Time-reversal-violating rotation of a polarization plane of light in gas placed in an electric field

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The rotation of a polarization plane of light in gas placed in an electric field is considered. Different factors causing this phenomenon are investigated. The angle of polarization plane rotation for the transition $6S_{1/2} \to 7S_{1/2}$ in cesium ($\lambda = 539$ nm) is estimated. The possibility of observing this effect experimentally is discussed.

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I. INTRODUCTION

Violation of time-reversal symmetry was observed in K_0 meson [1,2] and B_0 meson decay [3] and remains one of the greatest unsolved problems in elementary particle physics. A lot of attempts have been undertaken to observe time-reversal-violating phenomena in different processes experimentally. However, those experiments have not been successful. Among them are, for example, measurements of electric dipole moment (EDM) of neutrons [4], atoms and molecules [5–7]. No EDM was found but these experiments impose strong restrictions on the theory. During the search of the EDM of heavy atoms, in particular, tight limits for parameters of electron-nucleon *P*-, *T*-violating interactions and the value of the electron EDM [5] are set.

It is well known that essential progress in measurements of *P*-odd interaction constants was achieved during studies of the optical activity of atomic gases. High precision of optical measurements allow us to expect that the investigation of time-reversal invariance in photon interactions with atoms will provide new limits for constants of *T*-noninvariant weak interaction.

As was shown in Refs. [8–11] *T*-noninvariant interactions induce several new optical phenomena. They are (1) *T*-noninvariant rotation of a polarization plane of light in an electric field, (2) *T*-noninvariant birefringence, and (3) *T*-noninvariant rotation of a light polarization plane in diffraction grating with a noncentrosymmetrical elementary cell.

Experiments to observe the rotation of a polarization plane of light in an electric field are under preparation now [12,8]. Therefore, it is important to attract attention to the fact that two effects can contribute to *T*-noninvariant rotation of the polarization plane of light. They are as follows.

(1) Light polarization plane rotation caused by the pseudo-Zeeman splitting of atomic levels in atoms with nonzero EDM in the electric field [13,14,12].

(2) Light polarization plane rotation due to the interference of *P*-, *T*-odd and Stark-induced transition amplitudes [8–10].

The present paper is organized as follows. In Sec. II the general theory describing *P*-, *T*-noninvariant rotation of the polarization plane of light in atomic gas is examined. Polarization plane rotation caused by the interference of *P*-, *T*-odd and Stark-induced transitions is considered in detail in Sec. III. Polarization plane rotation caused by nonzero atomic EDM is considered in Sec. IV. Estimates of magnitudes of effects for different kinds of transitions are given in Sec. V. Section VI examines possible sources of *P*-, *T*-violating interactions in an atom and Sec. VII gives estimates of the angle of *P*-, *T*-odd polarization plane rotation for the $6S_{1/2} \to 7S_{1/2}$ transition in cesium. Section VIII briefly discusses the possibilities of observing this phenomenon experimentally and also summarizes general conclusions.

II. *P*-, *T*-ODD ROTATION OF THE POLARIZATION PLANE OF LIGHT

To illustrate the mechanism of polarization plane rotation due to the interference of *P*-, *T*-odd and Stark-induced transition amplitudes we consider a simple model at first. Let us take an atom in the $s_{1/2}$ state and place it in an electric field. Taking into account the admixture of the nearest $p_{1/2}$ state due to *P*- and *T*-odd interactions and the interaction with the electric field, one can represent the wave function of an atom in the form:

$$|\tilde{s}_{1/2}\rangle = \frac{1}{\sqrt{4\pi}}[R_0(r) - R_1(r)(\vec{\sigma}\vec{n})\eta - R_1(r)(\vec{\sigma}\vec{n}) \times (\vec{\sigma}\vec{E})\delta]|\chi_{1/2}\rangle. \tag{1}$$

Here $\vec{\sigma}$ are the Pauli matrices; $\vec{n} = \vec{r}/r$ is the unit vector along $\vec{r}$, $\vec{E}$ is the electric-field strength, R_0 and R_1 are the radial parts of $s_{1/2}$ and $p_{1/2}$ wave functions, respectively, $|\chi_{1/2}\rangle$ is the spin part of wave function, η and δ are the mixing coefficients describing *P*- and *T*-noninvariant interactions and electric field, respectively, and $\vec{E}$ is the electric-field strength.

