Ground-state properties of a dilute Bose-Fermi mixture

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We investigate the properties of a dilute boson-fermion mixture at zero temperature. The ground-state energy of the system is calculated to second order in the Bose-Fermi coupling constant. The Green's function formalism is applied to obtain results for the effect of Bose-Fermi interactions on the spectrum of phonon excitations, the momentum distribution, the condensate fraction, and the superfluid density of the bosonic component. A quantitative discussion of these effects is presented in the case of mixtures with attractive interspecies interaction.

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I. INTRODUCTION

Recent experimental achievements in the trapping and cooling of mixtures of bosonic and fermionic atoms have attracted much attention on the physics of dilute Bose-Fermi mixtures in the quantum degenerate regime. Stable Bose-Einstein condensates (BEC's) immersed in a degenerate Fermi gas have been realized with ^{7}Li in ^{6}Li [1], ^{23}Na in ^{6}Li [2], and very recently with ^{87}Rb in ^{40}K [3]. In particular, the ^{87}Rb-^{40}K mixture appears to be highly interesting because of the large and negative interspecies scattering length that favors the stability of the mixture. Current theoretical investigations of dilute Bose-Fermi mixtures have mainly addressed the determination of the density profiles of the two components in trapped systems [4], the problem of stability and phase separation [5,6], and the BEC-induced interaction between fermions [7]. The effect of boson-fermion interactions on the phonon excitation spectrum has been investigated in Refs. [8] and [9]. Further, perturbation theory has been applied to the calculation of the ground-state energy in the regime of high fermion concentration by Albus *et al.* [10], whereas the opposite regime of low fermion concentration has been investigated long ago by Saam [11].

In the present paper we consider a homogeneous dilute mixture of a normal Fermi gas and a Bose-Einstein condensed gas at zero temperature. We calculate the ground-state energy of the system to second order in the Bose-Fermi coupling constant, interpolating between the high and low fermion concentration regimes, and obtain results for the dispersion of phonon excitations, including corrections to the sound velocity and Landau damping. In addition to these results, we focus on the effect of boson-fermion interactions on the momentum distribution of the bosonic component, on the depletion of the condensate state, and on the superfluid density. We find that the boson momentum distribution can be strongly affected by the fermions, resulting in a suppression of the occupation of low momentum states and in a long tail at large momenta. We also point out the possible occurrence of striking features caused by the Bose-Fermi coupling, such as the increase of the condensate fraction compared to the pure bosonic case and the superfluid density becoming smaller than the condensate fraction.

The structure of the paper is as follows. In Sec. II we introduce the basic Hamiltonian of the system within the Bogoliubov approximation. In Sec. II A we calculate the ground-state energy by direct use of perturbation theory. In Secs. II B–II D we introduce the Green's function formalism which we apply to calculate the elementary excitation energies, the momentum distribution, the condensate fraction, and the superfluid density of the bosonic component. In Sec. III we comment on the stability of the mixture against phase separation and present numerical results of the boson momentum distribution, as well as of the condensate depletion and normal fluid fraction obtained for specific configurations of mixtures with attractive Bose-Fermi coupling. Finally, in Sec. IV we draw our conclusions.

We have kept Sec. II deliberately detailed and pedagogical. This was done in order to present all the general features of the system and to provide, we hope, a useful reference. We suggest the reader whose main interest is on the discussion of the results obtained for specific real configurations, to skip Secs. II A–II D and to read directly Sec. III.

II. THEORY

We consider a stable homogeneous mixture of a Bose gas and a spin-polarized Fermi gas. The Hamiltonian of the system can be written as the sum

$$H=H_F+H_B+H_{int} \tag{1}$$

of the pure fermionic (H_F) and bosonic (H_B) Hamiltonian, and of the interaction term (H_{int}) which accounts for the coupling between the two species. If one neglects p-wave interactions in the spin-polarized Fermi gas, H_F corresponds to the Hamiltonian of a free gas

$$H_F=\sum_{\mathbf{k}} \epsilon_k^F b_{\mathbf{k}}^\dagger b_{\mathbf{k}}, \tag{2}$$

where $b_{\mathbf{k}}$, $b_{\mathbf{k}}^\dagger$ are the annihilation and creation operators of fermions and $\epsilon_k^F=\hbar^2k^2/2m_F$ is the energy of a free particle of mass m_F. The pure bosonic component is described instead by the Bogoliubov Hamiltonian,

$$H_B=E_B+\sum_{\mathbf{k}} \omega_k \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}}, \tag{3}$$

written in terms of the quasiparticle annihilation and creation operators $\alpha_{\mathbf{k}}$, $\alpha_{\mathbf{k}}^\dagger$. These operators are related to the bosonic particle operators $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$ through the well-known canonical transformation,

$$a_{\mathbf{k}}=u_k\alpha_{\mathbf{k}}+v_k\alpha_{-\mathbf{k}}^\dagger,$$
$$a_{\mathbf{k}}^\dagger=u_k\alpha_{\mathbf{k}}^\dagger+v_k\alpha_{-\mathbf{k}}, \tag{4}$$

with coefficients $u_k^2=1+v_k^2=(\epsilon_k^B+g_{BB}n_0+\omega_k)/2\omega_k$ and $u_kv_k=-g_{BB}n_0/2\omega_k$. The elementary excitation energies obey the usual Bogoliubov spectrum,

$$\omega_k=\sqrt{(\epsilon_k^B)^2+2g_{BB}n_0\epsilon_k^B}, \tag{5}$$

where $\epsilon_k^B=\hbar^2k^2/2m_B$ is the free particle energy, n_0 is the condensate density, and $g_{BB}=4\pi\hbar^2a/m_B$ is the coupling constant between bosons fixed by the mass m_B and the s-wave scattering length a which we assume positive. The constant term

$$E_B=N_B\left[4\pi n_Ba^3+\frac{512}{15}\sqrt{\pi}(n_Ba^3)^{3/2}\right]\frac{\hbar^2}{2m_Ba^2} \tag{6}$$

is the ground-state energy of a dilute Bose gas expressed in terms of the gas parameter n_Ba^3, with $n_B=N_B/V$ the total density of bosons. Result (6) includes the zero-point motion of the elementary excitations. Finally, the Hamiltonian

$$H_{int}=g_{BF}\int d\mathbf{r}n_B(\mathbf{r})n_F(\mathbf{r}) \tag{7}$$

describes the interaction between fermions and bosons through the coupling constant $g_{BF}=\pm 2\pi\hbar^2b/m_R$, which can be either positive or negative, depending on whether the interactions are repulsive or attractive. The coupling constant g_{BF} is fixed by the reduced mass $m_R=m_Bm_F/(m_B+m_F)$ and by the modulus of the boson-fermion s-wave scattering length b. By introducing the bosonic and fermionic density fluctuation operators $\rho_{\mathbf{k}}^B=1/\sqrt{V}\int d\mathbf{r}e^{i\mathbf{k}\cdot\mathbf{r}}[n_B(\mathbf{r})-n_B]$ and $\rho_{\mathbf{k}}^F=1/\sqrt{V}\int d\mathbf{r}e^{i\mathbf{k}\cdot\mathbf{r}}[n_F(\mathbf{r})-n_F]$, where $n_F=N_F/V$ is the mean density of fermions, the interaction Hamiltonian can be rewritten in the following form:

$$H_{int}=g_{BF}N_Bn_F+g_{BF}\sum_{\mathbf{k}} \rho_{\mathbf{k}}^B\rho_{-\mathbf{k}}^F. \tag{8}$$

