# Uniform electromagnetic field as viscous medium for moving particles

M. Ya. Amusia,[1,2] A. S. Baltenkov,[1,3] Z. Felfli,[4] A. Z. Msezane,[4] and A. B. Voitkiv[3,5]
[1]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[2]*A. F. Ioffe Physical-Technical Institute, St. Petersburg 194021, Russia*
[3]*Arifov Institute of Electronics, 700125 Tashkent, Uzbekistan*
[4]*Center for Theoretical Studies of Physical Systems, Clark Atlanta University, Atlanta, Georgia 30314*
[5]*Max-Planck-Institute for Nuclear Physics, Heidelberg D-69117, Germany*
(Received 28 March 2002; published 31 December 2002)

The mechanism of transverse radiation viscosity acting on free charges, atomic, and small macroscopic particles in uniform electromagnetic fields is analyzed. It is shown that in the process of light scattering by these particles, besides the force accelerating them in the direction of propagation of the radiation, there is a force in the transverse direction slowing them down. The general expression for this force is obtained. It is considered how this force can influence: (i) the motion of ultrarelativistic electrons in transverse photon fluxes; (ii) the behavior of a beam of nonrelativistic electrons moving in a copropagating uniform electromagnetic field; (iii) the transverse motion of atoms under the action of resonant radiation and (iv) the motion of small macroscopic particles.

DOI: 10.1103/PhysRevA.66.063416 PACS number(s): 42.50.Vk, 31.30.Jv, 32.90.+a, 34.10.+x

## I. INTRODUCTION

There are two qualitatively different optical forces that act on an atomic particle in an electromagnetic field [1–3]. One of them is the force due to elastic scattering of photons, which accelerates a particle in the direction of propagation of the light wave. The other one is the dipole force due to the interaction of the induced atomic dipole moment with the electric-field gradient of the light wave; this leads to the focusing or defocusing of the atomic beam. These optical forces were used in experiments to deflect [4,5], focus [6–8], and accelerate [9] atomic beams, as well as in laser cooling [10,11] and the deceleration [12] of single atoms. For charges without a proper or induced dipole moment the second force is absent. Therefore, the single dissipative spontaneous force that acts on a charge in the field of a plane monochromatic electromagnetic wave is the force due to the elastic-scattering radiation [13]. It is normally assumed that the scattering of a photon by a free charge leads to its acceleration along the direction of propagation of the electromagnetic wave. In the present paper we analyze the other feature of elastic scattering of a photon by a charge, namely, the effect of the "viscosity" of light that influences the motion of the charge in the directions perpendicular to the axis of the beam of light.

The physical reason for this effect is obvious from the following qualitative consideration of the scattering process. Let us consider a charge that is initially at rest in the laboratory coordinate system. Under the action of a plane wave of light the charge makes small harmonic oscillations near this point. Assume that the velocity of the oscillatory motion of the charge in the electric field of the light wave is small compared to the velocity of light. Then the scattering process can be considered in the dipole approximation. In this case the scattering cross section of the unpolarized light wave by the charge in the solid angle $d\Omega$ is given by [13,14]

$$d\sigma=\frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\theta)d\Omega. \tag{1}$$

Here $m$ is the mass of the charge, $c$ is the velocity of light, $\theta$ is the angle between the directions of the incident and scattered light waves. According to Eq. (1), the total momentum of the scattered radiation is equal to zero in the coordinate system centered at the point relative to which the charge oscillates. Therefore, the total momentum of the incident radiation is absorbed by the particle, and the average force due to the pressure of light on the particle is [13]

$$\vec{F}_{\parallel}=\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 W\vec{n}, \tag{2}$$

where $W[\mathrm{J\,cm^{-3}}]$ is the energy density of the electromagnetic field, $\vec{n}$ is the unit vector determined by the direction of the incident wave. Under the action of this force the center of oscillations, initially at rest, starts to move along $\vec{n}$. Therefore, on the average, the charge itself is accelerated along the direction of the incident radiation. This is a manifestation of the pressure of electromagnetic radiation. The force, Eq. (2) is well known and is described in books of classical electrodynamics (see, for example Refs. [13,14]).

