only if the excess electron has a spin projection $\frac{1}{2}$ (in unit of $\hbar=1$). This Pauli-blocking mechanism has been observed experimentally in QDs [13,14] and can be used to produce entangled states. In Ref. [6], this Pauli blocking was used to yield a conditional phase gate, together with the Coulomb interaction between two neighboring QDs. In the single-particle picture, we define $|0\rangle_\nu=c^\dagger_{\nu,0,-1/2}|\mathrm{vac}\rangle$, $|1\rangle_\nu=c^\dagger_{\nu,0,1/2}|\mathrm{vac}\rangle$, and the excitonic state $|X^-\rangle_\nu=c^\dagger_{\nu,0,-1/2}c^\dagger_{\nu,0,1/2}d^\dagger_{\nu,0,-1/2}|\mathrm{vac}\rangle$, where $c^\dagger_{\nu,i,\sigma}$ ($d^\dagger_{\nu,i,\sigma}$) is the creation operator for a conduction-

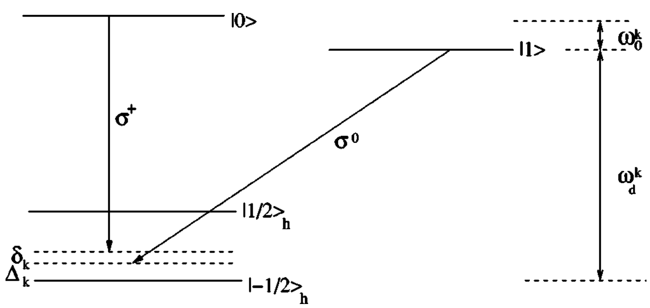

FIG. 1. Configuration of the quantum dot k in the near two-photon resonance process, where $|0\rangle=|-\frac{1}{2}\rangle_e$, $|1\rangle=|\frac{1}{2}\rangle_e$. ω_c and ω_L^k are frequencies of the cavity and the laser, respectively. The cavity light is σ^+ polarized and the laser beam is of linear polarization. Δ_k and δ_k are detunings defined in the text.

(valence-) band electron (hole) in the ith single-particle state of QD ν, with spin projection σ, and $|\mathrm{vac}\rangle$ accounts for the excitonic vacuum. The Hamiltonian of the QDs system is generally written as

$$H=\hbar\omega_c a^\dagger a+\sum_k H_k+\sum_k H_k^{\mathrm{int}}, \tag{1}$$

where ω_c is the cavity frequency, and $a^\dagger$ and a are creation and annihilation operators of the cavity. H_k is the single-QD Hamiltonian composed of H_k^0 and H_k^{co}, with $H_k^0=\sum_{i,\sigma=\pm 1/2}\epsilon_{i\sigma}^e c_{ki\sigma}^\dagger c_{ki\sigma}+\sum_{j,\sigma'=\pm 1/2}\epsilon_{j\sigma'}^h d_{kj\sigma'}^\dagger d_{kj\sigma'}$ describing the independent electrons and holes in the QDs, in which $\epsilon_{i\sigma}^e$ and $\epsilon_{j\sigma'}^h$ are, respectively, eigenenergies of an electron with spin projection σ in the ith single-particle state of QD k and a hole with spin projection σ' in the jth single-particle state of QD k. H_k^{co} is the electron-hole Coulomb interaction. $H_k^{\mathrm{int}}=H_k^L+H_k^c$ with H_k^L and H_k^c being the laser-QD interaction and cavity-QD interaction, respectively.

Two-qubit gate performance is the focus of various quantum computing proposals. As QDs are put into the cavity, the two spin states, employed as qubits, can be coupled via the cavity mode. Let us first consider the QD k, which is radiated by cavity light with σ^+ polarization and a laser beam with linear polarization, as shown in Fig. 1, where the energy difference between the conduction-band electron and the valence-band hole in the excitonic state $|X^-\rangle$ is $\hbar\omega_0^k$, the cavity frequency $\omega_c=\omega_d^k+\omega_0^k-\Delta_k-\delta_k$, and the laser frequency $\omega_L^k=\omega_d^k-\Delta_k$. Both δ_k and Δ_k are detunings, where δ_k can be written as $\omega_L^k+\omega_0^k-\omega_c$. If $\delta_k\to 0$ and Δ_k is large enough, then we have a typical resonance Raman transition between $|1\rangle$ and $|0\rangle$, whose interaction Hamiltonian in units of $\hbar=1$ is

$$H_{\mathrm{int}}=\frac{\Omega_k(t)}{2}[a\sigma_{01}^k e^{i\omega_L^k t}+\mathrm{H.c.}], \tag{2}$$

