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Nonlinear phase mismatch and optimal input combination in atomic four-wave mixing in Bose-Einstein condensates

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This work treats four-wave mixing (4WM) in Bose-Einstein condensates (BEC), focusing on the nonlinear phase mismatch, maximum output, and optimal input combination. We show that the nonlinear phase mismatch decreases the 4WM efficiency. It was found that the 4WM efficiency depends on both the coupling coefficient (i.e., the product of the total number of atoms, the scattering length, and the overlap integral) and the ratios among the three initial input beams. The 4WM efficiency increases with the increase of the coupling coefficient when it is small, then saturates, and finally decreases at high coupling coefficient due to both pump depletion and phase-modulation effects. A maximum output efficiency of about 50% in our case is predicted. In order to get the maximum output, the two pump beams should have equal amplitude and the probe beam should be as small as possible. In addition, a large coupling coefficient ($>\pi/2$), which is determined by the ratio of the probe beam to the total input, is required. On the other hand, when the coupling coefficient is fixed, a maximum output for this case can be obtained by optimizing the input ratios among the three input beams. Other ratio combinations will decrease the 4WM efficiency.

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I. INTRODUCTION

A recent spectacular experiment [1] demonstrated four-wave mixing (4WM) of matter waves in Bose-Einstein condensates (BEC). This important experiment opens up a new area in the study of BEC, and has triggered a flurry of theoretical activity [2–13]. Very recently, Ketterle's group reported new experimental results on 4WM with BEC [14]. A gain as high as 20 in atomic 4WM was obtained in their experiment. As a matter of fact, Goldstein *et al.* [15] proposed the idea of phase conjugation of matter waves. However, theoretical investigation of the 4WM of matter waves is still one of the main areas of interest in nonlinear atomic optics [2–11,16–18].

Trippenbach *et al.* presented the 4WM theory of matter waves in BEC [16]. Then they [2] developed a three-dimensional quantum-mechanical description for 4WM in Bose-Einstein condensates using the time-dependent Gross-Pitaevskii equation. Goldstein *et al.* [3] demonstrated that a trapped condensate could be used as a phase-conjugate mirror for a weak atomic beam. They also presented an exact quantum-mechanical analysis of collinear 4WM in a multicomponent BEC consisting of sodium atoms in the $F=1$ ground state [4]. In addition, many others works, such as the instabilities, self-oscillations [5], and fluctuation [6] of the number of atoms in BEC wave packets in atomic 4WM, have been studied.

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Research on 4WM of matter waves benefits a lot from nonlinear optics because of the similarities between the equations that govern each system. As is well known, both equations include a self-phase-modulation term, three cross-phase-modulation terms, and a wave-mixing term [2–6,19,20]. In optical 4WM, it was shown in the undepleted pump case that the unequal pump intensities introduce a phase mismatch in the wave-mixing term [19]. Two of the authors also have shown that self- and cross-phase-modulation effects do lead to nonlinear phase mismatch and influence the 4WM efficiency and phase-conjugation fidelity in optical 4WM [20,21]. Therefore, we expect that nonlinear phase mismatch also plays an important role in atomic 4WM. Trippenbach *et al.* [2] mentioned that the self-and cross-phase-modulation effects will lower the 4WM output, but they did not focus on the consequent nonlinear phase mismatch and its influence on the 4WM efficiency.

In this work, the self- and cross-phase-modulation induced nonlinear phase mismatch in atomic 4WM and its influence on the 4WM efficiency are studied. The maximum output and the optimal initial inputs for atomic 4WM are investigated. Section II describes our physical model and gives out the necessary formalism. In Sec. III, several aspects of atomic 4WM will be discussed. In Sec. III A, we discuss the nonlinear phase mismatch and its influence on the 4WM efficiency. In Sec. III B, we discuss the coupling coefficient dependence of the 4WM efficiency. In Secs. III C and III D, the maximum 4WM output and optimal initial conditions for atomic 4WM are investigated. Finally, in Sec. IV we present a summary and conclusion. We believe this work is useful for directing the practical atomic 4WM experiment. The results presented here are also valid for optical 4WM.

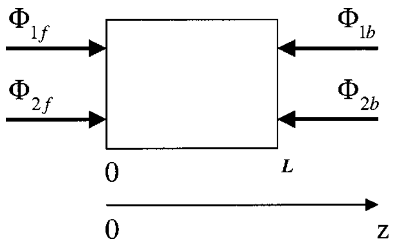

FIG. 1. Schematic of the system used for 4WM. The four matter waves interact with each other in the range of $0 \leqslant z \leqslant L$. The two forward (the two backward) propagating waves are distinguished by their internal state.