Interference of Stark and *P*-, *T*-odd terms changes the electron-spin direction as follows:

$$\Delta\vec{s}(\vec{r}) = \frac{\eta\delta}{8\pi}R_1^2(r)\langle\chi_{1/2}|(\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E}) + (\vec{\sigma}\vec{E}) \times (\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})|\chi_{1/2}\rangle = \frac{\eta\delta R_1^2(r)}{8\pi}[4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}]. \tag{2}$$

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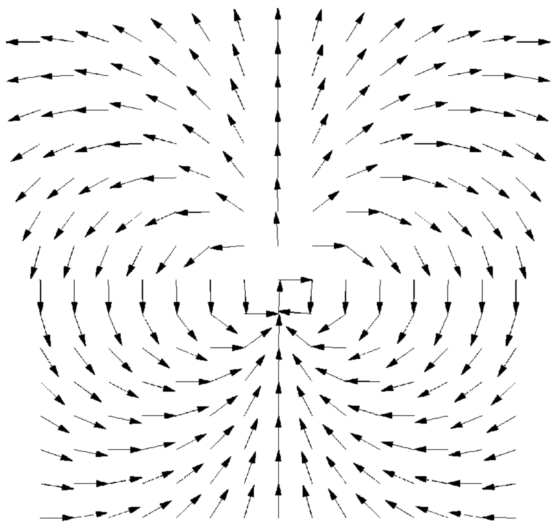

FIG. 1. Vector field $4\vec{n}(\vec{n}\vec{E})-2\vec{E}$. Vectors on figure shows direction of atomic spin in $s_{1/2}$ state if we take into account admixture of $p_{1/2}$ state due to *P*-, *T*-noninvariant interactions and external electric field.

The vector field $4\vec{n}(\vec{n}\vec{E})-2\vec{E}$ is shown in Fig. 1. Since $\Delta\vec{s}$ does not depend on the initial direction of the atomic spin, this spin structure appears even in a nonpolarized atom. The spin vector averaged over spatial variables differs from zero and is directed along $\vec{E}$. Photons with the angular moment parallel and antiparallel to $\vec{E}$ differently interact with such a gas. It causes rotation of the polarization plane of photons.

Let us also note that according to Ref. [10] the magnetization of the gas in an electric field induces the magnetic field $\vec{H}_{ind}(E)$. This magnetic field interacts with the magnetic moment of an atom giving additional contribution to the rotation of the polarization plane of light [9].

The refraction index of the gas is given by

$$n=1+\frac{2\pi N}{k^2}f(0), \tag{3}$$

where N is the number of atoms per cm^3, k is the photon wave number, $f(0)=f_{ik}e_i^{*'}e_k$ is the amplitude of elastic coherent forward scattering of photons by atoms. Here $\vec{e}$ and $\vec{e}\,'$ are the polarization vectors of the initial and scattered photons, respectively. Repeated indices imply summation. In the dipole approximation

$$f_{ik}=\omega^2\alpha_{ik}/c^2, \tag{4}$$

where α_{ik} is the tensor of dynamical polarizability of an atom, ω is the frequency of incident light. According to Refs. [8,15] the amplitude of light, scattering by nonpolarized atomic gas in an electric field is expressed by

$$f_{ik}=f_{ik}^{ev}+\frac{\omega^2}{c^2}[i\beta_s^P\epsilon_{ikl}n_{\gamma l}+i\beta_E^{PT}\epsilon_{ikl}n_{El}+\beta_{sE}^T(\vec{n}_\gamma\cdot\vec{n}_E)\delta_{ik}]. \tag{5}$$

Here f_{ik}^{ev} is the *P*- and *T*-invariant part of scattering amplitude, β_s^P is the *P*-odd but *T*-even scalar atomic polarizability [16], β_E^{PT} is the *P*- and *T*-odd scalar polarizability of an atom [8], β_{sE}^T is the *P*-even but *T*-odd atomic polarizability [15], $\vec{n}_\gamma=\vec{k}/k$ is the unit vector along the direction of photon propagation, $\vec{n}_E=\vec{E}/E$ is the unit vector along the direction of the electric field, and ϵ_{ijk} is the third-rank antisymmetric tensor.

The angle of rotation of the polarization plane is

$$\phi=\tfrac{1}{2}k\,\mathrm{Re}(n_+-n_-)l, \tag{6}$$

where n_+ and n_- are the refraction indices for the left and right circularly polarized photons. Vectors $\vec{e}_+$ and $\vec{e}_-$ describe the left and right circularly polarized photons, respectively, $\vec{e}_\pm=\mp(\vec{e}_x\pm i\vec{e}_y)/\sqrt{2}$. Using Eqs. (5) and (3) we can express the polarization plane rotation as follows:

$$\phi=-\frac{2\pi N\omega}{c}[\beta_s^P+\beta_E^{PT}(\vec{n}_E\vec{n}_\gamma)]l. \tag{7}$$

The term proportional to β_s^P describes the well-known phenomenon of the *P*-odd but *T*-even rotation of the polarization plane of light. The term proportional to β_E^{PT} corresponds to the *P*- and *T*-noninvariant light polarization plane rotation about the direction of the electric field [11].

In contrast to the *P*-odd, *T*-even rotation, the reversion of the electric field direction changes the sign of the *P*-, *T*-odd rotation of the light polarization plane. This contrast allows one to distinguish *P*-, *T*-odd effects from the other possible effects of polarization plane rotation.