Let us analyze how this physical picture changes when the charge is initially not at rest but is moving perpendicular to the axis of the beam of light with velocity $\vec{v}_\perp$. The motion of the charge in the electromagnetic field can be considered as a superposition of two motions. The first is the oscillatory motion under the action of the electric field of the light wave and the progressive motion with velocity $\vec{v}_\perp$ of the center of oscillations. The scattering cross section in the coordinate system relative to the center of oscillations is described by Eq. (1), in which the total momentum of the scattered radiation is zero. In the laboratory coordinate system the transverse momentum $\vec{v}_\perp\hbar\omega/c^2$ is transferred to each of the scattered photons with the mass $\hbar\omega/c^2$. This momentum transfer to the scattered photons decreases the velocity of the particle $\vec{v}_\perp$ in the direction perpendicular to $\vec{n}$, which is equivalent to the action of a transverse force $\vec{F}_\perp$ upon the charge. The magnitude of $\vec{F}_\perp$ is determined by the momentum $\vec{v}_\perp \hbar\omega/c^2$ and the time interval during which this momentum is transferred to the scattered photon. This time interval is defined by the flux density of the photon $Wc/\hbar\omega$ and the total cross section $(8\pi/3)(e^2/mc^2)^2$ of the scattered radiation. Therefore, the transverse force, that slows down the charge is given by

$$\vec{F}_\perp = -\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 W\frac{\vec{v}_\perp}{c}. \tag{3}$$

Thus, in addition to the longitudinal force $\vec{F}_\parallel$, a specific Stokes force $\vec{F}_\perp$ due to radiation viscosity acts on a charge that moves in a spatially uniform electromagnetic field slowing down the particle in the directions perpendicular to the propagation of the light. It is clear that the existence of this force is due to the Doppler effect, since it causes the photons scattered in the direction of the oscillatory motion to be harder than those emitted in the opposite direction.

In spite of the fact that the physical concept of the phenomenon of the viscosity of light is simple, the effect of $\vec{F}_\perp$ on the processes caused by the pressure of the electromagnetic radiation has, to our knowledge, never been considered. Perhaps, this is because this force is relatively weak in comparison with the longitudinal force. In the present paper we analyze what physical effects result from this force acting on free charges and atomic particles in strong fluxes of electromagnetic radiation.

In Sec. II we derive the general expressions for the forces of the pressure of light acting on particles moving with arbitrary velocities, including those close to the velocity of light. In Sec. III we consider some examples of the action of the force of viscosity on electrons, atoms, and macroscopic particles. Namely, in Secs. III A and III B we discuss the effect of the viscosity of light on ultrarelativistic electrons moving in an electromagnetic field and the possibility of collimating nonrelativistic electron beams in the coaxial fluxes of laser radiation, respectively. In Sec. III C we analyze the influence of the force of the viscosity of light on the behavior of atoms in resonant electromagnetic radiation. The effect of the viscosity of light on macroscopic particles is briefly touched upon in Sec. III D.

## II. GENERAL FORMULAS FOR THE FORCE OF LIGHT PRESSURE

For a charge moving with a velocity $v \ll c$ the force of viscosity Eq. (3) is small compared to the longitudinal force Eq. (2). As will be seen below the relation between the forces becomes quite different when $v \to c$. Let us derive the expressions for the longitudinal force and the force of the viscosity of light by calculating the electromagnetic pressure upon the particle in the coordinate system of the moving charge. Suppose that the particle is moving with velocity $\vec{v}$ relative to a radiating source which is at rest in the laboratory coordinate system, $\omega$ is the radiation frequency in this coordinate system and $\vec{k} = k\vec{n}$ is the wave vector of the incident electromagnetic wave located within the $xy$ plane at an angle $\alpha$ relative to the $x$ axis (Fig. 1). Thus, the four-dimensional wave vector of the incident radiation has in the laboratory coordinate system the components $\{\omega/c, k\cos\alpha, k\sin\alpha, 0\}$.

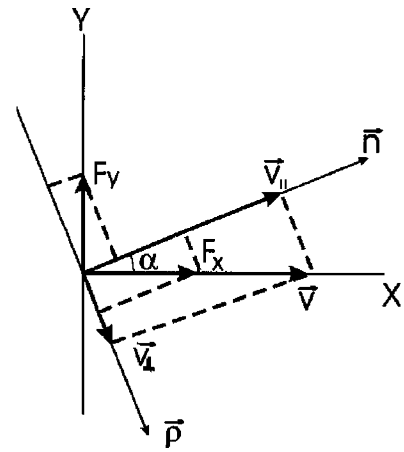


FIG. 1. Coordinate system of a particle moving with velocity $\vec{v}$ relative to a radiating source at rest in the laboratory reference frame.