II. MODEL

In our atomic 4WM system that we consider here, there are four separate matter waves that can be distinguished by their propagation direction and internal state [5], as shown in Fig. 1. For simplicity, we assume the matter beams can be described by plane waves. Then the total wave function can be written in the form

$$\Phi = [\Phi_1(x,y)\Phi_{1f}(z,t) + \Phi_2(x,y)\Phi_{2f}(z,t)]e^{i(kz-\omega t)} + [\Phi_1(x,y)\Phi_{1b}(z,t) + \Phi_2(x,y)\Phi_{2b}(z,t)]e^{-t(kz+\omega t)}, \tag{1}$$

where k is the wave vector, $\hbar k^2/2m = \omega$, and m is the atomic mass. For convenience, we call Φ_{1f} the forward pump beam, Φ_{2b} the backward pump beam, Φ_{2f} the probe beam, and Φ_{1b} the signal beam.

The matter waves are governed by the coupled Gross-Pitaevskii nonlinear Schrödinger equations [5],

$$i\hbar\frac{\partial \Phi_1}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_1 + N\hbar(g_1|\Phi_1|^2 + g_x|\Phi_2|^2)\right]\Phi_1 \tag{2}$$

and similarly for Φ_2 with $1 \leftrightarrow 2$, where $V_i(x,y)$ is the transverse potential, N is the total number of atoms in the condensate, and g_i is the scattering strength that is related to the corresponding s-wave scattering length a_i by $g_i = 4\pi\hbar a_i/m$.

Substituting Eq. (1) into Eq. (2) and using the slowly varying envelope approximation, i.e., the spatial envelopes of these beams vary slowly over a de Broglie wavelength, one can obtain the following coupling equations for the steady-state case [5]:

$$\frac{\partial \Phi_{1f}}{\partial z} = i\frac{m}{\hbar k}\left[g_1|\Phi_{1f}|^2 + 2g_1|\Phi_{1b}|^2 + g_x|\Phi_{2f}|^2 + g_x|\Phi_{2b}|^2 + g_x\frac{\Phi_{2f}\Phi_{1b}\Phi_{2b}^*}{\Phi_{1f}}\right]\Phi_{1f}, \tag{3}$$

$$\frac{\partial \Phi_{1b}}{\partial z} = -i\frac{m}{\hbar k}\left[g_1|\Phi_{1b}|^2 + 2g_1|\Phi_{1f}|^2 + g_x|\Phi_{2f}|^2 + g_x|\Phi_{2b}|^2 + g_x\frac{\Phi_{2b}\Phi_{1f}\Phi_{2f}^*}{\Phi_{1b}}\right]\Phi_{1b}, \tag{4}$$

$$\frac{\partial \Phi_{2f}}{\partial z} = i\frac{m}{\hbar k}\left[g_2|\Phi_{2f}|^2 + 2g_2|\Phi_{2b}|^2 + g_x|\Phi_{1f}|^2 + g_x|\Phi_{1b}|^2 + g_x\frac{\Phi_{1f}\Phi_{2b}\Phi_{1b}^*}{\Phi_{2f}}\right]\Phi_{2f}, \tag{5}$$

$$\frac{\partial \Phi_{2b}}{\partial z} = -i\frac{m}{\hbar k}\left[g_2|\Phi_{2b}|^2 + 2g_2|\Phi_{2f}|^2 + g_x|\Phi_{1f}|^2 + g_x|\Phi_{1b}|^2 + g_x\frac{\Phi_{1b}\Phi_{2f}\Phi_{1f}^*}{\Phi_{2b}}\right]\Phi_{2b}, \tag{6}$$

with

$$g_j \rightarrow Ng_j\eta_j = Ng_j\int dx\,dy|\Phi_j(x,y)|^4, \quad j=1,2,$$

$$g_x \rightarrow Ng_x\eta_x = Ng_x\int dx\,dy|\Phi_1(x,y)|^2|\Phi_2(x,y)|^2. \tag{7}$$

The overlap integral η is obviously determined by the radius of the condensate [3].

The first term on the right-hand side of each of the coupling equations describes the self-phase-modulation effect. The last term is a source term of 4WM that either creates or destroys atoms in the wave packet being propagated. The other terms account for cross-phase modulation. The self- and cross-phase-modulation terms do not lead to particle (intensity in nonlinear optics) exchange between the four beams directly. However, as two of the authors demonstrated in nonlinear optics [20,21], these terms do change the phases of the four beams, leading to phase mismatch in the 4WM process and a decrease of the 4WM efficiency.

For the sake of clarity, we decompose the field into a real amplitude and phase by the definition of $\Phi_{j\mu} = \sqrt{\rho_{j\mu}}e^{i\varphi_{j\mu}}$. Substituting it in Eqs. (3)–(6), one can find out readily that the nonlinear phase mismatch of the 4WM terms is given by $\Delta\varphi(z, I_{1f}(0), I_{2f}(0), I_{2b}(L), g) = \varphi_{2f} + \varphi_{1b} - \varphi_{1f} - \varphi_{2b}$, where L is the length of the interaction region as given in Fig. 1.