According to Refs. [8,9] the tensor of dynamical polarizability of an atom (molecule) in the ground state $|\tilde{g}_n\rangle$ has the form

$$\alpha_{ik}^n=\sum_m\left\{\frac{\langle\tilde{g}_n|d_i|\tilde{e}_m\rangle\langle\tilde{e}_m|d_k|\tilde{g}_n\rangle}{E_{em}-E_{gn}-\hbar\omega}+\frac{\langle\tilde{g}_n|d_k|\tilde{e}_m\rangle\langle\tilde{e}_m|d_i|\tilde{g}_n\rangle}{E_{em}-E_{gn}+\hbar\omega}\right\}, \tag{8}$$

where $|\tilde{g}_n\rangle$ and $|\tilde{e}_m\rangle$ are the wave functions of an atom in the ground and excited states perturbed by electric field and *P*-, *T*-noninvariant interactions, d is the operator of dipole transition, E_{em} and E_{gn} are the energies of atom states $|\tilde{g}_n\rangle$ and $|\tilde{e}_m\rangle$, respectively.

In general, atoms are distributed over the magnetic sublevels of ground state g_n with the probability $P(n)$. Therefore, α_{ik}^n should be averaged over all states n. As a result, the polarizability can be written as

$$\alpha_{ik}=\sum_n P(n)\alpha_{ik}^n. \tag{9}$$

In the present paper we discuss the nonpolarized atomic gas. In this case $P(n)=1/(2j_g+1)$ where j_g is the total moment of an atom in the ground state g.

Tensor α_{ik} can be decomposed into irreducible parts as

$$\alpha_{ik}=\alpha_0\delta_{ik}+\alpha_{ik}^s+\alpha_{ik}^a. \tag{10}$$

Here $\alpha_0=\frac{1}{3}\sum_i\alpha_{ii}$ is the scalar, and $\alpha_{ik}^s=\frac{1}{2}(\alpha_{ik}+\alpha_{ki})-\alpha_0\delta_{ik}$ is the symmetric tensor of rank two, $\alpha_{ik}^a=\frac{1}{2}(\alpha_{ik}-\alpha_{ki})$ is the antisymmetric tensor of rank two:

$$\alpha_{ik}^a=\frac{\omega}{(2j_g+1)\hbar}\sum_{m,n}\left\{\frac{\langle\tilde{g}_n|d_i|\tilde{e}_m\rangle\langle\tilde{e}_m|d_k|\tilde{g}_n\rangle-\langle\tilde{g}_n|d_k|\tilde{e}_m\rangle\langle\tilde{e}_m|d_i|\tilde{g}_n\rangle}{\omega_{em,gn}^2-\omega^2}\right\}, \tag{11}$$

where $\omega_{em,gn}=(E_{em}-E_{gn})/\hbar$.

If atoms are nonpolarized then in the absence of P- and T-odd interactions the antisymmetric part of polarizability is equal to zero. Therefore, a comparison of Eqs. (5) and (4) yields

$$\alpha_{ik}^a=i\epsilon_{ikl}(\beta_s^P n_{\gamma l}+\beta_E^{PT}n_{El}). \tag{12}$$

According to Refs. [9,11,16], a correct consideration of P-odd but T-even interactions requires one to take into account both $E1$ and $M1$ transition amplitudes. If only $E1$ transition operators are considered in Eq. (8), the P-odd but T-even polarizability β_s^P becomes equal to zero.

The evaluation of expression (12) for the left (or right) circular polarization of the incident light at $\vec{n}_E\|\vec{n}_\gamma$ yields $\alpha_{ik}^a e_i^{*(\pm)}e_k^{(\pm)}=\mp\beta_E^{PT}$. As a result we can represent P-, T-odd scalar polarizability of an atom as follows:

$$\beta_E^{PT}=\frac{\omega}{(2j_g+1)\hbar}\sum_{n,m}\left\{\frac{\langle\tilde{g}_n|d_-|\tilde{e}_m\rangle\langle\tilde{e}_m|d_+|\tilde{g}_n\rangle}{\omega_{em,gn}^2-\omega^2}-\frac{\langle\tilde{g}_n|d_+|\tilde{e}_m\rangle\langle\tilde{e}_m|d_-|\tilde{g}_n\rangle}{\omega_{em,gn}^2-\omega^2}\right\}, \tag{13}$$

where $d_\pm=\mp(d_x\pm id_y)/\sqrt{2}$.

For further analysis, more detailed expressions for wave functions of an atom are necessary. Since constants of P-, T-noninvariant interactions are very small we can use the perturbation theory. Let $|\bar{g}\rangle$ and $|\bar{e}\rangle$ be the wave functions of the ground and excited states of an atom (molecule) in electric field $\vec{E}$ in the absence of P-, T-odd interactions. Switch on P-, T-noninvariant interaction ($H_T\neq 0$). According to the perturbation theory the wave functions $|\tilde{g}\rangle$ and $|\tilde{e}\rangle$ take the form

$$|\tilde{g}\rangle=|\bar{g}\rangle+\sum_n |n\rangle\frac{\langle n|H_T|\bar{g}\rangle}{E_g-E_n},$$

$$|\tilde{e}\rangle=|\bar{e}\rangle+\sum_n |n\rangle\frac{\langle n|H_T|\bar{e}\rangle}{E_e-E_n}, \tag{14}$$

where H_T is Hamiltonian of P-, T-noninvariant interactions.

