Limiting temperature of sympathetically cooled ions in a radio-frequency trap

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The limiting temperature achieved by sympathetic cooling in an rf trap is calculated with a theoretical model in which no fitting parameters are used. The calculated result agrees well with observation. The dependence of the temperature on trapping parameters and ion mass is also analyzed. The results can be used for designing an rf trap system.

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I. INTRODUCTION

A radio-frequency trap which provides an ideally isolated ion system is used for application to frequency standards [1,2], high-resolution spectroscopy [3–5], and fundamental experiments such as observation of quantum jumps [6–9]. For these applications, laser cooling of ions is required [10,11]. Sympathetic cooling is employed for ions that cannot be directly cooled by lasers and has been demonstrated in a Penning trap [12–16] and in a linear rf trap [17–19]. Sympathetic cooling of molecular ions in a Penning trap [15] and crystallization of sympathetically cooled atomic ions in a linear rf trap [17] have been observed. Application of sympathetically cooled ions to quantum logic was also proposed [20]. Sympathetic cooling in an rf trap, in which no magnetic field is used, is preferred for application to frequency standards and high-resolution spectroscopy. It was considered that sympathetic cooling in an rf trap is difficult because of the presence of rf heating [21], and efficient sympathetic cooling was achieved only when the guest ion mass is close to that of the host ion. For example, MgH^+ and MgD^+ were cooled by the laser-cooled $^{24}Mg^+$ [18], and $^{112}Cd^+$ by the laser-cooled $^{114}Cd^+$ [19]. Recently, however, sympathetic cooling of much heavier ion species than the coolant ion species has been successfully carried out by using a technique in which guest ions are produced by charge transfer from the laser-cooled host ions [22].

It is interesting to know how low a temperature can be achieved by sympathetic cooling in an rf trap. The purpose of this paper is to construct a theoretical model of sympathetic cooling in an rf trap and to confirm the general applicability of the model by reproducing the experimental results [22]. In this model, complicated ion behavior involving the superposition of micromotion and secular motion is considered. This theoretical model predicts the final temperature of indirectly cooled ions as functions of the trap parameters and the ion mass, and provides the optimum trapping conditions.

II. EXPERIMENT AND RESULTS

We briefly summarize the experimental procedure and the result of the previous work [22]. $^{24}Mg^+$ is laser-cooled as the host ion species. The energy-level structure of $^{24}Mg^+$ is simple and only one laser at the wavelength $\lambda = 280$ nm is necessary for laser cooling. The cooling efficiency is high because of the high spontaneous emission rate ($\gamma = 4.3 \times 10^7$ s^{-1}). These features are advantageous in evaluating potential coolants for use in sympathetic cooling. The guest ion species is Ba^+, which is cooled indirectly through collisions with the host ions. The Ba^+ temperature is obtained by measuring the linewidth of its optical transition, and the Mg^+ temperature is determined by the fluorescence spectral line shape obtained by scanning the cooling laser frequency [23]. The trapped ion numbers are estimated from the fluorescence intensities of both ion species.

The observed temperature and the number of Ba^+ ions are 500 K and 5, respectively, while those of Mg^+ ions are 100 K and 100, respectively. The temperature of Ba^+ is much higher than that of Hg^+ sympathetically cooled by laser-cooled Be^+ in a Penning trap (0.4–1.8 K) [12]. The higher temperature in the rf trap is due to rf heating [21].

III. THEORETICAL MODEL

The rate equations of the thermal energies of both host ions and guest ions are expressed as

$$\frac{dU_H}{dt} = W_{H,H} + W_{H,G} - W_C, \tag{1}$$

$$\frac{dU_G}{dt} = W_{G,H} + W_{G,G}, \tag{2}$$

where $U_H = k_B T_H$ and $U_G = k_B T_G$ are the thermal energies per ion of the host (Mg^+) and the guest (Ba^+), respectively (k_B is the Boltzmann constant, and T_H and T_G are the temperatures of the host ions and the guest ions, respectively). W_C is the laser-cooling rate. $W_{a,b}$ is the rate of thermal energy change of the ion a caused by collisions with b, where a or b is either H (host) or G (guest). For example, $W_{H,H}$ is the thermal energy change rate of a host ion by collisions with other host ions, and $W_{G,H}$ is that of a guest ion by collisions with host ions. $W_{H,H}$ and $W_{G,G}$ are the rf heating rates, and $W_{G,H}$ and $W_{H,G}$ include the rates of both the rf heating and the sympathetic cooling. These rates can be calculated by a formula for Coulomb collisions.