For simplicity, we investigate the case of $g_1 = g_2 = g_x = g$ so that the coupling equations become

$$\frac{d\rho_{1f}}{dz} = \frac{d\rho_{1b}}{dz} = -\frac{d\rho_{2f}}{dz} = -\frac{d\rho_{2b}}{dz} = \frac{2mg}{\hbar k}\sqrt{\rho_{1f}\rho_{2f}\rho_{1b}\rho_{2b}}\sin(\Delta\varphi), \tag{8}$$

$$\frac{d\Delta\varphi}{dz}=-\frac{mg}{\hbar k}\Bigg[\rho_{2b}-\rho_{1f}+\rho_{2f}-\rho_{1b}+\Bigg(\sqrt{\frac{\rho_{1f}\rho_{1b}\rho_{2b}}{\rho_{2f}}}-\sqrt{\frac{\rho_{2f}\rho_{1b}\rho_{2b}}{\rho_{1f}}}+\sqrt{\frac{\rho_{1f}\rho_{2f}\rho_{1b}}{\rho_{2b}}}-\sqrt{\frac{\rho_{1f}\rho_{2f}\rho_{2b}}{\rho_{1b}}}\Bigg)\cos(\Delta\varphi)\Bigg]. \tag{9}$$

Equation (8) indicates clearly that the nonlinear phase mismatch $\Delta\varphi$ does affect the 4WM process. The phase-matching condition can be satisfied only if the nonlinear phase mismatch is negligible, for example $\Delta\varphi\approx-\pi/2$, as we pointed out in Ref. [20]. Combining Eqs. (8) and (9), one can get

$$\cos(\Delta\varphi)=\frac{\rho_{1f}(z)\rho_{1b}(z)+\rho_{2f}(z)\rho_{2b}(z)-\rho_{2b}(L)\rho_{2f}(L)}{2\sqrt{\rho_{1f}(z)\rho_{1b}(z)\rho_{2f}(z)\rho_{2b}(z)}}. \tag{10}$$

Substituting Eq. (10) into Eq. (8), one can get the analytical solutions for the atomic density as [5,22]

$$\rho_{1b}(z)=\frac{4\rho_{1f}(0)\rho_{2b}(L)[\rho_{2f}(0)+\rho_{1b}(0)]}{\rho^2-4\rho_{2b}(0)[\rho_{2f}(0)+\rho_{1b}(0)]\cos^2(\zeta_{\rm eff})}\times\sin^2\left(\zeta_{\rm eff}\frac{L-z}{L}\right), \tag{11}$$

$$\rho_{1f}(z)=\rho_{1f}(0)-\rho_{1b}(0)+\rho_{1b}(z), \tag{12}$$

$$\rho_{2f}(z)=\rho_{2f}(0)+\rho_{1b}(0)-\rho_{1b}(z), \tag{13}$$

$$\rho_{2b}(z)=\rho_{2b}(L)-\rho_{1b}(z), \tag{14}$$

with

$$\rho=\rho_{1f}(0)+\rho_{2f}(0)+\rho_{2b}(L), \tag{15}$$

$$\zeta_{\rm eff}=\zeta\sqrt{1-4\frac{\rho_{2b}(L)}{\rho}\left(\frac{\rho_{2f}(0)}{\rho}+\frac{\rho_{1b}(0)}{\rho}\right)}, \tag{16}$$

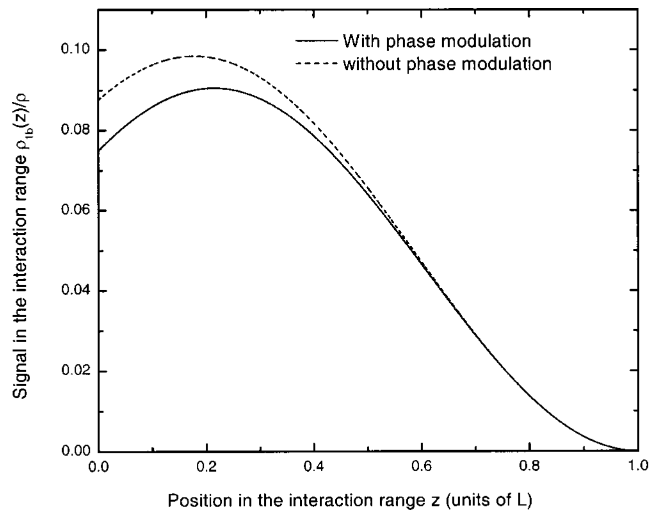

FIG. 2. The calculated 4WM signals in the interaction range with and without considering the phase-modulation effect, where the coupling coefficient is $\zeta=2.226$. The three input beams satisfy $\rho_{1f}^{(0)}=\rho_{2f}^{(0)}=5\rho_{1b}^{(L)}$.

$$\zeta=\frac{mg\rho}{2\hbar k}L=\frac{2\pi}{k}aN\eta, \tag{17}$$

where ζ is called the coupling strength in nonlinear optics [19]. However, in atomic 4WM, ζ becomes the coupling coefficient except a factor of 4 since $\rho L=1$ (the normalization condition) [3], therefore we will call it a coupling coefficient in this paper to emphasize the difference from nonlinear optics. In order to get these solutions, the boundary condition $\rho_{1b}(L)=0$ was used.