It should be reminded that the denominator of Eq. (13) contains shifts caused both by the interaction of the electric dipole moment of an atom with the electric field $\vec{E}$ and the magnetic moment of an atom with the T-odd induced magnetic field $H_{ind}(\vec{E})$ [10]. If H_T is small, one can represent total polarizability β_E^{PT} as the sum of two terms,

$$\beta_E^{PT}=\beta_{mix}+\beta_{split}. \tag{15}$$

Here

$$\beta_{mix}=\frac{\omega}{(2j_g+1)\hbar}\sum_{n,m}\left\{\frac{\langle\tilde{g}_n|d_-|\tilde{e}_m\rangle\langle\tilde{e}_m|d_+|\tilde{g}_n\rangle}{\omega_{\overline{em},\overline{gn}}^2-\omega^2}-\frac{\langle\tilde{g}_n|d_+|\tilde{e}_m\rangle\langle\tilde{e}_m|d_-|\tilde{g}_n\rangle}{\omega_{\overline{em},\overline{gn}}^2-\omega^2}\right\}, \tag{16}$$

where $\omega_{\overline{em},\overline{gn}}$ does not include the P-, T-noninvariant shift of atomic levels, and

$$\beta_{split}=\frac{\omega}{(2j_g+1)\hbar}\sum_{n,m}\left\{\frac{\langle\bar{g}_n|d_-|\bar{e}_m\rangle\langle\bar{e}_m|d_+|\bar{g}_n\rangle}{\omega_{em,gn}^2-\omega^2}-\frac{\langle\bar{g}_n|d_+|\bar{e}_m\rangle\langle\bar{e}_m|d_-|\bar{g}_n\rangle}{\omega_{em,gn}^2-\omega^2}\right\}, \tag{17}$$

$$\omega_{em,gn}=[E_{em}(\vec{E})-E_{gn}(\vec{E})]/\hbar,$$

where energy levels $E_{e,m}(\vec{E})$ and $E_{g,n}(\vec{E})$ contain shifts caused by the interaction of the electric dipole moment of an atom with the electric field $\vec{E}$ and the magnetic moment of an atom with the T-odd induced magnetic field $\vec{H}_{ind}(\vec{E})$.

Below we consider a small detuning of radiation frequency from resonance frequency of atomic transition. Therefore, Eqs. (16) and (17) can be written as follows:

$$\beta_{mix}=\frac{1}{2\hbar(2j_g+1)}\sum_{n,m}\left\{\frac{\langle\tilde{g}_n|d_-|\tilde{e}_m\rangle\langle\tilde{e}_m|d_+|\tilde{g}_n\rangle}{\omega_{\overline{em},\overline{gn}}-\omega}-\frac{\langle\tilde{g}_n|d_+|\tilde{e}_m\rangle\langle\tilde{e}_m|d_-|\tilde{g}_n\rangle}{\omega_{\overline{em},\overline{gn}}-\omega}\right\}, \tag{18}$$

$$\beta_{split}=\frac{1}{2\hbar(2j_g+1)}\sum_{n,m}\left\{\frac{\langle\overline{g}_n|d_-|\overline{e}_m\rangle\langle\overline{e}_m|d_+|\overline{g}_n\rangle}{\omega_{em,gn}-\omega}-\frac{\langle\overline{g}_n|d_+|\overline{e}_m\rangle\langle\overline{e}_m|d_-|\overline{g}_n\rangle}{\omega_{em,gn}-\omega}\right\}. \quad (19)$$

III. INTERFERENCE OF *P*-, *T*-ODD AND STARK-INDUCED AMPLITUDES

In this section we consider the effects associated with β_{mix}. Rotation associated with β_{split} is studied in Sec. IV.

Let us assume that the electric field is small enough. For atoms in the ground state, the energy of Stark interactions is usually less than the difference in energies of levels mixed by the electric field. In this case we can use the first order of the perturbation theory. Perturbed states $|\tilde{g}\rangle$ and $|\tilde{e}\rangle$ have the form

$$|\tilde{g}\rangle=|g\rangle+\sum_n |n\rangle\frac{\langle n|H_T|g\rangle}{E_g-E_n}+\sum_m |m\rangle\frac{\langle m|-\vec{d}\vec{E}|g\rangle}{E_g-E_m},$$

$$|\tilde{e}\rangle=|e\rangle+\sum_n |n\rangle\frac{\langle n|H_T|e\rangle}{E_e-E_n}+\sum_n |m\rangle\frac{\langle m|-\vec{d}\vec{E}|e\rangle}{E_e-E_m}. \quad (20)$$

Here H_T is the Hamiltonian of *P*-, *T*-noninvariant interactions, $|g\rangle$ and $|e\rangle$ are the unperturbed ground and excited states of an atom, and $\vec{E}$ is the external electric field. We assume that the electric field is directed along the z axis.