with $\Omega_k(t)=G_c G_{\mathrm{las}}^k(t)[1/\Delta_k+1/(\Delta_k+\delta_k)]$, G_c and $G_{\mathrm{las}}^k(t)$ being cavity-QD and laser-QD couplings, respectively, $\sigma_{01}^k=|1\rangle_k\langle 0|$, and no excitation in state $|-\frac{1}{2}\rangle_h$. From now on, we consider two identical QDs A and B, and set $\omega_d^A=\omega_d^B$ and $\omega_0^A=\omega_0^B=\omega_0$. If we set $\omega_L^A=\omega_L^B$, then we have $\Delta_A=\Delta_B$ and $\delta_A=\delta_B=\delta$. To suppress the cavity decay as much as we can, in the remainder of the paper we suppose that the cavity mode is in a vacuum state. By adjusting the cavity light and laser beam to make δ smaller than ω_0, but larger than both $\Omega_k(t)$ and the cavity linewidth, we will have a near two-photon resonance condition for two qubits, with the following effective Hamiltonian under the rotating-wave approximation [15]:

$$H_{\mathrm{eff}}=\frac{\tilde{\Omega}(t)}{2}(\sigma_{01}^A\sigma_{01}^{\dagger B}+\sigma_{01}^B\sigma_{01}^{\dagger A}), \tag{3}$$

where $\tilde{\Omega}(t)=\Omega_A(t)\Omega_B(t)/(2\delta)$. By means of Eq. (3), we may obtain the time evolution of the system,

$$|01\rangle_{AB}\to\cos\left[\frac{1}{2}\int_0^T\tilde{\Omega}(t)dt\right]|01\rangle_{AB}-i\sin\left[\frac{1}{2}\int_0^T\tilde{\Omega}(t)dt\right]|10\rangle_{AB} \tag{4}$$

and

$$|10\rangle_{AB}\to\cos\left[\frac{1}{2}\int_0^T\tilde{\Omega}(t)dt\right]|10\rangle_{AB}-i\sin\left[\frac{1}{2}\int_0^T\tilde{\Omega}(t)dt\right]|01\rangle_{AB}, \tag{5}$$

with $|\,\rangle_{AB}$ being the product of internal states of QDs A and B. It means that, *no matter whether QDs A and B are adjacent or not*, their internal states can be entangled by coupling to the same cavity mode, although the cavity mode is only virtually populated. Equation (3) is also called the *XY* model. Based on it, a universal EQC can be constructed by means of the nearest-neighbor and next-nearest-neighbor couplings [12]. The idea is to encode logical qubits in the state space of pairs of adjacent QDs: $|0_L\rangle_i:=|01\rangle_{i,i+1}$, $|1_L\rangle_i:=|10\rangle_{i,i+1}$. Given this encoding, Wu and Lidar showed in Ref. [12] how arbitrary qubits manipulations, i.e., universality, could be achieved just by time-dependent control of the *XY* Hamiltonian with nearest-neighbor and next-nearest-neighbor interactions. The necessity of the difficult single-qubits operation is relaxed in this way. This scheme fits in the general conceptual framework of *encoded universality* (see again [12] and references therein) in which one exploits the naturally available interactions in the system in such a way as to enact universality in a suitable subspace, i.e., the code, of the full physical state space. Notice that our scheme meets the requirement of EQC if Coulomb interaction can be neglected due to a large enough distance between two neighboring QDs. When EQC is performed in our scheme, however, the short decoherence time of the excitonic state must be seriously considered. Besides, the cavity decay also has a detrimental effect on our scheme, although the cavity mode is factorized from the computational subspace. This is because the fluctuation of the cavity mode would affect the ''bus'' role it plays and therefore affects the coupling of the two

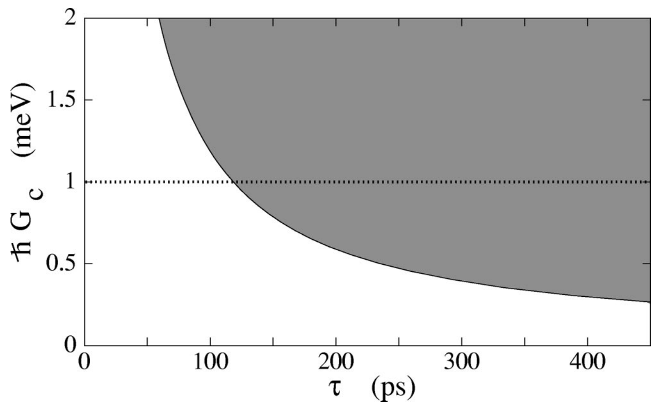

FIG. 2. Plot of the parameter space available (shaded area) for Eq. (3) in the implementation of $\int_0^T \tilde{\Omega}(t)dt = 2\pi$.