The velocity $v_{a,i}$ of the ith ion of the species denoted by a ($i=1,2,\ldots,N_a$, where N_a is the number of a ions) between the successive collisions is expressed as

$$v_{a,i}=\frac{d}{dt}\left\{A_{a,i}\cos(\omega_a t+\phi_{a,i})\left[1+\frac{q_a}{\sqrt{2}}\cos\Omega t\right]\right\}. \tag{3}$$

Here, $A_{a,i}$ is the amplitude of secular motion, $\omega_a/2\pi$ the secular frequency, $\phi_{a,i}$ the phase angle of the secular motion, q_a the trapping parameter proportional to the driving rf voltage, and $\Omega/2\pi$ the rf frequency. Equation (3) includes both the secular motion and the micromotion.

Now we consider the collision of the ith ion of a species with the jth ion of b species. It is assumed that the collision process is one-dimensional and that the period during which the particles interact is much shorter than the period of micromotion, so that the total momentum and kinetic energy of the collision are conserved. Hence the amplitude $A'_{a,i}$ and phase $\phi'_{a,i}$ after the collision are given as functions of those of ions before the collision ($A_{a,i}$, $\phi_{a,i}$, $A_{b,j}$, and $\phi_{b,j}$). The energy change of the ith ion ($\Delta E_{a,i}$) in a collision is

$$\Delta E_{a,i}=\tfrac{1}{2}\,m_a\omega_a^2({A'_{a,i}}^2-A_{a,i}^2), \tag{4}$$

where m_a is the ion mass. The energy change rate of the a ion by the collision with the b ion is given by

$$W_{a,b}=\langle n_b\sigma_{a,i,b,j}v_{a,i,b,j}\Delta E_{a,i}\rangle, \tag{5}$$

where n_b is the density of the ion b, and $\sigma_{a,i,b,j}$ is the collision cross section between the ith a and jth b ions. $v_{a,i,b,j}$ is the relative speed along the collision axis and is given by $\sqrt{(v_{a,i}-v_{b,j})^2+v_p^2}$, where v_p^2 is the sum of squares of the two relative velocity components perpendicular to the collision axis. In the case of a Coulomb collision, the product of the cross section $\sigma_{a,i,b,j}$ and the relative speed $v_{a,i,b,j}$ is given by [24]

$$\sigma_{a,i,b,j}v_{a,i,b,j}=\frac{e^4}{4\pi\varepsilon_0^2m_a^2u^2}\int_0^\infty\frac{v_p dv_p}{v_{a,i,b,j}^3}e^{-v_p^2/2u^2}\times\log\left[\left(\frac{4\pi\varepsilon_0 m_a r_b v_{a,i,b,j}^2}{e^2}\right)^2+1\right]. \tag{6}$$

Here, $u^2=k_B(T_a/m_a+T_b/m_b)$, e is the charge of the trapped ion, and ε_0 is the dielectric constant in vacuum. r_b is the radius of the b ion cloud, estimated as $\sqrt{(k_BT_b)/(m_b\omega_b^2)}$ [25].

The brackets in Eq. (5), which imply taking the average over phases and amplitudes of motion of the ith a and jth b ions, are expressed by the following integral:

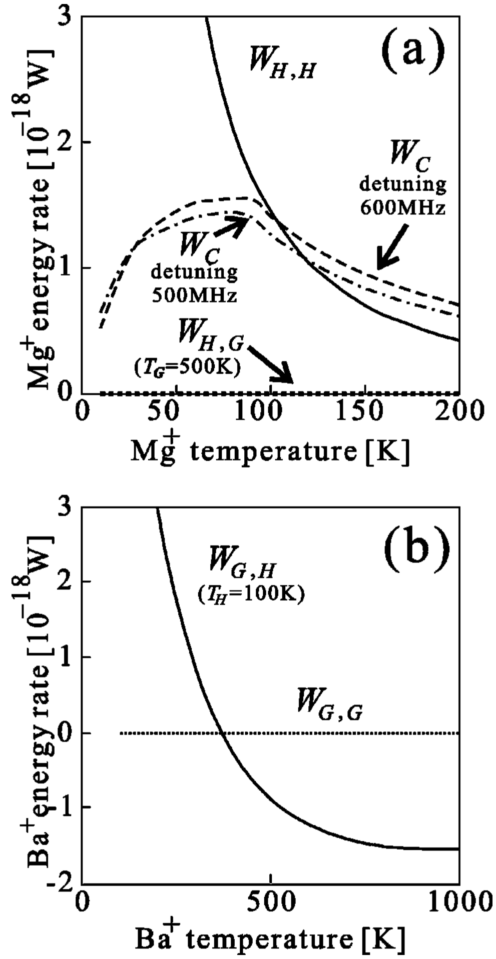

FIG. 1. Thermal energy change rates for (a) Mg^+ and (b) Ba^+. Absolute values of $W_{H,G}$ and $W_{G,G}$ are much smaller than others ($W_{H,G}\sim10^{-23}$ W and $W_{G,G}\sim10^{-22}$ W).