III. DISCUSSION

A. Influence of nonlinear phase mismatch

One should notice that $\rho_{1b}\to 0$ when $z\to L$, therefore Eq. (10) becomes

$$\cos[\Delta\varphi(z\to L)]=\lim_{\rho_{1b}(z\to L)\to 0}\frac{\rho_{1f}(z\to L)\rho_{1b}(z\to L)}{2\sqrt{\rho_{1f}(z\to L)\rho_{1b}(z\to L)\rho_{2f}(z\to L)\rho_{2b}(z\to L)}}=0.$$

From a physical consideration, $\partial\rho_{1b}/\partial z$ must be negative at $z=L$, as can be obtained from Eq. (8), namely, $\sin[\Delta\varphi(L)]<0$. Therefore, the phase of the signal beam at $z=L$ is given by $\varphi_{1b}(L)=-\pi/2+\varphi_{1f}(L)+\varphi_{2b}(L)-\varphi_{2f}(L)$. This relationship is also true at other z positions if the self- and cross-phase-modulation terms are neglected in the coupling equations [20]. This is the well-known result in nonlinear optics. It indicates that the phase-matching condition is satisfied completely [19]. However, it is not reasonable to neglect these terms, since the phase-modulation effect and the wave mixing have the same physical origin as is seen in the coupling equations. On the other hand, two of the authors argued that the phase-modulation effect on the 4WM efficiency has to be considered when the nonlinear phase mismatch is large in nonlinear optics [20]. In atomic 4WM, the same result, i.e., that the phase-modulation effect decreases the 4WM signal, is expected.

Figure 2 shows the calculated signal amplitude divided by

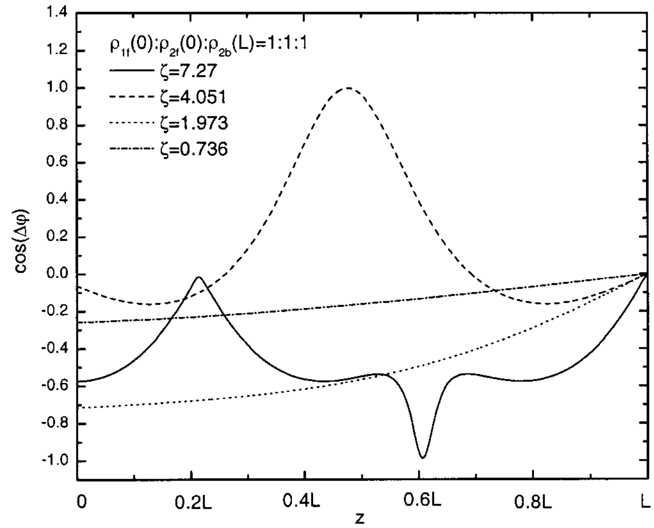

FIG. 3. Cosine values of the nonlinear phase mismatches in the interaction range for a different coupling coefficient, where the three input beams have equal amplitude.

the total input as a function of the position in the interaction range. The solid line represents the result with the phase-modulation effect. The dashed line indicates the result without including the phase-modulation effect. The parameters used in the calculation are given in the figure. Clearly, with the propagation of the signal beam in the interaction range, the nonlinear phase mismatch decreases the 4WM efficiency, thus the signal calculated with the phase-modulation effect becomes smaller than that without including the phase-modulation effect. Obviously, the influence of the nonlinear phase mismatch on the 4WM efficiency cannot be neglected. Therefore, all terms will be included in the following calculations.

It is worth noticing that the nonlinear phase mismatch varies with both the position z in the interaction range and the initial input atomic densities (input intensities in nonlinear optics). Consequently, the 4WM efficiency changes according to the position z and the initial input atomic densities.