The constant term in the above equation corresponds to the mean-field interaction energy; the second term describes the coupling between density fluctuations in the two species. Within the Bogoliubov approximation, the bosonic density fluctuation operator can be written as a linear combination of quasiparticle operators,

$$\rho_{\mathbf{k}}^B=\frac{1}{\sqrt{V}}\sum_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}+\mathbf{k}}\simeq\sqrt{n_0}(u_k+v_k)(\alpha_{\mathbf{k}}+\alpha_{-\mathbf{k}}^\dagger). \tag{9}$$

By using the above expression for $\rho_{\mathbf{k}}^B$, the total Hamiltonian (1) takes the form

$$H=E_B+g_{BF}N_Bn_F+\sum_{\mathbf{k}} \epsilon_k^F b_{\mathbf{k}}^\dagger b_{\mathbf{k}}+\sum_{\mathbf{k}} \omega_k\alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}$$
$$+g_{BF}\sqrt{n_0}\sum_{\mathbf{k}} (u_k+v_k)(\alpha_{\mathbf{k}}+\alpha_{-\mathbf{k}}^\dagger)\rho_{-\mathbf{k}}^F. \tag{10}$$

The Hamiltonian (10) is treated in perturbation theory. We write $H=H_0+H_{int}$, where $H_0=H_F+H_B$ is the unperturbed Hamiltonian which is diagonal in the fermionic particle operators and in the bosonic elementary excitation operators, and

$$H_{int}=g_{BF}N_Bn_F+g_{BF}\sqrt{n_0}\sum_{\mathbf{k}} (u_k+v_k)(\alpha_{\mathbf{k}}+\alpha_{-\mathbf{k}}^\dagger)\rho_{-\mathbf{k}}^F \tag{11}$$

is the perturbation term. Since we make use of the pseudopotential approximation for both the boson-boson and the boson-fermion interatomic potential, the present approach is valid if both gas parameters are small, i.e., if $n_Ba^3\ll 1$ and $n_Fb^3\ll 1$.

A. Ground-state energy

The unperturbed ground-state energy can be readily calculated from the Hamiltonian H_0. One finds

$$E_0=E_B+\tfrac{3}{5}N_F\epsilon_F, \tag{12}$$

where E_B is given by Eq. (6) and $\epsilon_F=\hbar^2k_F^2/2m_F=\hbar^2(6\pi^2n_F)^{2/3}/2m_F$ is the Fermi energy, k_F being the Fermi wave vector. The mean-field term $\Delta E^{(1)}=g_{BF}N_Bn_F$ provides the first-order correction to E_0. To second order in g_{BF}, one obtains the result

$$\Delta E^{(2)}=\sum_{n\neq 0}\frac{|\langle 0|H_{int}|n\rangle|^2}{E_0-E_n}+N_Bg_{BF}^2n_F\frac{1}{V}\sum_{\mathbf{k}}\frac{2m_R}{\hbar^2k^2}, \tag{13}$$

where $|n\rangle$ are the excited states of the unperturbed system with energy E_n. The first term is the standard result of second-order perturbation theory. As it stands, this term is ultraviolet divergent. The divergence is cured in a standard way (see, for instance, Ref. [12]) by introducing the second term in the above equation, which arises from $\Delta E^{(1)}$ due to the renormalization of the boson-fermion scattering length $g_{BF}\to g_{BF}+g_{BF}^2 1/V\Sigma_{\mathbf{k}}2m_R/\hbar^2k^2$. By directly calculating the matrix elements of the interaction Hamiltonian H_{int}, one finds the following result for the ground-state energy:

$$E=E_0+N_Bg_{BF}n_F+N_Bg_{BF}^2n_F\frac{1}{V}\sum_{\mathbf{k}}\frac{2m_R}{\hbar^2k^2}$$
$$-N_Bg_{BF}^2\frac{1}{V^2}\sum_{\mathbf{k},\mathbf{q}}(u_k+v_k)^2\frac{n_{\mathbf{q}}^F(1-n_{\mathbf{q}+\mathbf{k}}^F)}{\omega_k+\epsilon_{|\mathbf{q}+\mathbf{k}|}^F-\epsilon_q^F}, \tag{14}$$

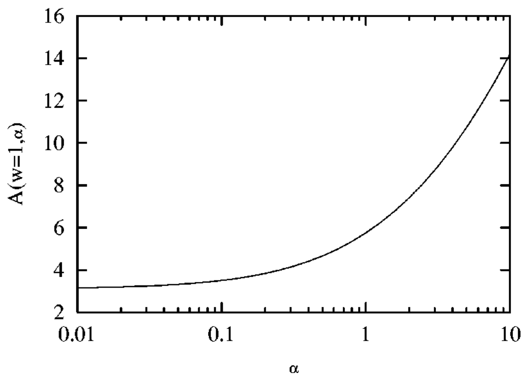

FIG. 1. Dimensionless function $A(w,\alpha)$ [Eq. (16)] as a function of $\alpha=2/(k_F\xi_B)^2$ for the value $w=1$ of the mass ratio.

where $n^F_{\mathbf{q}}=\theta(k_F-q)$ is the momentum distribution of the Fermi gas. We notice that the renormalization of the boson-fermion scattering length is crucial in order to cancel the ultraviolet divergence in the integral over the wave vector $\mathbf{k}$. The dependence on the relevant parameters becomes clearer by expressing the integrals over wave vectors in units of k_F. Result (14) takes the form

$$E=E_0+N_B g_{BF} n_F+N_B \epsilon_F (n_F b^3)^{2/3} A(w,\alpha), \qquad (15)$$

where the second-order energy shift is expressed in terms of the Fermi energy ϵ_F, the fermionic gas parameter $n_F b^3$, and the dimensionless function

$$A(w,\alpha)=\frac{2}{3}\left(\frac{6}{\pi}\right)^{2/3}\frac{1+w}{w}\int_0^\infty dk\int_{-1}^{+1}d\Omega\left[1-\frac{3k^2(1+w)}{\sqrt{k^2+\alpha}}\right.$$
$$\left.\times\int_0^1 dq\,q^2\frac{1-\theta(1-\sqrt{q^2+k^2+2kq\Omega})}{\sqrt{k^2+\alpha}+wk+2qw\Omega}\right]. \qquad (16)$$