$$\langle\cdots\rangle=\int_0^{2\pi}\frac{d\phi_{a,i}}{2\pi}\int_0^{2\pi}\frac{d\phi_{b,j}}{2\pi}\int_0^{2\pi}\frac{d(\Omega t)}{2\pi}\int_{-\infty}^{\infty}\sqrt{\frac{m_a\omega_a^2}{\pi k_BT_a}}\times\exp\left(-\frac{m_a\omega_a^2A_{a,i}^2}{k_BT_a}\right)dA_{a,i}\int_{-\infty}^{\infty}\sqrt{\frac{m_b\omega_b^2}{\pi k_BT_b}}\times\exp\left(-\frac{m_b\omega_b^2A_{b,j}^2}{k_BT_b}\right)dA_{b,j}. \tag{7}$$

The laser-cooling rate of the host ion W_C is given by

$$W_C=\int_0^{2\pi}\frac{d\phi_{H,i}}{2\pi}\int_0^{2\pi}\frac{d(\Omega t)}{2\pi}\int_{-\infty}^{\infty}\sqrt{\frac{m_H\omega_H^2}{\pi k_BT_H}}\times\exp\left(-\frac{m_H\omega_H^2A_{H,i}^2}{k_BT_H}\right)dA_{H,i}\times\frac{1}{6}\frac{(\hbar kv_{H,i}+2R)\gamma x^2}{(\Delta-kv_{H,i})^2+\gamma^2+x^2}, \tag{8}$$

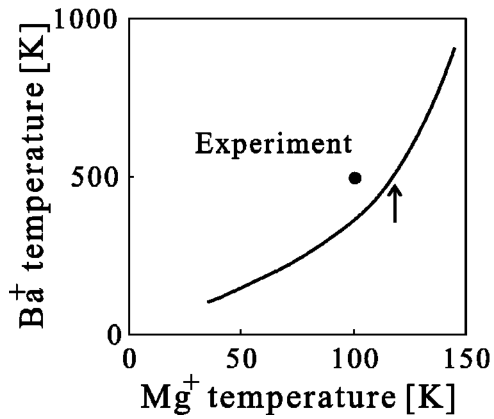

FIG. 2. Relation between the temperatures of Mg^+ and Ba^+. A solid circle is the experimental result. The solid line and the arrow imply the relation between the temperatures and Mg^+ temperature obtained by the calculation, respectively.

where $2\pi\hbar$ is Planck's constant, $k=2\pi/\lambda$, $R=(\hbar k)^2/2m_H$, and x and $\Delta/2\pi$ are the Rabi frequency and the detuning of the cooling laser, respectively.

The rates appearing in Eqs. (1) and (2) are numerically calculated and shown in Fig. 1. Here, the values of the constants are given in Ref. [22]. As seen in Fig. 1(a), $|W_{H,G}|$ ($\sim 10^{-23}$ W) is much smaller than either $|W_C|$ or $|W_{H,H}|$ ($\sim 10^{-18}$ W), and in Fig. 1(b), $|W_{G,G}|$ ($\sim 10^{-22}$ W) is much smaller than $|W_{G,H}|$ ($\sim 10^{-18}$ W). The negligibly small values of $W_{H,G}$ and $W_{G,G}$ are due to the low density of Ba^+ caused by both the shallow trap potential for Ba^+ and the small number of Ba^+ in the experiment. Therefore, the stationary state solution of Eq. (1) is obtained for $W_{H,H}=W_C$, and thus from Fig. 1(a) the temperature of $^{24}Mg^+$ (T_H) is determined to be 110 K for $\Delta/2\pi=500$ MHz. Similarly, the temperature of Ba^+ (T_G) is determined to be 380 K from setting Eq. (2) to zero and neglecting $W_{G,G}$ (namely, $W_{G,H}=0$), as shown in Fig. 1(b). The calculated temperatures agree roughly with the observed temperatures of 100 K ($^{24}Mg^+$) and 500 K (Ba^+) [22]. Generally the guest ion temperature can be calculated as a function of the host ion temperature by the relation of $W_{G,H}=0$, as shown in Fig. 2.

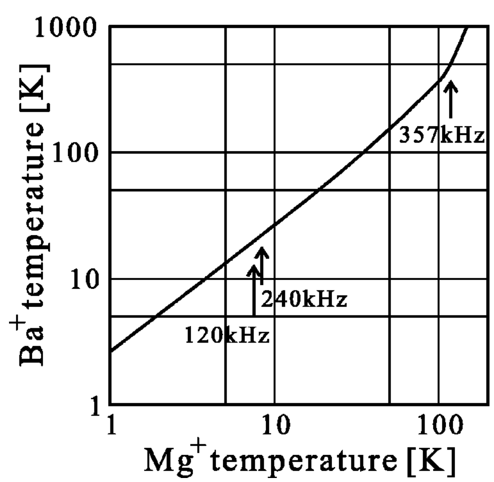

FIG. 3. Relation between the temperatures of Mg^+ (T_H) and Ba^+ (T_G) for secular frequencies of Mg^+ at 120, 240, and 357 kHz. The three curves corresponding to these three frequencies are indistinguishable. Each arrow implies the calculated temperature of Mg^+ ions at the corresponding secular frequency.