Using Eq. (20), we can rewrite β_{mix} as follows:

$$\beta_{mix}=\frac{1}{\hbar(2j_g+1)}\mathrm{Re}\times\sum_{m_g,m_e}\frac{\langle g|d_+^{PT}|e\rangle\langle e|d_-^{St}|g\rangle-\langle g|d_-^{PT}|e\rangle\langle e|d_+^{St}|g\rangle}{\omega_{\overline{em},\overline{gn}}-\omega}, \quad (21)$$

where

$$\langle g|d_\pm^{PT}|e\rangle=\sum_m \frac{\langle g|H_T|m\rangle\langle m|d_\pm|e\rangle}{E_m-E_g}+\frac{\langle g|d_\pm|m\rangle\langle m|H_T|e\rangle}{E_m-E_e} \quad (22)$$

and

$$\langle g|\vec{d}^{St}\vec{e}|e\rangle=\Lambda_{ik}e_iE_k \quad (23)$$

is the Stark-induced amplitude of the transition between the states g and e in the constant electric field $\vec{E}$ and $\vec{e}$ is the polarization vector of the photon:

$$\Lambda_{ik}=\sum_n \frac{\langle g|d_k|n\rangle\langle n|d_i|e\rangle}{E_n-E_g}+\frac{\langle g|d_i|n\rangle\langle n|d_k|e\rangle}{E_n-E_e}. \quad (24)$$

Representation of the second-rank tensors Λ_{ik} and e_iE_k in terms of their irreducible spherical components yields [17]

$$\langle e|\vec{d}^{St}\vec{e}|g\rangle=\sum_{q,q'}(-1)^{q+q'}\Lambda_{q,q'}E_{-q}e_{-q'}=\sum_{K,Q}(-1)^Q\Lambda_Q^K(E\otimes e)_{-Q}^K, \quad (25)$$

where subscripts q and q' refer to the spherical vector components and Λ_Q^K and $(E\otimes e)_{-Q}^K$ are the components of irreducible spherical tensors.

Using the Wigner-Ekhard theorem we can represent Λ_Q^K as follows:

$$\Lambda_Q^K=(-1)^{j_e-m_e}\begin{pmatrix} j_e & K & j_g \\ -m_e & Q & m_g\end{pmatrix}\Lambda^K. \quad (26)$$

Reduced matrix elements Λ^K ($K=0,1,2$) are proportional to the scalar, vector, and tensor transition polarizabilities, respectively. Substituting Eqs. (25) and (26) into Eq. (21), representing the matrix element of *P*-, *T*-odd $E1$ transition (22) in terms of the reduced matrix element $\langle e||d^{PT}||g\rangle$ and performing summation over m_g, m_e one obtains the following expression for β_{mix}:

$$\beta_{mix}=-\frac{2}{3\hbar(2j_g+1)}\frac{\mathrm{Re}\langle e||d^{PT}||g\rangle\Lambda^1E}{\sqrt{2}(\omega_{\overline{em},\overline{gn}}-\omega)}. \quad (27)$$

We assume here that electric field $\vec{E}$ is parallel to the direction of light propagation and use the expression $(\vec{E}\otimes\vec{e}_\pm)_\pm^1=E/\sqrt{2}$. Due to the orthogonality of $3j$ symbols, only terms proportional to the vector part of transition polarizability remain in Eq. (27) after summation over magnetic sublevels.

Equations (27) and (7) give the angle of polarization plane rotation without considering the Doppler broadening in gas. Because of the Doppler shift, the resonance frequency of transition for a single atom depends on atom velocity. In order to obtain the expression for the angle of polarization plane rotation in this case we should average Eq. (27) over the Maxwell distribution of atom velocity.

If a nucleus has a nonzero spin, we should take into account the hyperfine structure. After routine calculations the angle of *P*-, *T*-odd rotation of the polarization plane can be expressed as

$$\phi=4\pi N_F l\frac{\omega}{\hbar c\Delta_D}g(u,v)\frac{1}{3(2F_g+1)}K^2\times\mathrm{Re}\left(\langle g||d^{PT}||e\rangle\Lambda^1E\frac{1}{\sqrt{2}}\right). \quad (28)$$

For completeness, we give here the expression for the absorption length of light in atomic gas [16],

$$L^{-1}=4\pi N_F\frac{\omega}{\hbar c\Delta_D}f(u,v)\frac{1}{3(2F_g+1)}K^2|\langle g||A||e\rangle|^2. \quad (29)$$

Here F_g, F_e are the total angular moments of an atom in the ground and exited states, respectively, j_g and j_e are the total-electron moments in these states, and i is the nuclear spin.