In the above expression $w=m_B/m_F$ is the mass ratio and $\alpha=2/(k_F\xi_B)^2$ is a dimensionless parameter fixed by the product of the Fermi wave vector k_F and the Bose healing length $\xi_B=1/\sqrt{8\pi n_0 a}$. To gain more insight into the physical meaning of α, notice also that $\alpha=2w(g_{BB}n_0/\epsilon_F)$, $g_{BB}n_0$ being the chemical potential of the bosons. In Fig. 1 we show the dependence of the function $A(w,\alpha)$ on the parameter α for a fixed value $w=1$ of the mass ratio. Two regimes are worth studying at this point. The regime $k_F\xi_B \gg 1$ ($\alpha \ll 1$) corresponds to a system where the Fermi energy is much larger than the chemical potential of the bosons $\epsilon_F \gg g_{BB}n_0$ (if $w\simeq 1$). In this regime, expression (16) can be simplified as it depends only on the mass ratio w: $A(w,\alpha\to 0)=A_1(w)$, where

$$A_1(w)=\frac{2}{3}\left(\frac{6}{\pi}\right)^{2/3}\frac{1+w}{w}\int_0^\infty dk\int_{-1}^{+1}d\Omega\left[1-3k(1+w)\right.$$
$$\left.\times\int_0^1 dq\,q^2\frac{1-\theta(1-\sqrt{q^2+k^2+2kq\Omega})}{(1+w)k+2qw\Omega}\right]. \qquad (17)$$

In this regime, result (15) reads

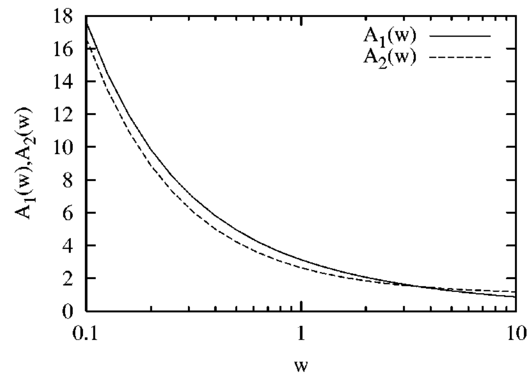

FIG. 2. Dimensionless functions A_1 [Eq. (17)] (solid line) and A_2 [Eq. (19)] (dashed line) of the mass ratio $w=m_B/m_F$.

$$E=E_0+N_B g_{BF} n_F+N_B \epsilon_F (n_F b^3)^{2/3} A_1(w), \qquad (18)$$

and coincides with the finding of Ref. [10] obtained using the T-matrix approach. Notice that the effect of the Bose-Fermi interaction on the ground-state energy is independent of the Bose-Bose interaction. In fact, in the limit $k_F\xi_B \gg 1$, the relevant contribution to the integral in Eq. (14) comes from the large momentum region where the bosons behave as independent particles. The opposite regime $k_F\xi_B \ll 1$ ($\alpha \gg 1$) corresponds to a Fermi energy $\epsilon_F \ll g_{BB}n_0$ (if $w\simeq 1$). One finds $A(w,\alpha\to\infty)=2\sqrt{\alpha}A_2(w)$, where

$$A_2(w)=\frac{2}{3}\left(\frac{6}{\pi}\right)^{2/3}\frac{1+w}{w}\int_0^\infty dk\left[1-\frac{(1+w)k^2}{\sqrt{1+k^2}(\sqrt{1+k^2}+wk)}\right]. \qquad (19)$$

In this regime, the ground-state energy of the system can be written as

$$E=E_0+N_B g_{BF} n_F+\frac{8}{6^{1/3}\pi^{1/6}}N_B \epsilon_F (n_F b^3)^{2/3}\left(\frac{n_B}{n_F}\right)^{1/3}$$
$$\times(n_B a^3)^{1/6}A_2(w). \qquad (20)$$

In contrast to result (18), Bose-Bose interactions are important if $k_F\xi_B \ll 1$ and the second-order correction depends explicitly on the Bose gas parameter. Result (20) has been first derived by Saam [11] for a dilute mixture with $a=b$. The dependence of A_1 and A_2 on the mass ratio w is presented in Fig. 2.

B. Phonon spectrum

In this subsection we calculate the effect of fermions on the dispersion of the bosonic phonon branch. To this aim, it is convenient to use the Green's function formalism. The bosonic quasiparticle Green's function is defined as

$$D(\mathbf{k},t)=-\frac{i}{\hbar}\langle T(\alpha_{\mathbf{k}}(t)\alpha^\dagger_{\mathbf{k}}(0))\rangle, \qquad (21)$$

where $T(\cdots)$ is the time-ordered product. Perturbation theory provides us with a precise recipe for calculating the Green's function $D(\mathbf{k},t)$ to a given order in H_{int},

$$D(\mathbf{k},t) = -\frac{i}{\hbar}\frac{1}{\langle S\rangle}\langle T(\alpha_{\mathbf{k}}(t)\alpha_{\mathbf{k}}^{\dagger}(0)S)\rangle, \tag{22}$$

where the time evolution operator S is defined through the series expansion

$$S = \sum_{n=0}^{\infty}\left(-\frac{i}{\hbar}\right)^n \frac{1}{n!}\int_{-\infty}^{+\infty}dt_1\cdots\int_{-\infty}^{+\infty}dt_n \times T[H_{int}(t_1),\ldots,H_{int}(t_n)]. \tag{23}$$

In Fourier space the unperturbed quasiparticle Green's function is given by

$$D_0(\mathbf{k},\omega) = \frac{1}{\hbar\omega - \omega_k + i\eta}, \tag{24}$$

where $\eta>0$ and infinitesimally small. To second order in H_{int}, a direct calculation gives the result

$$D(\mathbf{k},\omega) = D_0(\mathbf{k},\omega) + g_{BF}^2 n_0 (u_k+v_k)^2 D_0^2(\mathbf{k},\omega)\Pi_0^F(\mathbf{k},\omega). \tag{25}$$

In the above equation $\Pi_0^F(\mathbf{k},\omega)$ is the density-density response function of an ideal Fermi gas, defined as

$$\Pi_0^F(\mathbf{k},\omega) = \frac{1}{V}\sum_{\mathbf{q}} n_{\mathbf{q}}^F(1-n_{\mathbf{q}+\mathbf{k}}^F)\left(\frac{1}{\hbar\omega + \epsilon_q^F - \epsilon_{|\mathbf{q}+\mathbf{k}|}^F + i\eta} - \frac{1}{\hbar\omega + \epsilon_{|\mathbf{q}+\mathbf{k}|}^F - \epsilon_q^F - i\eta}\right). \tag{26}$$

By solving Eq. (25) for $D^{-1}(\mathbf{k},\omega)$ one gets the following result for the excitation energy to order g_{BF}^2:

$$\hbar\omega = \omega_k + g_{BF}^2 n_0 \frac{\epsilon_k^B}{\omega_k}\Pi_0^F(\mathbf{k},\hbar\omega=\omega_k). \tag{27}$$

The real and imaginary parts of the fermionic response function Π_0^F give rise, respectively, to a frequency shift and a damping of the quasiparticle. We are interested in the collective modes of the boson system which correspond to low momenta $\omega_k \ll g_{BB}n_0$ and $k \ll k_F$. In the regime $k_F\xi_B \gg 1$ the fermionic density-density response function can be approximated by [13]

$$\Pi_0^F(\mathbf{k},\hbar\omega=\omega_k) = -\frac{m_F k_F}{2\pi^2\hbar^2}\left(1 + i\frac{\pi}{2\sqrt{2}wk_F\xi_B}\right), \tag{28}$$

and one gets the following result for the phonon excitation energy:

$$\hbar\omega = \hbar k c_B\left[1 - \frac{6^{1/3}(1+w)^2}{4\pi^{1/3}w}\frac{b}{a}(n_F b^3)^{1/3} - i\frac{\sqrt{\pi}(1+w)^2}{4w^2}\frac{b^2}{a^2}(n_B a^3)^{1/2}\right]. \tag{29}$$