Figure 3 shows the cosines of the nonlinear phase mismatches versus the position z in the interaction range. Here we present the results for cases with different nonlinear coupling coefficient ζ but equal initial amplitudes for the three input beams, i.e., $\rho_{1f}(0):\rho_{2f}(0):\rho_{2b}(L)=1:1:1$. The initial values at $z=L$ of the cosine functions are 0, as we mentioned above. With the propagation of the signal beam, they deviate from 0 due to phase-modulation effects. We notice that the wave-mixing term as well as the self- and cross-phase-modulation terms contribute to the phase changes of the four beams. In addition, the phases are coupled with the real amplitudes. Thus, a complex relationship exists between the nonlinear phase mismatch and the position z, as shown in Fig. 3. However, the amplitudes vary in a simple way with the position z in the interaction range. As is shown in Fig. 4, the signal grows until a maximum is obtained, then it decreases to zero. After that, the process will be repeated. The larger the coupling coefficient, the shorter the interaction

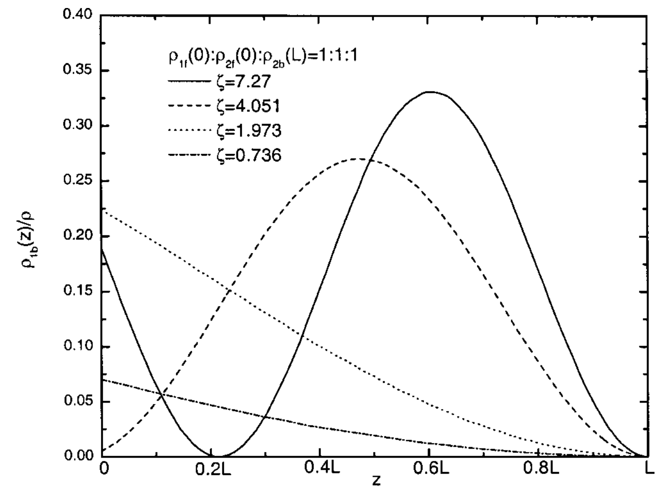

FIG. 4. Distribution of the 4WM signals in the interaction range for a different coupling coefficient, where the three input beams have equal amplitude.

length needed to arrive at the maximum. Of course, the maximum cannot be arrived at in the whole interaction range when the coupling coefficient is small. The calculations indicate that the maximum signal arrives when the absolute value of the cosine function of the nonlinear phase mismatch is the maximum.

We also note that the nonlinear phase mismatch and the corresponding 4WM signal not only depend on the coupling coefficient, but also on the ratios of the three initial input atomic densities. For convenience, we define two ratios as $r_b=\rho_{2f}(0)/\rho$ and $r_p=\rho_{1f}(0)/[\rho-\rho_{2f}(0)]$. Figure 5 shows the nonlinear phase mismatch in the interaction range when the coupling coefficient is 2.111. The associated 4WM signals in the interaction range are shown in Fig. 6. The parameters used in the calculations are shown in the figures. Obviously, the nonlinear phase mismatch and the 4WM output change significantly with the ratios r_b and r_p, even though the total input is fixed (the coupling coefficient is fixed).

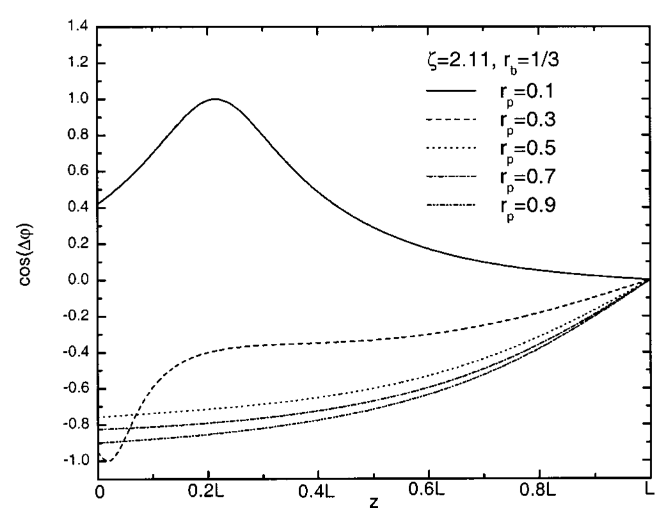

FIG. 5. Cosine values of the nonlinear phase mismatches in the interaction range for a different input combination when the coupling coefficient is fixed.

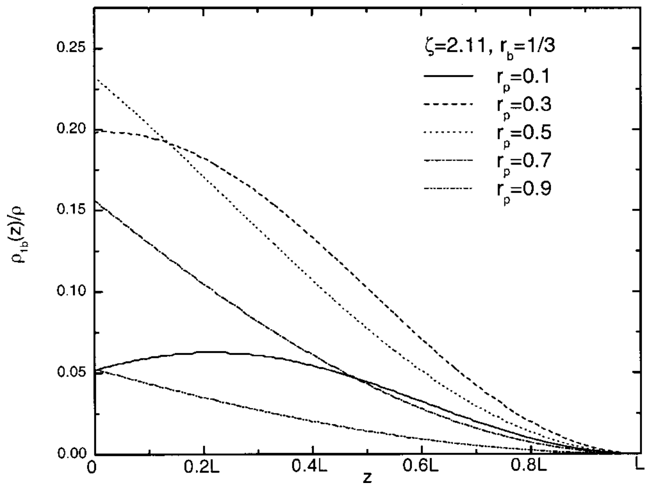

FIG. 6. The 4WM signals in the interaction range for different input combinations when the coupling coefficient is fixed.