$$N_F = N\frac{2F_g+1}{(2i+1)(2j_g+1)}$$

is the density of atoms with the total moment F_g,

$$K^2 = (2F_g+1)(2F_e+1)\begin{Bmatrix} i & j_g & F_g \\ 1 & F_e & J_e \end{Bmatrix},$$

$\Delta_D = \omega_0\sqrt{2kT/Mc^2}$ is the Doppler linewidth,

$$\left.\begin{matrix} g(u,v) \\ f(u,v) \end{matrix}\right\} = \left\{\begin{matrix} \mathrm{Im} \\ \mathrm{Re} \end{matrix}\right\}\{\sqrt{\pi}e^{-w^2}[1-\Phi(-iw)]\}, \qquad (30)$$

where $w = u + iv$, $\Phi(z) = (2/\sqrt{\pi})\int_0^z dt e^{-t^2}$, $u = (\omega - \omega_0)/\Delta_D$, $v = \Gamma/2\Delta_D$, Γ is the recoil line width, $\langle g||A||e\rangle$ is the reduced matrix element of dipole transition between states $|e\rangle$ and $|g\rangle$.

IV. ROTATION OF THE POLARIZATION PLANE DUE TO AN ATOMIC EDM

The presence of EDM in the ground or excited state of the atom also causes rotation of the polarization plane of light. We can derive the expression for the angle of the polarization plane rotation performing the calculations similar to those described in Sec. III, but using β_{split} instead of β_{mix}. But in this case the calculations can be appreciably simplified if we note that *P*-, *T*-noninvariant rotation caused by the atomic EDM is similar to the Faraday rotation of the photon polarization plane in a weak magnetic field. Indeed, according to Refs. [16,18] a weak magnetic field affects the refractive index of atomic gas in two ways: through the change of the magnetic sublevels energies and through the mixing of hyperfine states.

If we consider only terms proportional to the magnetic-field strength H and neglect the terms of higher orders, then the level shift is [18]:

$$\Delta E_i = -H\langle i|\mu_z|i\rangle.$$

The magnetic field H mixes states of the same F_z but different F, so the state $|j\rangle$ becomes

$$|\overline{j}\rangle = |j\rangle - \sum_{k\neq j} H_z\frac{|k\rangle\langle k|\mu_z|j\rangle}{E_k - E_j}.$$

If the atom has an EDM then the electric field similarly affects the refraction index [see Eq. (17)] and leads to a shift in atomic levels,

$$\Delta E_i = -E\langle i|d_z|i\rangle.$$

It also mixes the hyperfine states of the atom with the same F_z but different F,

$$|\overline{j}\rangle = |j\rangle - \sum_{k\neq j} E_z\frac{|k\rangle\langle k|d_z|j\rangle}{E_k - E_j}.$$

As a result we can use the expression describing rotation of the polarization plane of light in a weak magnetic field [16,18] for calculations of the effect of the polarization plane rotation in an electric field. For this we must substitute $H \to E$, $\mu_g \to d_g$, $\mu_e \to d_e$, where d_e, d_g are the EDM of an atom in the ground and excited states and μ_i is the magnetic moment of state i.

If we neglect quadrupole transition amplitudes (it is possible, for example, for $6S_{1/2} \to 7S_{1/2}$ transition in cesium), then the angle of polarization plane rotation has the form

$$\phi = \frac{2\pi Nl}{(2i+1)(2j_g+1)}\frac{\omega}{\Delta_D\hbar c}\frac{E_z}{\hbar\Delta_D}|\langle g||A||e\rangle|^2 \times\left(\frac{\partial g(u,v)}{\partial u}\delta_1 + 2g(u,v)\gamma_1\right). \qquad (31)$$

The expressions for parameters γ_1 and δ_1 are given below:

$$\gamma_1 = \frac{(2F_g+1)(2F_e+1)}{\sqrt{6}}(-1)^i\begin{Bmatrix} i & j_g & F_g \\ 1 & F_e & j_e \end{Bmatrix}\Bigg[d_e(-1)^{j_e+F_g}\sqrt{\frac{(j_e+1)(2j_e+1)}{j_e}}\Bigg(\frac{\Delta_D}{\Delta_{hf}(F_e,F_e-1)}(2F_e-1) \times\begin{Bmatrix} i & j_g & F_g \\ 1 & F_e-1 & j_e \end{Bmatrix}\begin{Bmatrix} i & j_e & F_e \\ 1 & F_e-1 & j_e \end{Bmatrix}\begin{Bmatrix} F_g & 1 & F_e \\ 1 & F_e-1 & 1 \end{Bmatrix} + \frac{\Delta_D}{\Delta_{hf}(F_e,F_e+1)}(2F_e+3)\begin{Bmatrix} i & j_g & F_g \\ 1 & F_e+1 & j_e \end{Bmatrix} \times\begin{Bmatrix} i & j_e & F_e \\ 1 & F_e+1 & j_e \end{Bmatrix}\begin{Bmatrix} F_g & 1 & F_e \\ 1 & F_e+1 & 1 \end{Bmatrix}\Bigg) - (j_e\leftrightarrow j_g, F_e\leftrightarrow F_g, d_e\leftrightarrow d_g,)\Bigg]$$