In the above expression the real part in the square bracket gives the correction to the Bogoliubov sound velocity $c_B = \sqrt{g_{BB}n_0/m_B}$, and the imaginary part gives the Landau damping of the phonon excitation due to collisions with the fermions. We notice that the correction to the sound velocity (second term in the square bracket) can be rewritten as $N(0)g_{BF}^2/2g_{BB}$, with $N(0)=m_F k_F/(2\pi^2\hbar^2)$ being the density of states at the Fermi surface. The stability condition requiring that the real part of the excitation energy be positive, corresponds to the linear stability condition $N(0)g_{BF}^2/g_{BB}<1$ (see Sec. III). In the regime $k_F\xi_B \gg 1$ the Bogoliubov phonon becomes soft when approaching the condition where the system phase separates. In the opposite regime $k_F\xi_B \ll 1$, the imaginary part of Π_0^F vanishes at low momenta and the renormalized velocity of sound of the undamped phonon mode becomes

$$\hbar\omega = \hbar k c_B\left[1 + \frac{w(1+w)^2}{8}\frac{b^2}{a^2}\frac{n_F}{n_B}\right]. \tag{30}$$

The dispersion of phonons in dilute mixtures has also been discussed by Yip [8] under more general conditions using a numerical approach. He also predicts a shift towards lower frequencies with Landau damping for $k_F\xi_B \gg 1$, and towards higher frequencies with no damping for $k_F\xi_B \ll 1$.

C. Condensate fraction and momentum distribution

The effect of the Bose-Fermi interaction on the momentum distribution of the bosons can be readily obtained by applying the perturbative approach to the bosonic single-particle Green's function,

$$G_{11}(\mathbf{k},t) = -\frac{i}{\hbar}\langle T(a_{\mathbf{k}}(t)a_{\mathbf{k}}^{\dagger}(0))\rangle. \tag{31}$$

The unperturbed single-particle Green's function is given by

$$G_{11}^0(\mathbf{k},\omega) = u_k^2 D_0(\mathbf{k},\omega) + v_k^2 D_0(\mathbf{k},-\omega) = \frac{u_k^2}{\hbar\omega - \omega_k + i\eta} - \frac{v_k^2}{\hbar\omega + \omega_k - i\eta}. \tag{32}$$

To second order in the Bose-Fermi coupling constant one gets the result

$$G_{11}(\mathbf{k},\omega) = G_{11}^0(\mathbf{k},\omega) + g_{BF}^2 n_0 (u_k+v_k)^2 \Pi_0^F(\mathbf{k},\omega) \times [u_k^2 D_0^2(\mathbf{k},\omega) + v_k^2 D_0^2(\mathbf{k},-\omega) + 2u_k v_k D_0(\mathbf{k},\omega)D_0(\mathbf{k},-\omega)]. \tag{33}$$

The momentum distribution of the bosons can be obtained from the single-particle Green's function through the following relation

$$n_{\mathbf{k}}^B = \langle a_k^{\dagger}a_k\rangle = \frac{i\hbar}{2\pi}\int_{-\infty}^{+\infty}d\omega G_{11}(\mathbf{k},\omega)e^{i\omega\eta}, \tag{34}$$

which gives the result

$$n_{\mathbf{k}}^B = v_k^2 + g_{BF}^2 n_0 (u_k+v_k)^2 \frac{1}{V} \sum_{\mathbf{q}} n_{\mathbf{q}}^F (1-n_{\mathbf{q}+\mathbf{k}}^F) \times \left(\frac{u_k^2}{(\omega_k + \epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F)^2} + \frac{v_k^2}{(\omega_k + \epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F)^2} + \frac{2u_k v_k}{\omega_k(\omega_k + \epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F)} \right). \tag{35}$$

By integrating $n_{\mathbf{k}}^B$ over momenta, one obtains the fraction of atoms that are scattered out of the condensate due to interaction. The quantum depletion of the condensate is given by

$$\frac{N'}{N_B} = \frac{8}{3\sqrt{\pi}} (n_B a^3)^{1/2} + g_{BF}^2 \frac{1}{V} \sum_{\mathbf{k}} \frac{\epsilon_k^B}{\omega_k} \frac{1}{V} \times \sum_{\mathbf{q}} \frac{n_{\mathbf{q}}^F (1-n_{\mathbf{q}+\mathbf{k}}^F)}{(\omega_k + \epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F)^2} \left(\frac{\epsilon_k^B}{\omega_k} - \frac{g_{BB} n_0}{\omega_k^2} (\epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F) \right). \tag{36}$$

The first term in the above equation corresponds to the Bogoliubov depletion present also in a pure bosonic system which is due to interaction effects among the bosons. The second term accounts instead for boson-fermion scattering processes. The condensate fraction is obtained from the difference $N_0/N_B = 1 - N'/N_B$. By writing the integrals over wave vectors in units of k_F, result (36) reads

$$\frac{N'}{N_B} = \frac{8}{3\sqrt{\pi}} (n_B a^3)^{1/2} + (n_F b^3)^{2/3} B(w,\alpha). \tag{37}$$

As for result (15), the dimensionless function B depends on the mass ratio $w = m_B/m_F$ and on the parameter $\alpha = 2/(k_F \xi_B)^2$, and is given by

$$B(w,\alpha) = \frac{12}{6^{1/3}\pi^{2/3}} (1+w)^2 \int_0^\infty dk \int_{-1}^{+1} d\Omega \times \frac{k^2}{\sqrt{k^2+\alpha}} \int_0^1 dq q^2 \frac{1-\theta(1-\sqrt{q^2+k^2+2kq\Omega})}{(\sqrt{k^2+\alpha}+wk+2qw\Omega)^2} \times \left(\frac{1}{\sqrt{k^2+\alpha}} - \frac{\alpha(wk+2qw\Omega)}{2k^2(k^2+\alpha)} \right). \tag{38}$$

The dependence of the function B on the parameter α is shown in Fig. 3 for the mass ratio $w=1$. In the regime $k_F \xi_B \gg 1$ ($\alpha \ll 1$) the quantum depletion due to scattering processes with the fermions becomes independent of the Bose-Bose coupling constant and one finds

$$\frac{N'}{N_B} = \frac{8}{3\sqrt{\pi}} (n_B a^3)^{1/2} + (n_F b^3)^{2/3} B_1(w), \tag{39}$$

where the function B_1 of the mass ratio is given by

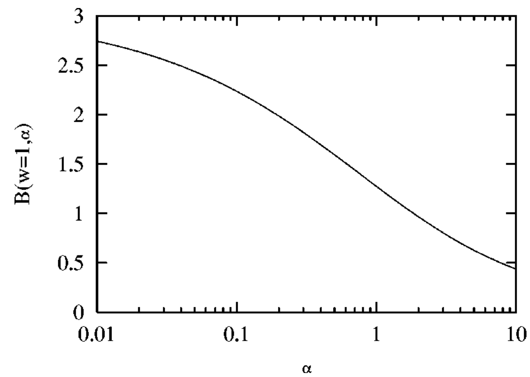

FIG. 3. Dimensionless function $B(w,\alpha)$ [Eq. (38)] as a function of $\alpha = 2/(k_F \xi_B)^2$ for the value $w=1$ of the mass ratio.