In the moving (primed) reference system, connected to the point relative to which the charge oscillates, this four-dimensional vector is defined by [13]

$$\frac{\omega'}{c} = \frac{\omega}{c}\gamma(1-\beta\cos\alpha); \quad k'_x = \frac{\omega}{c}\gamma(\cos\alpha - \beta);$$

$$k'_y = \frac{\omega}{c}\sin\alpha; \quad k'_z = 0, \tag{4}$$

which follows from the Lorentz transformation of four vectors. Here $\beta = v/c$ and $\gamma = (1-\beta^2)^{-1/2}$. The components of the unit vector $\vec{n} = \vec{k}'/k'$, which defines the direction of the wave-vector $\vec{k}'$ in the primed coordinate system, are

$$\vec{n}'\left\{\frac{k'_x}{k'}, \frac{k'_y}{k'}, \frac{k'_z}{k'}\right\} = \vec{n}'\left\{\frac{(\cos\alpha-\beta)}{1-\beta\cos\alpha}, \frac{\sin\alpha}{\gamma(1-\beta\cos\alpha)}, 0\right\}. \tag{5}$$

The energy density of the electromagnetic field in the primed coordinate system is related to the radiation density in the laboratory system through [13]

$$W' = W\gamma^2(1-\beta\cos\alpha)^2. \tag{6}$$

If the velocity of the oscillatory motion of the charge is small compared to $c$, the total momentum of the scattered radiation in the dipole approximation is zero in the primed coordinate system. Hence, the average force of light pressure acting on the charge in this coordinate system is [15]

$$\vec{F}' = W'\sigma(\omega')\vec{n}', \tag{7}$$

where $\sigma(\omega')$ is the total cross section of the scattered light by the free charge. The components of the four-vector force

$g'$ in the primed coordinate system are expressed in terms of the three-dimensional force Eq. (7) as [13,14]

$$g' = \{\vec{F}' \cdot \vec{v}\,\gamma/c^2, F'_x\gamma/c, F'_y\gamma/c, F'_z\gamma/c\}. \tag{8}$$

Transforming $g'$ into the laboratory coordinate system, we obtain for the Cartesian components of the usual three force

$$F_x = F'_x = W\sigma(\omega')\gamma^2(1-\beta\cos\alpha)(\cos\alpha-\beta),$$

$$F_y = F'_y/\gamma = W\sigma(\omega')(1-\beta\cos\alpha)\sin\alpha,$$

$$F_z = F'_z/\gamma = 0. \tag{9}$$

According to Eq. (9), the longitudinal and transverse forces of the light pressure are expressed as

$$\vec{F}_\parallel = (F_x\cos\alpha + F_y\sin\alpha)\vec{n}$$
$$= W\sigma(\omega')\gamma^2(1-\beta\cos\alpha-\beta^2\sin^2\alpha)(1-\beta\cos\alpha)\vec{n},$$

$$\vec{F}_\perp = (F_x\sin\alpha - F_y\cos\alpha)\vec{\rho}$$
$$= -W\sigma(\omega')\gamma^2\beta\sin\alpha(1-\beta\cos\alpha)^2\vec{\rho}. \tag{10}$$

Here $\vec{\rho}$ is the unit vector. This vector is located in the $xy$ plane and is perpendicular to $\vec{n}$. In the limiting case $v \ll c$, Eqs. (10) reduce to Eqs. (2) and (3), because $\beta\vec{\rho}\sin\alpha = \vec{v}_\perp/c$.