Therefore, it is very necessary to determine the ratios r_b and r_p to get the maximum output in the 4WM experiments for a fixed coupling coefficient case. This will be discussed further in Sec. III C.

B. Atomic number dependence

Figure 7 shows the coupling coefficient dependence of the 4WM efficiency. In this calculation, we assume $\rho_{1f}(0){:}\rho_{2\,f}(0){:}\rho_{2b}(L)=1{:}1{:}1$. As we expected, when the coupling coefficient is small, the signal increases with the increase of the coupling coefficient. With the increase of the coupling coefficient, the 4WM efficiency will be saturated due to both the pump depletion and the nonlinear phase-mismatch effects. When the coupling coefficient is very high, the 4WM efficiency begins to decrease. If one increases the coupling coefficient, the bistable and multistable results will be obtained [5,22]. Because the coupling coefficient is the product of the atomic number, the scattering length, and the overlap integral, when the latter two are constants, Fig. 7 just shows the 4WM efficiency versus the atomic number. Of course, Fig. 7 shows that when the total atomic number and the scattering length (or the overlap integral) are constant, the 4WM efficiency serves as a function of the overlap integral (or scattering length).

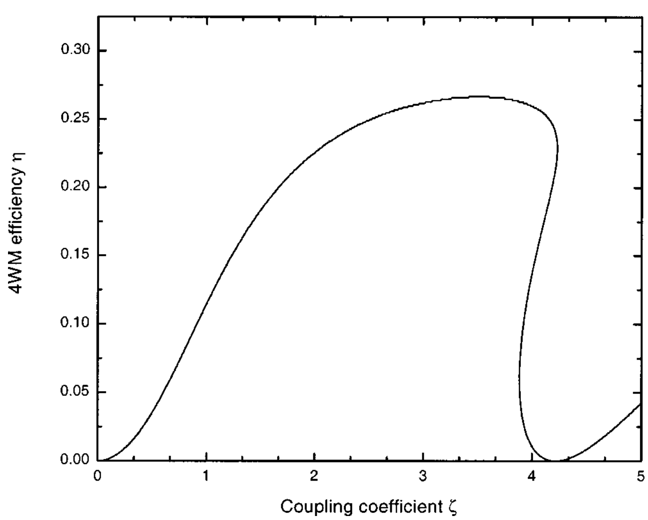

FIG. 7. The 4WM efficiency vs the coupling coefficient for the case in which the three input beams have equal amplitude.

C. Maximum output

As shown in Figs. 4 and 6, the signal in the interaction range not only increases but also decreases with the interaction length. Therefore, it is useful and possible to set the initial input beam amplitudes and adjust the coupling coefficient (change the total atomic number, the scattering length, or the overlap integral) to get the maximum signal output in atomic 4WM. We discuss the conditions for maximum signal output from the 4WM in this subsection. Then, in the following subsection, we give out the optimal conditions for atomic 4WM when the coupling coefficient is fixed.

The output signal can be obtained from Eq. (11) by setting $z=0$. It reads

$$\rho_{1b}(0)=\frac{\beta-\sqrt{\beta^2-16\rho_{1f}(0)\rho_{2\,f}(0)\rho_{2b}^2(L)\sin^2(2\zeta_{\rm eff})}}{8\rho_{2\beta}(L)\cos^2(\zeta_{\rm eff})} \tag{18}$$

with

$$\beta=\rho^2-4\rho_{2\,f}(0)\rho_{2b}(L)+4[\rho_{2\,f}(0)-\rho_{1f}(0)]\rho_{2b}(L)\sin^2(\zeta_{\rm eff}). \tag{19}$$

One can rewrite Eq. (18) by defining the 4WM efficiency

$$\eta=\frac{\rho_{1b}(0)}{\rho}=\frac{\xi-\sqrt{\xi^2-16r_br_p(1-r_b)^3(1-r_p)^2\sin^2(2\zeta_{\rm eff})}}{8(1-r_b)(1-r_p)\cos(\zeta_{\rm eff})} \tag{20}$$

with

$$\xi=1-4(1-r_b)(1-r_p)[r_b\cos^2(\zeta_{\rm eff})+r_p(1-r_b)\sin^2(\zeta_{\rm eff})]. \tag{21}$$

One finds from Eqs. (20) and (21) that the 4WM efficiency is dependent on three parameters: r_b, r_p, and $\zeta_{\rm eff}$. One should notice that the last parameter depends on the other two and the coupling coefficient ζ through Eq. (16). Therefore, the 4WM efficiency is determined by both the coupling coefficient ζ and the ratios of the three input beams to the total input.