$$B_1(w) = \frac{12}{6^{1/3}\pi^{2/3}} (w+1) \int_{-1}^{+1} d\Omega \int_0^1 dq q^2 \times \frac{1}{(w+1)\sqrt{q^2(\Omega^2-1)+1} + (w-1)q\Omega}. \tag{40}$$

In the opposite regime $k_F \xi_B \ll 1$ ($\alpha \gg 1$), the quantum depletion takes important contributions from the region of low momenta $k < mc_B/\hbar$ and interaction effects among the bosons become crucial. In this regime, one finds $B(w,\alpha \to \infty) = 2B_2(w)/(3\sqrt{\alpha})$, where the function B_2 is given by

$$B_2(w) = \frac{12}{6^{1/3}\pi^{2/3}} (1+w)^2 \int_0^\infty dk \frac{k^2}{1+k^2} \times \frac{1}{(\sqrt{1+k^2}+wk)^2} \left(1 - \frac{w}{2k\sqrt{1+k^2}} \right) \tag{41}$$

and the quantum depletion takes, consequently, the form

$$\frac{N'}{N_B} = \frac{8}{3\sqrt{\pi}} (n_B a^3)^{1/2} + \frac{\pi^{1/6}}{6^{2/3}} \frac{b^2}{a^2} (n_B a^3)^{1/2} \frac{n_F}{n_B} B_2(w). \tag{42}$$

In Fig. 4 we plot the functions B_1 and B_2 of the mass ratio w. We notice that, while B_1 is always positive, B_2 is positive for small values of w and becomes large and negative when $m_B \gg m_F$. Thus, if the fermions are light enough compared to the bosons, their effect in the regime $k_F \xi_B \ll 1$ is to enhance the occupation of the condensate state resulting in a decrease of the quantum depletion. Of course, when B_2 is negative, result (42) is valid only if the second term is small compared to the first one.

Another quantity of interest is the momentum distribution of the bosons given by Eq. (35). For a pure system, the momentum distribution is given by the well-known Bogoliubov result

$$n_{\mathbf{k}}^B|_{pure} = \frac{x^2+1-x\sqrt{x^2+2}}{2x\sqrt{x^2+2}}, \tag{43}$$

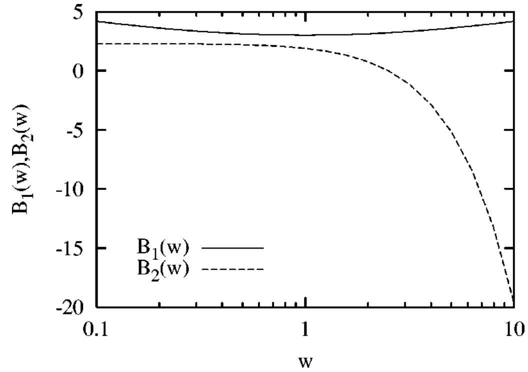

FIG. 4. Dimensionless functions B_1 [Eq. (40)] (solid line) and B_2 [Eq. (41)] (dashed line) of the mass ratio $w=m_B/m_F$.

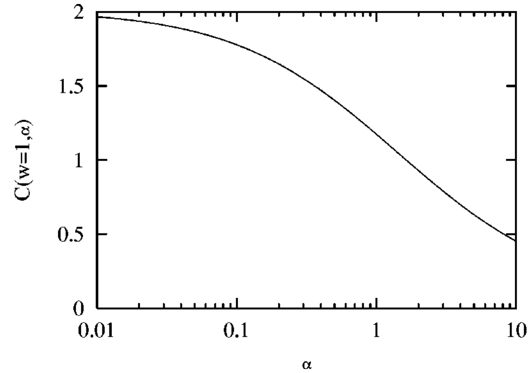

FIG. 5. Dimensionless function $C(w,\alpha)$ [Eq. (55)] as a function of $\alpha=2/(k_F\xi_B)^2$ for the value $w=1$ of the mass ratio.

which has been written in units of the inverse healing length $x=k\xi_B$. At small momenta $x\ll 1$, the momentum distribution is dominated by the quantum fluctuations of phonons which give rise to the infrared divergence $n_{\mathbf{k}}^B|_{pure}\sim 1/\sqrt{8}x$. At large momenta $x\gg 1$, one finds instead the algebraic decay $n_{\mathbf{k}}^B|_{pure}\sim 1/4x^4$. The important question arises concerning the effect of the fermionic component on $n_{\mathbf{k}}^B$. In units of the inverse healing length, result (35) reads

$$n_{\mathbf{k}}^B=n_{\mathbf{k}}^B|_{pure}+\frac{4(1+w)^2}{(6\pi^2)^{1/3}}(n_Fb^3)^{2/3}\frac{n_B}{n_F}\times\frac{k_F\xi_B}{\sqrt{x^2+2}}\int_{-1}^{+1}d\Omega\int_0^{k_F\xi_B}dyy^2\times\frac{1-\theta(k_F\xi_B-\sqrt{y^2+x^2+2xy\Omega})}{(\sqrt{x^2+2}+wx+2yw\Omega)^2}\times\left(\frac{1}{\sqrt{x^2+2}}-\frac{wx+2yw\Omega}{x^2(x^2+2)}\right). \quad (44)$$

In a mixture there are two characteristic wave numbers in the problem: ξ_B^{-1} and k_F. The structure of the distribution is consequently richer and depends on how ξ_B^{-1} compares with k_F. Nevertheless, very interesting conclusions can be drawn in the two limits $k\ll\xi_B^{-1},k_F$ and $k\gg\xi_B^{-1},k_F$. At low momenta ($k\ll\xi_B^{-1},k_F$) the effect of the Bose-Fermi coupling is to enhance the occupation of the condensate by stimulating scattering of particles from $\mathbf{k}\neq 0$ states into the condensate. The contribution to the momentum distribution is thus negative and reduces the coefficient of the $1/x$ divergent term

$$n_{\mathbf{k}}^B\sim\frac{1}{\sqrt{8}x}\left[1-\frac{6^{1/3}(1+w)^2}{4\pi^{1/3}w}\frac{b}{a}(n_Fb^3)^{1/3}\times f(2w/\sqrt{\alpha})\right], \quad (45)$$

where

$$f(t)=\frac{2+t}{1+t}-\frac{2}{t}\ln(1+t). \quad (46)$$

Notice that the modulus of the Bose-Fermi contribution can be written as $N(0)g_{BF}^2/2g_{BB}\cdot f(2w/\sqrt{\alpha})$ [see comment after Eq. (29)]. Phase stability of the mixture requires that in any case $N(0)g_{BF}^2/g_{BB}<1$. Since, moreover, $0<f(t)<1$ for all $t>0$, we conclude that the Bose-Fermi term is always less, in modulus, than the Bose-Bose one and thus Eq. (45) is well defined. It is also worth noticing that in the regime $k_F\xi_B\gg 1$ the function $f\to 1$ and $n_{\mathbf{k}}^B\sim(c/c_B)/\sqrt{8}x$, where c is the renormalized sound velocity from Eq. (29).