Let us compare the forces in Eqs. (10) in the relativistic case. For the parallel motion, $\alpha=0$, of the ultrarelativistic particles ($\gamma \gg 1$) and photons both forces are equal to zero. For $\alpha=\pi$ (opposite to the motion of photons and charges), the transverse force is zero. The longitudinal force is defined by $F_\parallel = 4W\sigma(2\omega\gamma)\gamma$. For angles $0<\alpha<\pi/2$ we have, as in the nonrelativistic case, $|\vec{F}_\perp|/|\vec{F}_\parallel| \approx |\vec{v}_\perp|/c \leqslant 1$. For the perpendicular crossing, $\alpha=\pi/2$, of the ultrarelativistic charges and photon beams we have $|\vec{F}_\perp|/|\vec{F}_\parallel| \approx \gamma^2|\vec{v}_\perp|/c \gg 1$. Thus, in the latter case the force $\vec{F}_\perp$ strongly dominates the force $\vec{F}_\parallel$. This is connected with the so-called "projector" effect according to which an ultrarelativistic charged particle emits radiation into a small cone in the direction of its motion [13].

## III. SOME EXAMPLES OF THE EFFECT OF THE LIGHT VISCOSITY ON THE MOTION OF ELECTRONS, ATOMS, AND MACROSCOPIC PARTICLES

### A. Ultrarelativistic electron motion in transverse photon fluxes

Let us assume that the ultrarelativistic electron, which moves along a circular trajectory in a magnetic field, crosses many times a radial light beam. Moving in the area under irradiation, as in a medium with viscosity, the electron loses part of its kinetic energy. If this energy is comparable to the energy of the synchrotron radiation [13,14] emitted by the electron on this part of its trajectory, we can expect significant corrections to the rate of change of the energy, which can be experimentally observed.

The change of the electron's kinetic energy due to the force of the viscosity of light in the area under irradiation is

$$\Delta E = F_\perp D = W\sigma\gamma^2 D. \tag{11}$$

Here, $\sigma=(8\pi/3)r_e^2$ is the Thomson elastic-scattering cross section of the photon by the free electron, $r_e=e^2/mc^2$ is the classical electron radius and $D$ is the distance traversed. Let us estimate the energy density $W$ of the irradiation field for the $\Delta E$ to be of the same order as the energy of emitted synchrotron radiation on the same part of the electron orbit. The intensity of synchrotron radiation of the ultrarelativistic electron is defined by [13]

$$I = \frac{2e^2c}{3R^2}\gamma^4. \tag{12}$$

Here $R$ is the radius of the circular orbit of the electron in the magnetic field. On the side of the trajectory with length equal to $D$, the energy emitted by the electron moving with velocity $v \sim c$ is $ID/c$. Equating these energies, we obtain the energy density $W$ of the radiation field

$$W = \frac{2e^2\gamma^2}{3R^2\sigma}. \tag{13}$$

For estimation, let us assume that $R=10^3$ cm and $\gamma=10$. Then, from Eq. (13) we obtain for the energy density of the light field $W=0.23\times10^{-5}$ J cm$^{-3}$, which corresponds to a radiation intensity $P\approx10^5$ W cm$^{-2}$. Such an intensity is easily accessible in current experiments with laser radiation, particularly if we take into account that the resonator mirrors can effectively increase $W$.

### B. Viscosity effect on beams of nonrelativistic electrons moving in a copropagating uniform electromagnetic field

How can the force of the viscosity of light act on beams of nonrelativistic electrons moving in a copropagating uniform strong electromagnetic field? Such a problem may have practical applications [16]. Suppose that the electrons are injected into the light beam along its axis (taken as the x axis) with an initial longitudinal velocity $v_x$ [$v_y(0)=v_z(0)=0$]. Let us consider, for example, the electron motion along the $y$ axis. In the preceding section we have derived the formulas for the forces acting on particles which appear due to the transformation of the incoming electromagnetic wave into the scattered one. This derivation was based on the classical picture for the scattering of electromagnetic radiation. Since the quantum nature of the electromagnetic radiation was not taken into account, the forces given by Eqs. (10) correspond to the mean (classical) action of the field on the particles. In fact, because of the quantum character of the scattering, the particle in every scattering act gets a recoil momentum which is randomly orientated in space. Because of this reason the transverse motion of the electrons in the uniform electromagnetic field is quite similar to the motion of the Brownian particles and can be classically described by the Langevin equation (see e.g., Ref. [17])

$$m\frac{dv_y}{dt}=-\eta v_y+f_y(t). \tag{14}$$

The first term on the right-hand side of Eq. (14) is the force of viscosity with $\eta=W\sigma/c$. $f_y(t)$ is the stochastic force which is introduced to describe that part of the action of radiation which is related to the statistical nature of the spontaneous emission in the process of the Thomson scattering. The latter force has the well-known properties (see e.g., Ref. [17]):

$$\langle f_y(t)\rangle=0,\langle f_y(t+\tau)f_y(t)\rangle=D_{yy}\delta(\tau). \tag{15}$$

Here and below $\langle\,\rangle$ denotes the averaging over the electron ensemble. Further, $D_{yy}$ is the coefficient of the electron diffusion in the momentum space which, in this case, is approximated by $D_{yy}\sim\hbar kW\sigma$.