distant spin qubits. Consequently the implementation time of Eq. (3) is required to be shorter than the decoherence time of the cavity mode and the excitonic state. In order for Eq. (3) to work, the following adiabatic conditions must be fulfilled:

$$\Delta_k \gg \delta_k \gg \max\left(\frac{\Omega_k}{2}, \frac{1}{\tau}\right), \tag{6}$$

$$\Delta_k + \delta_k \gg \max\left(G_c, \frac{1}{\tau}\right), \tag{7}$$

$$\Delta_k \gg \max\left(G_{\rm las}^k, \frac{1}{\tau}\right), \tag{8}$$

where τ is the characteristic time associated with a Gaussian laser pulse of the form $G_{\rm las}^k(t) = G_{\rm las}^k \exp(-t^2/2\tau^2)$. By analyzing the whole parameter space while imposing (i) conditions (6)–(8) and (ii) $\int_0^T \tilde{\Omega}(t)dt = 2\pi$, we obtain that the points available to our computation in the parameter plane (G_c, τ) are those corresponding to the shaded region in Fig. 2. In particular, if we consider a coupling strength G_c of the order of 1 meV, we see that the characteristic time associated with the implementation of Eq. (3) will be of the order of 150 ps. Fortunately, in the implementation of Eq. (3), both the cavity mode and the exciton are only virtually excited. If we suppose that the probability of their excitations is less than 1% [9], the coherent implementation time of Eq. (3) can be at least 100 times longer than the decoherence time of the cavity and the exciton themselves, i.e., as long as 1 ns. This implies that Eq. (3) will work well.

We will now compare our scheme with previous ones involving spin qubits. The obvious difference of our scheme from Ref. [6] is that the two QDs interact via the cavity mode, instead of the Coulomb interaction. So the biexcitonic shift produced in Ref. [6] by the Coulomb interaction between two QDs is not necessary anymore and the external in-plane electric field applied to enlarge the biexcitonic shift can be removed. Moreover, the two-qubit gate implemented on two non-neighboring QDs makes our scheme of quantum computing more efficient than those proposals based on the nearest-neighbor coupling [6,7]. It is also the prerequisite of EQC performance in our scheme. Furthermore, our scheme is different from Ref. [9]. As the Pauli blocking is introduced, we employ the spin states of $m_z = \pm\frac{1}{2}$ to be qubits. Due to this fact, we can perform Eq. (3) without any external magnetic field [16].

For achieving the scheme experimentally, III–V semiconductor material is a suitable candidate because of the low spin decoherence rate of a conduction electron. Each QD must be initially cooled and prepared to contain one excess electron only. As far as we know, this has been experimentally achieved [17]. Moreover, individual addressing of QDs by a laser beam is necessary, which is a challenge for almost all proposals of semiconductor quantum computing. But in our scheme, since Coulomb interaction is not necessary, a possible way to avoid this difficulty is to enlarge the spacing between two adjacent QDs and to use near-field techniques. Furthermore, to perform quantum computing in parallel in cavity QED, it is generally required that the decoherence time of the cavity photon must be very long. However, this requirement can be removed because the cavity mode is only virtually populated throughout our scheme. For the measurement of the final result, we can adopt the method proposed in [9] by employing the Raman transition between $|1\rangle$ and $|0\rangle$. If the QD spin state is initially in $|1\rangle$, and the transition is induced between states $|1\rangle$ and $|0\rangle$, a photon would be created in the cavity and eventually leak out of the cavity. So by detecting the single-photon signal, we can judge whether the QD spin state is in $|1\rangle$ or $|0\rangle$.

The quantum gate based on our scheme can be carried out with high fidelity. To our knowledge, possible sources of error are as follows. (i) There is probably a small admixture of the heavy-hole component to the light-hole wave function, which yields the excitonic state $|m_J^e = -\frac{1}{2}, m_J^h = \frac{3}{2}\rangle$ in each cavity radiation with the σ^+ polarization when the spin projection of the only excess electron is $\frac{1}{2}$. To avoid this situation, we can adjust the strength of the magnetic field to make the radiated light nonresonant with the undesired transition. So it is expected that the probability of this error would be very small. (ii) When EQC is performed, Förster processes [18] happening in the nearest-neighbor coupled QDs could take place. However, due to both spin-selection rules and energy-conservation requirements, and in particular to the relatively large distance between two neighboring QDs required in our scheme in order to reduce Coulomb interaction, this kind of process would be largely inhibited.