At large momenta ($k\gg\xi_B^{-1},k_F$) the Bose-Fermi coupling provides instead further depletion of the condensate and one finds

$$n_{\mathbf{k}}^B\sim\frac{1}{4x^4}\left[1+\frac{32}{3(6\pi^2)^{1/3}}(n_Fb^3)^{2/3}\frac{n_B}{n_F}(k_F\xi_B)^4\right]. \quad (47)$$

In the regime $k_F\xi_B\gg 1$ the shape of $n_{\mathbf{k}}^B$ can be very different compared to the pure case. In fact, the cut-off momentum increases from ξ_B^{-1} to k_F and the momentum distribution acquires a long tail at large momenta (see Fig. 5). The effect of the Bose-Fermi interaction on the momentum distribution and condensate fraction of the bosons has been calculated using variational methods in the hypernetted-chain scheme for the strongly correlated liquid ^{3}He-^{4}He mixture [14]. However, weak effects are found due to the low ^{3}He concentration attainable in stable mixtures.

D. Superfluid density

An important aspect of Bose-Fermi mixtures concerns their superfluid behavior. In a pure bosonic system at zero temperature the density of the normal component vanishes, $\rho_n=0$, and the system is completely superfluid. In a Bose-Fermi mixture the normal density does not vanish and can be written as $\rho_n=m_F^*n_F$ in terms of the effective mass m_F^* of the fermionic particles. In the absence of Bose-Fermi coupling, the effective mass coincides with the bare mass of the fermions, $m_F^*=m_F$, and the normal density of the mixture coincides with the mass density of the fermionic component. Interaction effects dress the fermions, resulting in an effective mass larger than the bare mass. A well-known example is provided by liquid ^{3}He-^{4}He solutions, where due to the strong correlations the effective mass of ^{3}He atoms is

$m_F^*/m_F \simeq 2.3$ [15]. An equivalent way of interpreting the excess of normal density in Bose-Fermi mixtures is to write $\rho_n = m_F n_F + \rho_n^B$, where ρ_n^B is the contribution arising from the excitation of bosonic quasiparticles due to the Bose-Fermi coupling. In terms of the mass difference $\delta m_F = m_F^* - m_F$, the bosonic component of ρ_n is given by $\rho_n^B = n_F \delta m_F$. The superfluid density of the system is defined as $\rho_s = \rho_B - \rho_n^B$. The bosonic component of the normal density can be calculated through the long-wavelength behavior of the static transverse current-current response function [16],

$$\rho_n^B = -\lim_{\mathbf{q}\to 0} G_T(\mathbf{q},\omega=0). \tag{48}$$

In the above equation $G_T(\mathbf{q},t) = -i/\hbar \langle T(j_{\mathbf{q}}^x(t) j_{-\mathbf{q}}^x(0))\rangle$ is the bosonic transverse current-current response function, with $\mathbf{q}=(0,0,q)$ directed along z and the x component of the current operator defined by $j_{\mathbf{q}}^x = 1/\sqrt{V}\Sigma_{\mathbf{k}} \hbar k_x a_{\mathbf{k}}^\dagger a_{\mathbf{k}+\mathbf{q}}$. In terms of single-particle Green's functions, $G_T(\mathbf{q},\omega)$ can be written as

$$G_T(\mathbf{q},\omega) = \frac{i\hbar}{2\pi}\frac{1}{V}\sum_{\mathbf{k}} (\hbar k_x)^2 \int_{-\infty}^{+\infty} d\omega' \times [G_{11}(\mathbf{k},-\omega')G_{11}(\mathbf{k}+\mathbf{q},\omega-\omega') - G_{21}(\mathbf{k},\omega')G_{12}(\mathbf{k}+\mathbf{q},\omega-\omega')], \tag{49}$$

where $G_{12}(\mathbf{k},t) = -i/\hbar \langle T(a_{\mathbf{k}}(t) a_{-\mathbf{k}}(0))\rangle$ and $G_{21}(\mathbf{k},t) = -i/\hbar \langle T(a_{\mathbf{k}}^\dagger(t) a_{-\mathbf{k}}^\dagger(0))\rangle$ are the anomalous single-particle Green's functions, which are peculiar of Bose condensed systems. The unperturbed anomalous single-particle Green's functions can be readily obtained and one finds

$$G_{12}^0(\mathbf{k},\omega) = G_{21}^0(\mathbf{k},\omega) = \frac{u_k v_k}{\hbar\omega - \omega_k + i\eta} - \frac{u_k v_k}{\hbar\omega + \omega_k - i\eta}. \tag{50}$$

To second order in g_{BF}, we find the result

$$G_{12}(\mathbf{k},\omega) = G_{21}(\mathbf{k},\omega) = G_{12}^0(\mathbf{k},\omega) + g_{BF}^2 n_0 (u_k+v_k)^2 \Pi_0^F(\mathbf{k},\omega) \times [(u_k^2+v_k^2) D_0(\mathbf{k},\omega) D_0(\mathbf{k},-\omega) + u_k v_k D_0^2(\mathbf{k},\omega) + u_k v_k D_0^2(\mathbf{k},-\omega)]. \tag{51}$$

From the perturbation expansions (33) and (51) one gets the following result for the long-wavelength limit of the static current-current response function:

$$\lim_{\mathbf{q}\to 0} G_T(\mathbf{q},\omega=0) = \frac{i\hbar}{2\pi}\frac{2}{V}\sum_{\mathbf{k}} (\hbar k_x)^2 g_{BF}^2 n_0 (u_k+v_k)^2 \times \int_{-\infty}^{+\infty} d\omega \Pi_0^F(\mathbf{k},\omega) D_0^3(\mathbf{k},\omega), \tag{52}$$

valid to order g_{BF}^2. The fraction of Bose particles that contribute to the normal density of the system is thus given by the following expression

$$\frac{\rho_n^B}{\rho_B} = \frac{4}{3} g_{BF}^2 \frac{1}{V}\sum_{\mathbf{k}} \frac{(\epsilon_k^B)^2}{\omega_k} \frac{1}{V}\sum_{\mathbf{q}} \frac{n_{\mathbf{q}}^F (1-n_{\mathbf{q}+\mathbf{k}}^F)}{(\omega_k + \epsilon_{|\mathbf{k}+\mathbf{q}|}^F - \epsilon_q^F)^3}. \tag{53}$$