and

$$\delta_1 = \frac{(2F_g+1)(2F_e+1)}{\sqrt{6}}(-1)^i\begin{Bmatrix} i & j_g & F_g \\ 1 & F_e & j_e \end{Bmatrix}^2\Bigg[d_e(-1)^{j_e+F_g}\sqrt{\frac{(j_e+1)(2j_e+1)}{j_e}}(2F_e+1)\begin{Bmatrix} i & j_e & F_e \\ 1 & F_e & j_e \end{Bmatrix}\begin{Bmatrix} F_g & F_e & 1 \\ 1 & 1 & F_e \end{Bmatrix} + (j_e\leftrightarrow j_g, F_e\leftrightarrow F_g, d_e\leftrightarrow d_g,)\Bigg].$$

Here Δ_{hf} is the hyperfine level splitting. The first term in Eq. (31) arises from the level splitting in the electric field. It describes an effect similar to the Macaluso-Corbino rotation of the photon polarization plane in a magnetic field. The second term is caused by a mixing of hyperfine levels with the different total moment F but the same F_z in the electric field. It describes the T-noninvariant analog of the polarization plane rotation due to the Van-Vleck mechanism.

V. ESTIMATES

Let us compare the angle of P-, T-odd polarization plane rotation for different kinds of transitions. The angle of rotation of the polarization plane per absorption length due to the interference of the P-, T-odd and Stark-induced transition amplitudes according to Eq. (28) is expressed by

$$\phi(L_{abs}) = \frac{g(u,v)}{f(u,v)} \frac{\mathrm{Re}\langle g||d^{PT}||e\rangle \Lambda^1 E}{\sqrt{2}|\langle g||A||e\rangle|^2}. \tag{32}$$

If detuning $\Delta \sim \Delta_D$, then $g \sim f \sim 1$.

The value of the transition matrix element depends on the kind of transition. For the allowed $E1$ transition, $\langle g||A||e\rangle \sim \langle d\rangle \sim ea_0$; for the allowed $M1$ transition, $\langle g||A||e\rangle \sim \langle \mu\rangle \sim \alpha\langle d\rangle$. For the strongly forbidden $M1$ transition in the electric field, dominant contribution to the angle of rotation gives the Stark-induced $E1$ transition. Its amplitude can be estimated as $\langle g||A||e\rangle \sim \langle d\rangle^2 E_z/\Delta E$, where ΔE is the typical difference between the opposite parity levels in the atom. For the transition $6S_{1/2} \to 7S_{1/2}$ in Cs and feasible electric field $E \sim 10^3 - 10^4$ V/cm, the value of $\langle d\rangle E_z/\Delta E \sim 10^{-3} - 10^{-4}$. Here E_z is the electric-field strength, e is the electron charge, a_0 is the Bohr radius, $\Delta E \sim \mathcal{R}$ is the typical difference between the energy levels of the opposite parity states, $\mathcal{R} = 13.6$ eV is the Rydberg energy constant, and $\alpha \simeq 1/137$ is the fine-structure constant.

The numerator of Eq. (32) has the same order of magnitude for all kinds of transitions considered above: $\langle g||d^{PT}||e\rangle \Lambda^1 E \sim \langle d\rangle^2 \langle H_T\rangle \langle d\rangle E_z/(\Delta E)^2$. Now we can estimate the angle of polarization plane rotation per absorption length.

For allowed $E1$ transition

$$\phi(L_{abs}) \sim \frac{\langle H_T\rangle}{\Delta E} \frac{\langle d\rangle E_z}{\Delta E}, \tag{33}$$

where $\langle H_T\rangle$ is the typical value of the matrix element of P-, T-odd Hamiltonian.

The angle of rotation per absorption length near allowed $M1$ transition is larger than in the case of $E1$,

$$\phi(L_{abs}) \sim \frac{\langle H_T\rangle}{\alpha^2 \Delta E} \frac{\langle d\rangle E_z}{\Delta E}. \tag{34}$$

The largest value of the angle of rotation per absorption length can be observed near strongly forbidden $M1$ transition because the absorption of light is the lowest in this case,

$$\phi(L_{abs}) \sim \frac{\langle H_T\rangle}{\Delta E} \frac{\Delta E}{\langle d\rangle E_z}. \tag{35}$$

It is interesting to compare these estimates with the angle of rotation of the polarization plane caused by the nonzero EDM of the atom. In the absence of hyperfine structure the angle of rotation per absorption length can be estimated using Eqs. (28) and (31) as follows:

$$\phi_{EDM}(L_{abs}) = \frac{1}{2(2j_g+1)} \frac{E_z \delta}{f \Delta_D} \frac{\partial g}{\partial u} \sim \frac{d_{at} E_z}{\Delta_D} \sim \frac{\langle d\rangle E_z}{\Delta_D} \frac{\langle H_T\rangle}{\Delta E}, \tag{36}$$

where $d_{at} \sim \langle d\rangle \langle H_T\rangle/\Delta E$ is the EDM of the atom, $\Delta_D \sim (10^{-5} \sim 10^{-6})\Delta E$ is the Doppler linewidth. Here the value $\phi(L_{abs})$ does not depend on the transition amplitude $\langle g||A||e\rangle$ and has the same order of magnitude for all kinds of transitions considered above.