Using Eqs. (14) and (15) and the initial condition $v_y(0)$, one can obtain that the mean-squared deviation of the electrons from their initial positions, $\sqrt{\langle(y(t)-y(0))^2\rangle}$, is given by

$$\langle[y(t)-y(0)]^2\rangle=\frac{D_{yy}}{\eta^2}\left[t-\frac{2m}{\eta}[1-\exp(-\eta t/m)]+\frac{m}{2\eta}[1-\exp(-2\eta t/m)]\right]. \tag{16}$$

There are two different limiting cases where the solution Eq. (16) has an especially simple form. These are the cases of small ($t\ll m/\eta$) and large ($t\gg m/\eta$) time $t$. In the former one has

$$\langle[y(t)-y(0)]^2\rangle\approx\frac{D_{yy}}{3m^2}t^3,\quad t\ll m/\eta, \tag{17}$$

whereas in the latter one obtains

$$\langle[y(t)-y(0)]^2\rangle\approx\frac{D_{yy}}{\eta^2}t,\quad t\gg m/\eta. \tag{18}$$

It is clear that the solution similar to Eqs. (16) and (18) holds also for the electron motion along the $z$ axis.

The mean-squared deviation in the electron coordinates given by Eq. (16) (and the similar equation for the $z$ component) give an estimate for the size of the electron spot on the target in the case if initially the electron beam is narrow enough. Neglecting the viscosity of light, one would obtain that the spot size is given by Eq. (17), because only the region of the small time can be realized when $\eta=0$. On the other hand when the light viscosity is present and the time of the electron propagation, estimated as $t_0=L/v_\parallel$ ($L$ is the distance which the electrons travel before they reach the target), is comparable or higher than $m/\eta=mc/(W\sigma)$ the spot size is estimated by Eq. (18). Assuming that the latter condition is fulfilled, we obtain that under the action of the radiation viscosity the radius of the electron spot decreases by a factor $\lambda\sim\eta t/m$. Taking that $L=10^6$ cm (such beam lengths were considered e.g. in Ref. [16]), $v_x=0.1c$ and $P=Wc\approx10^{15}\ \mathrm{W\,cm^{-2}}$, we obtain $\lambda\sim\sqrt{3}\approx1.7$. Thus, in the case under consideration the effect of the viscosity of light is quite substantial. Of course, the value of the field intensity was taken to be quite large. However, modern lasers make it possible to reach intensities of $P\approx10^{18}-10^{20}\ \mathrm{W\,cm^{-2}}$ [18,19] and even higher. This, in principle, permits an experimental observation of the predicted effect of the viscosity of light for electrons. Already, the scattering of laser radiation by free electrons under laboratory conditions (where the size of the region of the electron and photon interaction is thousands times smaller than that we considered) was observed experimentally [20–23].

### C. Viscosity effect on atomic particles moving in resonant electromagnetic field

The reason for the appearance of the forces in Eq. (10) is the fact that the scattering indicatrix is symmetric in the reference system connected with the scattering particle. Consequently, our consideration is applicable also to the scattering of the monochromatic radiation by a single atomic particle. In this case, instead of the differential cross section Eq. (1) we have [24]

$$d\sigma=\frac{1}{2}\left(\frac{\omega}{c}\right)^4|\alpha_d(\omega)|^2(1+\cos^2\vartheta)d\Omega. \tag{19}$$

Here, $\omega$ is the frequency of the scattered radiation, $\alpha_d(\omega)$ is the dynamic polarizability of the atom. According to Eq. (19), in this case the total momentum flux of the scattered photons is zero in the reference system centered on the atomic nucleus. Replacing in Eqs. (2), (3), and (10) the cross section for the scattering of light by the charge by the total cross section of the scattering of a photon by an atom

$$\sigma(\omega)=\frac{8\pi}{3}\left(\frac{\omega}{c}\right)^4|\alpha_d(\omega)|^2, \tag{20}$$

we obtain the longitudinal force and the force of the viscosity of light that act on a single atomic particle in a uniform field of light.