By expressing the integrals in units of the Fermi wave vector k_F, one finds the result

$$\frac{\rho_n^B}{\rho_B} = (n_F b^3)^{2/3} C(w,\alpha), \tag{54}$$

in terms of the Fermi gas parameter and of the dimensionless function

$$C(w,\alpha) = \frac{16}{6^{1/3}\pi^{2/3}}(1+w)^2 \int_0^\infty dk \int_{-1}^{+1} d\Omega \times \frac{k^2}{\sqrt{k^2+\alpha}} \int_0^1 dq q^2 \frac{1-\theta(1-\sqrt{q^2+k^2+2kq\Omega})}{(\sqrt{k^2+\alpha}+wk+2qw\Omega)^3}. \tag{55}$$

In Fig. 5 we show the dependence of the above function on the parameter $\alpha = 2/(k_F\xi_B)^2$ for the value $w=1$ of the mass ratio. In the regime $k_F\xi_B \gg 1$ the function C becomes independent of the parameter α and result (54) takes the form

$$\frac{\rho_n^B}{\rho_B} = (n_F b^3)^{2/3} C_1(w), \tag{56}$$

where C_1 is given by

$$C_1(w) = \frac{16}{6^{1/3}\pi^{2/3}} \int_{-1}^{+1} d\Omega \int_0^1 dq q^2 \times \frac{(1+w)\sqrt{q^2(\Omega^2-1)+1} - q\Omega}{[(1+w)\sqrt{q^2(\Omega^2-1)+1} + (w-1)q\Omega]^2}. \tag{57}$$

In the opposite regime, $k_F\xi_B \ll 1$, the function C takes the following asymptotic value: $C(w,\alpha\to\infty) = 2C_2(w)/(3\sqrt{\alpha})$ and one finds

$$\frac{\rho_n^B}{\rho_B} = \frac{\pi^{1/6}}{6^{2/3}} \frac{b^2}{a^2} (n_B a^3)^{1/2} \frac{n_F}{n_B} C_2(w), \tag{58}$$

where the function C_2 is given by

$$C_2(w) = \frac{16(1+w)^2}{6^{1/3}\pi^{2/3}} \int_0^\infty dk \frac{k^2}{\sqrt{1+k^2}(\sqrt{1+k^2}+wk)^3}. \tag{59}$$

Results (58)–(59) are in agreement with the finding of Ref. [11] where the fermion effective mass has been calculated for $k_F\xi_B \ll 1$ in the case of equal scattering lengths $a=b$. In Fig. 6 we show both C_1 and C_2 as a function of the mass ratio.

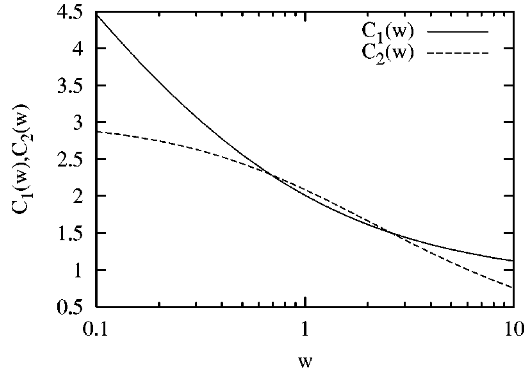

FIG. 6. Dimensionless functions C_1 [Eq. (58)] (solid line) and C_2 [Eq. (59)] (dashed line) of the mass ratio $w=m_B/m_F$.

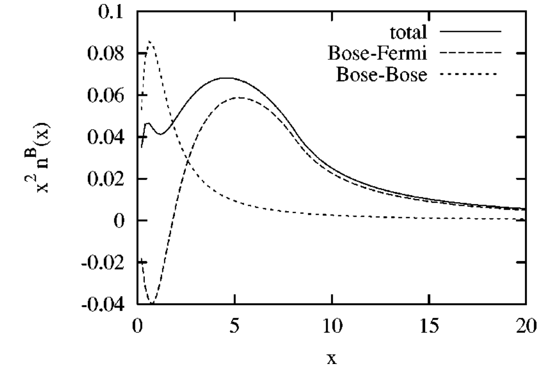

FIG. 7. Boson momentum distribution [Eq. (44)] of a ^{87}Rb-^{40}K mixture (solid line). The momentum distribution of a pure system (short-dashed line) and the Bose-Fermi contribution (long-dashed line) are also shown. In the figure, as in the text, x stands for $k\xi_B$.

III. APPLICATIONS TO REAL SYSTEMS

In deriving the above results we have been completely general. In this section instead, we look at their implications on actual physical systems and we discuss more quantitatively the striking consequences of the presence of fermions on the Bose-Einstein condensate which we have pointed out.

First of all, we have to spend a few words on phase stability. The obvious requirement for all of the above to be valid is that bosons and fermions mix in the first place. According to the findings of Ref. [5], one has to separately analyze two cases: attractive Bose-Fermi interaction, i.e., $g_{BF}<0$, and repulsive one, i.e., $g_{BF}>0$. The former case is simpler, as the only requirement is that arising from linear stability: $N(0)g_{BF}^2/g_{BB}<1$, where $N(0)=m_F k_F/(2\pi^2\hbar^2)$ is the density of states at the Fermi surface. This condition fixes an upper limit on the fermionic density irrespective of the bosonic one,

$$n_{F,max}=\frac{4\pi}{3}\frac{w^3}{(1+w)^6}\left(\frac{a}{b}\right)^3\frac{1}{b^3}. \tag{60}$$

If instead g_{BF} is greater than zero, while the fermionic density cannot in any case be larger than $n_{F,max}$, yet for every $n_F<n_{F,max}$ there is a critical $n_{B,c}(n_F)$ at which the system phase separates anyway. The function $n_{B,c}(n_F)$ is nontrivial, and one has to check specifically for a given pair of elements and densities whether the system is stable or not.

For this reason it is certainly easier to observe important effects in a mixture where g_{BF} is negative, as one can reach higher densities without running into problems. On the other hand, since our calculations are to order g_{BF}^2, all effects computed here are identical for both signs of g_{BF}.

A mixture with negative g_{BF}, and supposingly large b, is already available experimentally. This is the ^{40}K-^{87}Rb mixture recently reported in Ref. [3]. According to the authors $b=300(100)a_0$ in units of the Bohr radius.

We consider a mixture of ^{87}Rb and ^{40}K with densities $n_B=n_F=3\times10^{14}$ cm^{-3}. For the scattering lengths we use $a=110a_0$ and $b=300a_0$, and the mass ratio is $w=2.175$. The value of the parameter $k_F\xi_B$ is 3.9.

In Fig. 7 we show results for the boson momentum distribution of the mixture, obtained from Eq. (44) with the parameters given above, compared with the one of a pure system [Eq. (43)]. The total value of the quantum depletion is $\sim3\%$ for this configuration and the Bose-Bose and Bose-Fermi contributions are $\sim1\%$ and $\sim2\%$, respectively (see Fig. 8). We see a large effect due to boson-fermion interactions. The momentum distribution at low momenta is depressed while at large momenta is considerably enhanced and a tail appears which extends to very large momenta.

In Fig. 8 we report results on the dependence of the condensate depletion and normal fluid fraction as a function of the fermion density n_F. The boson density has been fixed to 3×10^{14} cm^{-3}. With the above values of the parameters, the system becomes unstable at a fermion density of 5.2×10^{14} cm^{-3}. We see that, by increasing the fermion density, both the condensate depletion and the normal fluid fraction increase. The prediction for fermion densities close to the instability threshold gives $N'/N_B\sim4\%$ and $\rho_n^B/\rho_B\sim2\%$.