In the case in which r_b and r_p are fixed, when the condition:

$$\sin^2(\zeta_{\rm eff})=1 \tag{22}$$

is satisfied, the maximum of the 4WM efficiency is obtained as

$$\eta_{\rm fixed\ \ ratios}^{\max}=\frac{4r_br_p(1-r_b)^2(1-r_p)}{(1-r_b)^2(2r_p-1)^2+2r_b-r_b^2}. \tag{23}$$

This equation indicates that the maximum 4WM efficiency is determined only by the ratios r_b and r_p. Obviously, when r_b is fixed, it is necessary that the two pump waves have an equal initial amplitude, i.e., $r_p=0.5$, to obtain the maximum output. It is readily apparent in Eq. (23) that the theoretical maximum output of the 4WM can be obtained when $r_b \to 0$. That is,

$$\eta^{\max} = \frac{(1-r_b)^2}{2-r_b} \overset{r_b \to 0}{\approx} \frac{1}{2}. \tag{24}$$

This result, which gives out the maximum 4WM efficiency of 50%, is completely different from the result for the undepleted pump approximation case [3,19].

In order to get the 4WM described by Eqs. (23) and (24), one has to adjust the coupling coefficient to satisfy the condition $\sin^2(\zeta_{\mathrm{eff}})=1$. For a certain allocation of the three input beams, i.e., r_b and r_p are constants, the associated coupling coefficient may be obtained by a combination of Eqs. (16) and (22),

$$\zeta^{\mathrm{optimal}}_{\mathrm{fixed\ ratios}} = \frac{\pi}{2}\sqrt{\frac{(1-r_b)^2(2r_p-1)^2+2r_b-r_b^2}{(1-r_b)^2(2r_p-1)^2+2r_b-r_b^2-4r_b(1-r_b)(1-r_p)}}. \tag{25}$$

Clearly, the coupling coefficient $\zeta^{\mathrm{optimal}}_{\mathrm{fixed\ ratios}}$ that is needed for the maximum output is determined only by the ratios r_b and r_p. On the other hand, this parameter relates with the total input, the scattering length, and the overlap integral by

$$\zeta^{\mathrm{optimal}}_{\mathrm{fixed\ ratios}} = \frac{2\pi}{k} aN\eta. \tag{26}$$

Therefore, one can satisfy the condition of Eq. (25) by changing the total input, the scattering length, and/or the overlap integral. That means the maximum 4WM output for a fixed allocation of the three input beams can be obtained by adjusting these parameters. However, if the coupling coefficient given by Eq. (25) is exceeded, the output signal will decrease, instead of increase, due to the phase-modulation effect. Of course, the signal will increase again, if the change of the coupling coefficient is large enough, because of the sine function in Eq. (20). However, the maximum output will not increase, as is shown in Eq. (23). Therefore, Eq. (25) gives the smallest coupling coefficient to get the maximum output for fixed r_b and r_p.

Figure 8 shows the 4WM efficiency as a function of r_b and r_p. As we mentioned above, the maximum 4WM efficiency of about 50% can be obtained when $r_p = \frac{1}{2}$ and $r_b \to 0$; this is shown clearly in Fig. 8. In order to reach this theoretical maximum 4WM efficiency, one has to use a very small probe and a large coupling coefficient, as shown in Fig. 9. The latter condition means that a large total input and/or a large overlap integral are required if the scattering length is fixed.

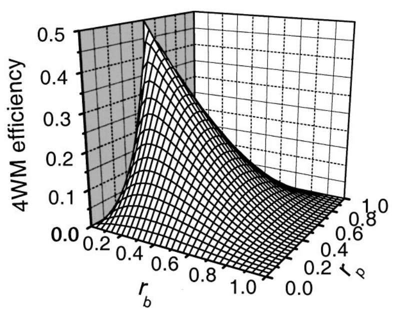

FIG. 8. Maximum 4WM efficiency vs the ratios r_b and r_p.

When the ratio of the probe to the total r_b is fixed, a maximum output can be obtained by setting $r_p = \frac{1}{2}$. The deviation of r_p from 0.5 decreases the 4WM output. When r_p is fixed, one can also find an r_b to get the maximum output.

The required coupling coefficient for obtaining the maximum output for the fixed r_b and r_p case is shown in Fig. 9. One can find that a large coupling coefficient ($>\pi/2$) is necessary for the maximum 4WM efficiency. However, this condition can be satisfied easily. For example, the coupling coefficient must be 6.8 in order to get the maximum output when $r_b = 0.1$ and $r_p = 0.5$. The typical value of the scattering length is 2.75 nm, the wave vector is $k \approx 10^7$, and the overlap integral is about 10^{10} m^2 [3]. Using these parameters, one can get the total number of trapped atoms of 4×10^5, which is smaller than that used in the first atomic 4WM experiment [1].