In experiments aimed at observing the effects connected with the pressure of light on the atomic particles, resonance laser radiation (with $\omega\approx\omega_0$), which saturates the atomic transition, is usually used. To reach this saturation the average intensity of the laser radiation $P$ of about tenths of $\mathrm{W\,cm^{-2}}$ is sufficient. As we shall see further, this limitation of $P$ substantially restricts the possibility of observing experimentally the effects of the viscosity of light.

Within the range of resonance frequencies for the dynamical polarizability of the atom $\alpha_d(\omega)$, Eq. (20), we can take into consideration only the pole term corresponding to a given resonance excitation of the atom. In this case the longitudinal force of the pressure of light, Eq. (2) can be represented in the form [1,25]

$$\vec{F}_\parallel^A=\frac{\hbar\vec{k}G\Gamma}{[1+G+(\omega-\omega_0-\vec{k}\cdot\vec{v})^2/\Gamma^2]}. \tag{21}$$

Here, $G=0.5(\vec{d}\cdot\vec{E}_0/\hbar\Gamma)^2$, where $\vec{d}$ is the atomic dipole moment, $\vec{E}_0$ is the amplitude of the field strength of the radiation field. Further, $\Gamma=\tau_{sp}^{-1}$ is the natural width of the resonance line, $\vec{v}$ is the velocity of the atom and $\omega$ is the radiation frequency. For the transverse force of the viscosity of light acting on the atom in the field of resonance radiation we have now the following expression:

$$\vec{F}_{\perp}^{A}=-|\vec{F}_{||}^{A}|\frac{\vec{v}_{\perp}}{c}. \tag{22}$$

The forces, Eqs. (21) and (22), have the Lorentz dependence on the projection of the velocity of the atom $\vec{v}$ onto the wave vector of the photon $\vec{k}$, typical for the resonance interaction with monochromatic radiation. These forces reach maximal values in the case of resonance when $v_x k=\omega-\omega_0$. As before, we assume here that the beam of light propagates along the $x$ axis. The maximal values of the forces Eqs. (21) and (22) are restricted, owing to the saturation effect, by $\hbar\vec{k}\Gamma$ and $\hbar k\Gamma\vec{v}/c$, respectively.

The evolution of the atomic velocity distribution for an atomic beam moving in a copropagating laser beam, when only the longitudinal force Eq. (21) was taken into account, was studied in great detail (for a review see Ref. [24,26]). There it was shown that, for sufficiently long time of the interaction of the radiation with the atomic beam, the force Eq. (21) transforms an initially wide velocity distribution into a narrow one. Thus, the action of this force results in the monochromatization of the atoms in the velocity space. Owing to the existence of the diffusion of the velocity due to the statistical nature of spontaneous radiation of the excited atoms, the monochromatization of the velocities of the atom along the axis of the beam of light axis proves to be possible only up to some definite limit. Because of the same reason the motion of the atom transverse to the beam of light, as the motion of the electron considered earlier, has the diffusive character as well.

Let us analyze how the force of the viscosity of light Eq. (22) can influence the transverse motion of atoms in the beam coaxial with the beam of laser radiation. Assume that both the beams propagate along the $x$ axis and that the initial atomic velocities along the $y$ axis are equal to zero. Then, owing to the diffusion of the atoms their velocity distribution along this axis $\Phi(v_y,t)$, initially having the form $\Phi(v_y,0)\sim\delta(v_y)$, obeys the following Fokker-Planck equation

$$\frac{\partial\Phi}{\partial t}-\frac{\partial}{\partial v_y}(Bv_y\Phi)=C_{yy}\frac{\partial^2\Phi}{\partial v_y^2}. \tag{23}$$

Here $B=|\vec{F}_{||}^{A}|/Mc$, $M$ is the atomic mass, $C_{yy}\sim(\hbar k/M)^2\Gamma$ is the coefficient of the velocity diffusion [25]. If there would be no light viscosity, then instead of Eq. (23), one would have the usual equation of diffusion in the velocity space. According to this equation the mean-squared deviation of the atomic velocities from their initial ones would increase linearly with time: $\langle v_y^2\rangle=C_{yy}t$, i.e., the "temperature" $T_y$ of the atoms $T_y\sim M\langle v_y^2\rangle$ (here and below the temperature $T_y$ is measured in energy units) would increase linearly with time until the longitudinal motion of the atoms along the $x$ axis moves them out of the resonance with the laser radiation.