In Fig. 9 we show the condensate depletion and the normal fluid fraction of a hypothetical mixture with mass ratio $m_B/m_F=0.1$. We also assume $g_{BF}<0$ and we have fixed the scattering lengths to $a=100a_0$ and $b=50a_0$, and the boson density to $n_B=3\times10^{14}$ cm^{-3}. Such system becomes unstable for $n_F\geqslant1.0\times10^{18}$ cm^{-3}. We see that an increase of n_F is followed by an increase of both the condensate depletion and the normal fluid fraction. However, at the fermion density $n_F\sim2\cdot10^{17}$ cm^{-3}, the normal fluid fraction becomes larger than the condensate depletion. This effect has been first discussed in connection with disordered dilute Bose gases by Huang and Meng [17] and very recently has been explicitly proved in a Bose gas with quenched impurities using quantum Monte-Carlo techniques [18]. It is striking that the same effect is found for Bose-Fermi mixtures in regimes of high fermion concentration when $m_F\gg m_B$.

Finally, in Fig. 10 we present results of the condensate depletion and normal fluid fraction of a different hypothetical mixture with mass ratio $m_B/m_F=20$. We assume again an attractive Bose-Fermi coupling ($g_{BF}<0$) and we use the following parameters: $a=100a_0$, $b=150a_0$, and $n_B=3\times10^{14}$ cm^{-3}. The mixture becomes unstable for $n_F\geqslant2.3\times10^{14}$ cm^{-3}. In this case we see a different striking behavior: for low fermion concentration the effect of the Bose-

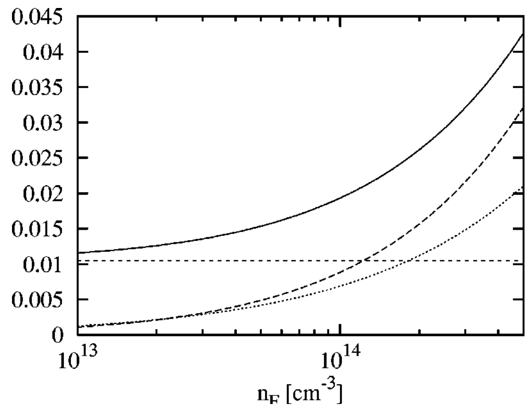

FIG. 8. Condensate depletion and normal fluid fraction of a ^{87}Rb-^{40}K mixture as a function of the fermion density. The total condensate depletion is shown with the solid line. The Bogoliubov and the fermion depletion are shown, respectively, with the horizontal short-dashed line and the long-dashed line. The dotted line corresponds instead to the normal fluid fraction.

Fermi coupling is to stimulate the occupation of the condensate, resulting in a quantum depletion smaller than in the pure case.

Due to the smallness of the effects, the interest of the results reported in Figs. 9 and 10 is to show that they can actually occur in realistic mixtures, though it might be difficult to observe them with present techniques.

In the present paper we have studied the ground-state properties of homogeneous Bose-Fermi mixtures. An interesting question arises as to how one can extend these results to investigate trapped mixtures. This will be the object of future studies. The problem of trapped mixtures is more complicated because the Bose-Fermi coupling can also strongly affect the density profiles of the two species [4]. However, it is worth pointing out here that the parameter $k_F\xi_B$, which has been a key parameter in our previous discussion, takes on a very simple form in terms of the characteristic parameters of trapped systems. Within the local density approximation the value of $k_F\xi_B$ in the center of the trap can be expressed as

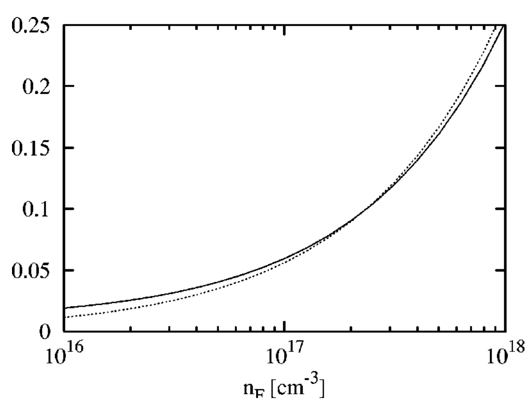

FIG. 9. Condensate depletion (solid line) and normal fluid fraction (dotted line) of a hypothetical mixture with mass ratio $m_B/m_F=0.1$ as a function of the fermion density.

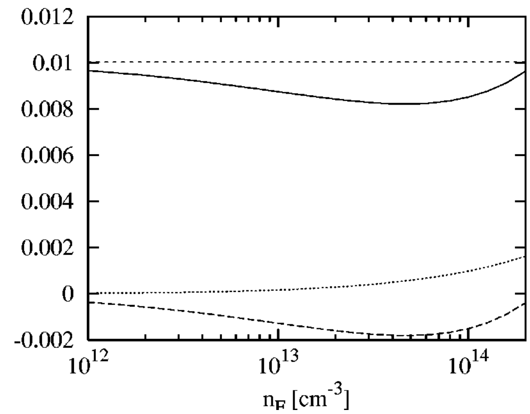

FIG. 10. Condensate depletion and normal fluid fraction of a hypothetical mixture with mass ratio $m_B/m_F=20$ as a function of the fermion density. The line codes used are the same as for Fig. 8.

$$k_F\xi_B=\frac{1}{\sqrt{w}}\frac{R_F}{R_B}, \tag{61}$$

in terms of the mass ratio $w=m_B/m_F$ and the Thomas-Fermi radii $R_B=\ell_B(15N_Ba/\ell_B)^{1/5}$ and $R_F=\ell_F\sqrt{2}(6N_F)^{1/6}$, corresponding, respectively, to the boson and fermion cloud. The boson (fermion) harmonic oscillator length is defined as $\ell_{B,F}=\sqrt{\hbar/m_{B,F}\omega_{B,F}}$, where $\omega_{B,F}$ are the geometric averages of the boson (fermion) trapping frequencies. To obtain the above result, we have assumed that the harmonic potential energies of the two species are the same at the same distance from the trap center: $\omega_B=(m_F/m_B)^{1/2}\omega_F$, as is the case, for instance, in the ^{87}Rb-^{40}K experiment of Ref. [3]. From Eq. (61), the regimes $k_F\xi_B\gg1(\ll1)$ discussed above have a simple interpretation in terms of the ratio of the corresponding radii of the fermion and boson cloud.

IV. CONCLUSIONS

We have studied the properties of a dilute Bose-Fermi mixture at zero temperature using a perturbation approach. We have investigated both equilibrium properties, such as the ground-state energy, the boson momentum distribution, and the normal fluid fraction, and dynamic properties such as the dispersion law and damping of phonon excitations. The system is very rich. By varying the mass ratio, the densities and the scattering lengths of the two components one can obtain very different regimes where striking effects due to boson-fermion interactions can occurr. These include: the strong suppression of the boson momentum distribution at low momenta, localization effects exhibited by the superfluid density which becomes smaller than the condensate fraction and stimulated scattering of bosons into the condensate.

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