D. Optimal initial inputs

As we have seen in Sec. III C, the maximum output can be obtained when the pump inputs are equal and the probe is very small. This requires a very large coupling coefficient, as we mentioned above. Because the coupling coefficient depends on the total atomic number, the scattering length, and the overlap integral, in principle one can adjust these parameters to satisfy the maximum output conditions. However, in some practical cases, only a small coupling coefficient is valid. Therefore, it is necessary and important to adjust the

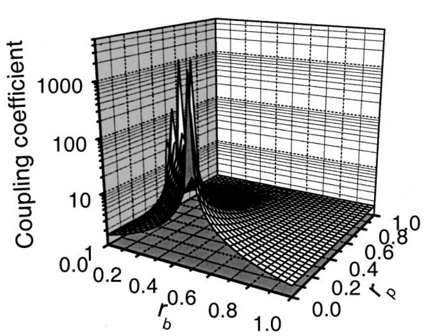

FIG. 9. Optimal coupling coefficient to get the maximum 4WM efficiency at different input beam ratios r_b and r_p.

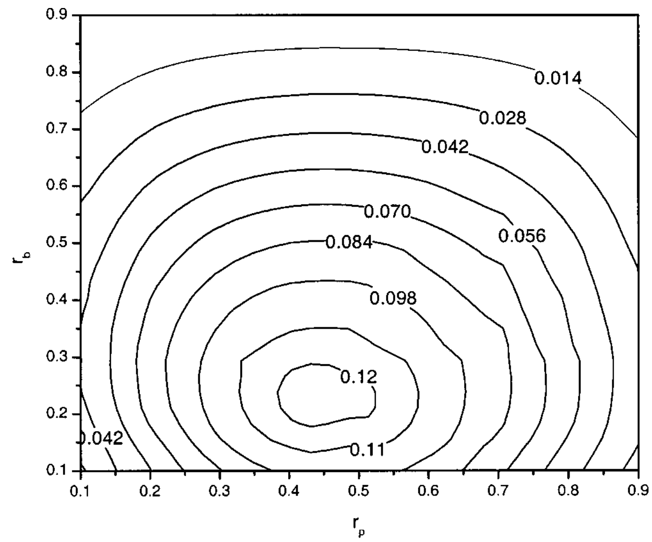

FIG. 10. Maximum 4WM efficiency vs the input beam ratios r_b and r_p, where the coupling coefficient is $\zeta = 1$.

three input beam amplitudes (r_b and r_p) to get a maximum output for the fixed coupling coefficient case.

Figures 5 and 6 show that the nonlinear phase mismatch and the 4WM output depend on the ratios r_b and r_p. Therefore, it is possible to find the optimal initial inputs to get the maximum output for a fixed coupling coefficient system. Unfortunately, we cannot get an analytical expression for the output signal as a simple function of the input beam ratios. Thus, one has to do a lot of calculations in order to find the optimal inputs. Figure 10 gives an example for the case of $\zeta = 1$. From the calculation, we find that a maximum 4WM efficiency of 12.36% can be obtained when $\rho_{2f}(0) = 0.23\rho$, $\rho_{2b}(L) = 0.3465\rho$, and $\rho_{1f}(0) = 0.4235\rho$. The 4WM efficiency for other input combinations can be obtained easily from the contours in Fig. 10. If the loss of atoms is taken into account, the 4WM efficiency will decrease [7].

IV. CONCLUSION

This work presented a detailed discussion of the nonlinear phase mismatch, maximum output, and optimal initial inputs for atomic 4WM in Bose-Einstein condensation. It was found that the nonlinear phase mismatch decreases the 4WM efficiency. In addition, the 4WM efficiency is determined by both the coupling coefficient and the ratios r_b and r_p. Depending on the magnitude of the coupling coefficient, the signal of the 4WM will increase, saturate, and decrease with the increase of the coupling coefficient when the ratios r_b and r_p are constant. On the other hand, when the coupling coefficient is fixed, the 4WM efficiency depends on the ratios r_b and r_p, thus one can adjust these two parameters to get a maximum 4WM output.

The maximum 4WM efficiency of about 50% can be obtained when the probe beam is very small and the two pump beams have equal amplitude. However, a large coupling coefficient is required to get this maximum output.

The general case is that the coupling coefficient is fixed or in a certain range. Therefore, we also discussed the optimal initial inputs for the case in which the coupling coefficient is constant. The results show that the maximum output for this case can be obtained by properly distributing the three input beams. The maximum 4WM efficiency of 12.36% was obtained for $\zeta = 1$.

The results given in this work should help experimentalists to obtain a perfect 4WM experimental setup to get the maximum output.

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