We now show how the above conclusion changes after the force of viscosity Eq. (22) has been taken into account. Let us consider the stationary solution of Eq. (23) that corresponds to the case when the diffusive expansion in the velocity space is compensated by the viscosity of light. Setting the time derivative in Eq. (23) to zero we obtain the following equation:

$$C_{yy}\frac{d^2\Phi}{dv_y^2}+Bv_y\frac{d\Phi}{dv_y}+B\Phi=0. \tag{24}$$

The solution of this equation is the Maxwellian distribution $\Phi(v_y)=A\exp(-v_y^2/2T_y)$, where $A$ is the normalization factor. Taking into account the explicit form of $C_{yy}$ and $B$ the temperature can be estimated as $T_y\approx\hbar\omega$. (The same, of course, holds for the motion along the $z$ axis: $T_z=T_y\approx\hbar\omega$.) Thus, the viscosity of light of a *spatially uniform electromagnetic wave* can lead to the stabilization of the transverse velocity distribution of the atomic beam. The time, which is necessary for the transverse distribution to stabilize, can be estimated as $\tau=(Mc/|\vec{F}_{||}^{A}|)$. From this expression it follows that, when the resonance transition saturates ($G\gg1$) and the longitudinal force reaches its maximal value, one has $\tau\approx(Mc^2/\hbar\omega)\tau_{sp}$. For example, in the case of Na atoms irradiated by an electromagnetic wave with resonant frequency to the $3s$-$3p$ transition ($\hbar\omega\approx2.1$ eV, $\tau_{sp}\approx1.6\times10^{-8}$ s) one obtains $\tau\approx10^2$ s. The reasons this time turned out to be so long are twofold. The first is, as was already mentioned, the effective limitation on the intensity of the laser radiation $P$ set by the saturation of the atomic transition at $G\gg1$. The second reason is a huge value, compared to the case with electrons, of atomic masses. Such large values of $\tau$, of course, restrict the possibilities of observing the effects of the viscosity of light in experiments with atomic beams.

### D. Effect of the viscosity of light on macroscopic particles

The scattering of light by macroscopic spherical particles, when the radius of the latter is much smaller than the wavelength of the radiation, is analogous to nonresonant scattering of light by isolated atoms [27]. The shape of the scattering indicatrix for these Rayleigh particles is symmetric as seen from Eq. (19). Therefore, these particles move in strong electromagnetic fields as in a viscous medium. The effect of the viscosity of light can be particularly important for radiation frequencies corresponding to the Mie resonances [27]. Obviously, the effect considered can influence the behavior of the cosmic dust, the shape of a comet's tail, etc. Note that a similar phenomenon is well known in astrophysics [28], where it acts as a mechanism of decelerating small celestial bodies moving around the Sun or other sources of strong radiation. Owing to this effect, a body in space moves along a spiral trajectory, which gradually approaches the source of radiation.

## IV. CONCLUSIONS

In this paper we have shown that when the scattering indicatrix of the electromagnetic radiation is such that the total momentum of the scattered radiation is equal to zero, then two forces act upon the particle moving relative to the radiation source. The first accelerates the particle in the direction of propagation of the radiation and is well known [13,14]. The second acts on the particle as a specific Stokes force and decelerates the moving particle across the radiation field. Therefore, the particle moves in a strong electromagnetic field as in a viscous medium. The force due to the viscosity of light is a factor $\beta$ smaller for nonrelativistic particles, but it can be much larger than the first one in the case of ultrarelativistic charges. We have analyzed and proposed plausible experimental conditions for the observation of the viscosity of light and estimated the magnitude of the effects due to this phenomenon.