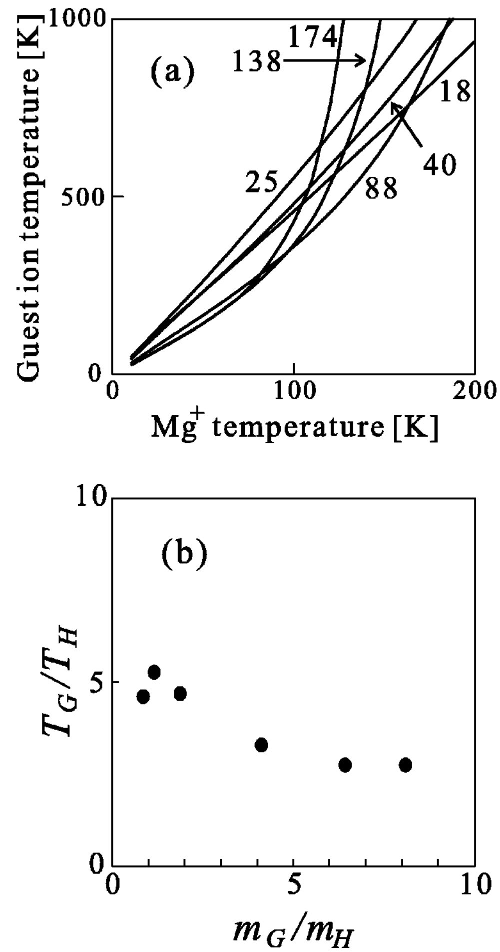

FIG. 4. (a) Relation between the temperatures of $^{24}Mg^+$ and specific guest ion species. The numbers in the figure are the masses of the ions in atomic mass units. (b) Dependence of the ratio between the temperatures of Mg^+ and of a specific guest ion (T_G/T_H) in the linear (low T_H) region of (a) on the mass ratio m_G/m_H.

Further features of the sympathetic cooling process in the rf trap are calculated by using the present model for Mg^+ as the host ion species and the trap rf frequency of 2.06 MHz [22]. Figure 3 shows the relation between T_H (Mg^+) and T_G (Ba^+) calculated at the secular frequencies of the host ion of 120, 240, and 357 kHz. No appreciable difference of the T_H–T_G curves is obtained for these secular frequencies, while the host ion temperature and hence the guest ion temperature depend on the secular frequency as indicated by three arrows in Fig. 3. The lower temperature is obtained for the lower secular frequency because of the weaker effect of the rf heating [26].

The guest ion temperature depends on its mass, as shown in Fig. 4(a). The numbers beside the T_H–T_G curves are the guest ion masses in atomic mass units. The heavier guest ion is cooled more efficiently in the low T_H region, while T_G increases rapidly with increasing mass in the high T_H region. In Fig. 4(b), the T_G/T_H ratio in the low T_H region is depicted as a function of the mass ratio m_G/m_H. The lowest guest ion temperature achieved in sympathetic cooling is predicted to be 2.5 times larger than the host ion temperature. When the guest ion mass is close to the host ion mass, the cooling efficiency becomes poor.

IV. CONCLUSION

We have constructed a theoretical model of sympathetic cooling in an rf trap without using any fitting parameters, and the calculated result agrees with the experimental results reported in Ref. [22]. This model is generally applicable to estimating the limiting temperature of sympathetically cooled ions in an rf trap. The relationship between the temperatures of the host ions and the guest ions and its dependence on the trapping voltage were calculated. It was found that the trapping voltage does not effect the temperature relation between the host ions and the guest ions. The host ions are effectively laser-cooled at low trapping voltage because of a low rate of the rf heating [26], and accordingly the temperature of the guest ions also becomes low. The dependence of the relation between the host ion temperature and the guest ion temperature on the ion mass ratio m_G/m_H was also calculated. The guest ion temperature depends linearly on T_H in the low T_H region and increases nonlinearly in the higher T_H region. The nonlinearity appears more remarkable for the heavier guest ions. The temperature ratio in the linear (low T_H) region becomes largest when the guest ion mass is close to the host ion mass. The temperature ratio T_G/T_H becomes as small as 2.5 and almost independent of m_G/m_H, if m_G/m_H is larger than 5.

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