In summary, we have reported an EQC scheme of quantum computing with semiconductor QDs in a high-Q single-mode cavity. The experimental feasibility of implementing our scheme has been discussed based on our numerical estimate for the adiabatic manipulation of a two-qubit gate. To minimize the gating time, a stronger coupling between the dots and the cavity is expected. In principle, our scheme can be generalized to the many-qubit case, in which quantum gates are performed in parallel. However, it is still experimentally challenging to place many QDs into a microcavity, although a scheme with the microdisk structure of tens of doped QDs was proposed [9]. Difficulties span from how to avoid the mismatch between the QD spacings and the standing-wave pattern of the cavity mode, to how to keep a large coupling between QDs and the cavity mode with the increase of QDs, and how to reduce decoherence when more QDs are located in the cavity. Moreover, we should note that to implement EQC, we need double qubits and more operations compared to nonencoded quantum computing schemes, which is also a challenge for current cavity QED experiment. For carrying out EQC in our system, we need the external magnetic field [16]. Actually, EQC is more useful for the systems in which the single-qubit operation is difficult to be performed. But for the system under consideration, we may easily perform the single-qubit rotation [9]. So the use of EQC is not the only choice for our system. We may alternatively implement usual quantum computing schemes with Eqs. (4) and (5), along with the single-qubit operation. The single-qubit operation can be done easily by two lasers with different polarizations and suitable frequencies [9] to meet the Raman-resonance condition between $|1\rangle$ and $|0\rangle$. Alternatively, we may rotate the spins by laser pulses, assisted by a magnetic field, as proposed recently in an ultrafast manipulation method [19]. Therefore, our approach resulting in Eq. (3) is useful not only for EQC but also for various nonencoded quantum computing schemes.

The work is supported by the European Commission through the Research Project SQID within the FET Program.

[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[2] P.W. Shor, in *Proceeding of the 35th Annual Symposium on Foundations of Computer Science* (IEEE Computer Society Press, Los Alamitos, CA, 1994), p. 116; A. Ekert and R. Jozsa, Rev. Mod. Phys. **68**, 733 (1996).
[3] L.K. Grover, Phys. Rev. Lett. **79**, 325 (1997); **79**, 4709 (1997); **80**, 4329 (1998).
[4] E. Biolatti, R. Lotti, P. Zanardi, and F. Rossi, Phys. Rev. Lett. **85**, 5647 (2000); E. Biolatti, I. D'Amico, P. Zanardi, and F. Rossi, Phys. Rev. B **65**, 075306 (2002).
[5] X. Li and Y. Arakawa, Phys. Rev. A **63**, 012302 (2000).
[6] E. Pazy, E. Biolatti, T. Calarco, I. D'Amico, P. Zanardi, F. Rossi, and P. Zoller, e-print cond-mat/0109337.
[7] D. Loss and D.P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); D.P. DiVicenzo, D. Bacon, J. Kempe, G. Burkard, and K.B. Whaley, Nature (London) **408**, 339 (2000).
[8] J.M. Kikkawa, I.P. Smorchkova, N. Samarth, and D.D. Awschalom, Science **277**, 1284 (1997).
[9] A. Imamog̃lu, D.D. Awschalom, G. Burkard, D.P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999).
[10] M.S. Sherwin, A. Imamog̃lu, and T. Montroy, Phys. Rev. A **60**, 3508 (1999).
[11] K.R. Brown, D.A. Lidar, and K.B. Whaley, Phys. Rev. A **65**, 012307 (2001).
[12] D.A. Lidar and L.-A. Wu, Phys. Rev. Lett. **88**, 017905 (2002).
[13] N.H. Bonadeo, G. Chen, D. Gammon, D.S. Katzer, D. Park, and D.G. Steel, Phys. Rev. Lett. **81**, 2759 (1998).
[14] G. Chen, N.H. Bonadeo, D.G. Steel, D. Gammon, D.S. Katzer, D. Park, and L.J. Sham, Science **289**, 1906 (2000).
[15] P.N. Butcher and D. Cotter, *The Elements of Nonlinear Optics* (Cambridge University Press, Cambridge, 1990); C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley-Interscience, New York, 1992).
[16] In the text above and in Fig. 1, the effect of the external magnetic field is taken into account, because the magnetic field along the z axis is needed to remove the degeneracy of the qubit states when EQC is carried out. But if we only consider the implementation of Eq. (3) or any other nonencoded quantum computing scheme, such a magnetic field may be unnecessary.
[17] J.J. Finley, M. Skalitz, M. Arzberger, A. Zrenner, G. Böhn, and G. Abstreiter, Appl. Phys. Lett. **73**, 2618 (1998).
[18] L. Quiroga and N.F. Johnson, Phys. Rev. Lett. **83**, 2270 (1999).
[19] J.A. Gupta, R. Knobel, N. Samarth, and D.D. Awschalom, Science **292**, 2458 (2001).