## ACKNOWLEDGMENTS

A.S.B. is grateful to Professor V. S. Letokhov and Dr. A. V. Zinoviev for useful comments. The work of A.S.B. was supported by Grant No. ZP1-2449-TA-02 of the Civilian Research and Development Foundation for the Independent States of the former Soviet Union (CRDF). A.S.B. acknowledges the hospitality of Professor V. Gontar from the Ben-Gurion University of the Negev (Israel). The U.S. DOE, Division of Chemical Sciences, Office of Basic Energy Sciences, Office of Energy Research, and NSF supported the research of A.Z.M. and Z.F.

[1] A. Ashkin, Phys. Rev. Lett. **25**, 1321 (1970).
[2] T. Hänschen and A. Schawlow, Opt. Commun. **13**, 68 (1975).
[3] A. Ashkin, Phys. Rev. Lett. **40**, 729 (1978).
[4] R. Schieder, H. Walter, and L. Woste, Opt. Commun. **5**, 337 (1972).
[5] P. Moscowitz, P.L. Gould, S.R. Atlas, and D.E. Pritchard, Phys. Rev. Lett. **51**, 370 (1983).
[6] J.E. Bjorkholm, R.R. Freeman, A. Ashkin, and D.B. Pearson, Phys. Rev. Lett. **41**, 1361 (1978).
[7] V.I. Balykin, V.S. Letokhov, V.G. Minogin, and T.V. Zueva, Appl. Phys. B: Photophys. Laser Chem. **35**, 149 (1984).
[8] A. Aspect, J. Dalibard, A. Heidmann, C. Salomon, and C. Cohen-Tannoudji, Phys. Rev. Lett. **57**, 1688 (1986).
[9] E. Riis, L.-U.A. Andersen, O. Poulsen, H. Simonsen, and T. Worm, Phys. Rev. A **37**, 2958 (1988).
[10] V.I. Balykin, V.S. Letokhov, and V.I. Mushin, Pis'ma Zh. Éksp. Teor. Fiz. **29**, 614 (1979) [JETP Lett. **29**, 1688 (1979)].
[11] W.D. Phillips and H. Metcalf, Phys. Rev. Lett. **48**, 596 (1982).
[12] J. Prodan, A. Migdall, W.D. Phillips, I. So, H. Metcalf, and J. Dalibard, Phys. Rev. Lett. **54**, 992 (1985).
[13] L.D. Landau and E.M. Lifshitz, *Theory of Fields* (Pergamon Press, Oxford, 1968).
[14] J.D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).
[15] W. Pauli, *Theory of Relativity* (Pergamon Press, Oxford, 1958).
[16] C.A. Robinson, Jr., Aviat. Week Space Technol. April **2**, 12 (1979).
[17] A. Isihara, Statistical Physics (Academic Press, New York, 1971).
[18] U. Teubner, D. Altenbernd, P. Gibbon, E. Föster, A. Mysyrowicz, P. Audebert, J.-P. Geindre, J.C. Gauthier, R. Lichters, and J. Meyer-ter-Vehn, Opt. Commun. **144**, 217 (1997).
[19] J. Zhang, M. Zepf, P.A. Norreys, A.E. Dangor, M. Bakarezos, C.N. Danson, A. Dayson, A.P. Fews, P. Gibbon, M.H. Key, P. Lee, P. Loukakos, S. Moustaizis, D. Neely, F.N. Walsh, and J.S. Wark, Phys. Rev. A **54**, 1597 (1996).
[20] G. Fiocco and E. Thompson, Phys. Rev. Lett. **10**, 89 (1963).
[21] H.J. Kunze, E. Fünfer, B. Kronast, and W.H. Kegel, Phys. Lett. **11**, 42 (1964).
[22] S.A. Ramsden and W.E.R. Davies, Phys. Rev. Lett. **13**, 227 (1964); **16**, 303 (1966).
[23] W.E.R. Davies and S.A. Ramsden, Phys. Lett. **8**, 179 (1964).
[24] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon Press, Oxford, 1974).
[25] V.G. Minogin and V.S. Letokhov, *Laser Light Pressure on Atoms* (Gordon and Breach, New York, 1987).
[26] H. J. Metcalf and P. van der Straten, *Laser Cooling and Trapping* (Springer-Verlag, New York, 1999).
[27] M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Academic Press, New York, 1969).
[28] H. Robertson, Mon. Not. R. Astron. Soc. **97**, 423 (1937).