For allowed $E1$ transition, $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D/\Delta E \ll 1$ and dominant contribution to the angle of P-, T-odd polarization plane rotation gives the level splitting caused by atomic EDM.

Near allowed $M1$ transition, $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D/\alpha^2 \Delta E \sim 1$ and both mechanisms contribute comparably.

Near the strongly forbidden $M1$ transition $\phi(L_{abs})/\phi_{EDM}(L_{abs}) \sim \Delta_D \Delta E/(\langle d\rangle E_z)^2 \gg 1$ and the interference of P-, T-odd and Stark-induced transitions contributes the most to the angle of polarization plane rotation.

VI. *P*- AND *T*-ODD INTERACTIONS IN ATOMS

Several mechanisms can induce the violation of P- and T-invariance in an atom. According to Ref. [16] they are (1) P-, T-odd weak interactions of electron and nucleon, (2) interaction of the electric dipole moment of an electron with the electric field inside the atom, (3) interaction of electrons with the electric dipole and magnetic quadrupole moments of the nucleus, and (4) P-, T-odd electron-electron interaction.

We consider effects that according to Ref. [16] give the dominant contribution in our case: P-, T-odd electron-nucleon interaction and the interaction of electron EDM with the electric field inside the atom.

According to Refs. [16,19,20] the Hamiltonian of T-violating interaction between the electron and hadron has the form

$$H_T = C_s \frac{G}{\sqrt{2}} (\overline{e} i \gamma_5 e)(\overline{n} n) + C_t \frac{G}{\sqrt{2}} (\overline{e} i \gamma_5 \sigma_{\mu\nu} e)(\overline{n} \sigma^{\mu\nu} n), \tag{37}$$

where $G = 1.055 \times 10^{-5} m_p^{-2}$ is the Fermi constant, m_p is the proton mass, e and n are the electron and hadron field operators respectively, C_s and C_t are the dimensionless constants characterizing the strength of T-violating interactions relative to T-conserving weak interaction. The first term in Eq. (37) describes the coupling of the scalar hadronic current with the pseudoscalar electronic current, and the second one describes the coupling of the tensor hadronic current with the pseudotensor electronic current.

The matrix element of the T-odd Hamiltonian according to Ref. [16] is equal to

$$\langle s_{1/2}||H_T||p_{1/2}\rangle=\frac{Gm_e^2\alpha^2Z^2R}{2\sqrt{2}\pi}\frac{\mathcal{R}}{\sqrt{\nu_s\nu_p}^3}2\gamma C_sA, \quad (38)$$

where m_e is the electron mass, ν_i is the effective principal quantum number of state i, A is the atomic number, $\gamma=\sqrt{(j+1/2)^2-Z^2\alpha^2}$, j is the total angular moment of the atom. The "relativistic enhancement factor" R is given by

$$R=4\frac{(a_0/2Zr_0)^{2-2\gamma}}{\Gamma^2(2\gamma+1)}.$$

Here $r_0=A^{1/3}1.2\times10^{-13}$ cm is the approximate nuclear radius. For cesium, $R=2.8$. We neglect the tensor part of the interaction for simplicity.

The Hamiltonian of interaction of the electron EDM and the electric field inside the atom that mix opposite parity atomic states has the form [16]

$$H_d=\sum_k(\gamma_{0k}-1)\vec{\Sigma}_k\vec{E}_k, \quad (39)$$

where

$$\Sigma_k=-\gamma_5\gamma_0\gamma_k=\begin{pmatrix}\sigma_k & 0\\ 0 & \sigma_k\end{pmatrix} \quad (40)$$

σ_k are the Pauli matrices, E_k is the electric-field strength acting upon electron k. If summation in Eq. (39) is performed over one valence electron and the electric-field strength near the nucleus approximately equals $\vec{E}=Z\alpha\vec{r}/r^3$, the matrix element of operator H_d can be written as follows [16]:

$$\langle j,l=j+1/2||H_d||j,l'=j-1/2\rangle=-\frac{4(Z\alpha)^3}{\gamma(4\gamma^2-1)(\nu_l\nu_{l'})^{3/2}a_0^2}, \quad (41)$$

where l and l' are the orbital angular moments.

VII. ESTIMATES FOR THE $6S_{1/2}\rightarrow7S_{1/2}$ TRANSITION IN CESIUM

Let us estimate the P-, T-odd rotation of the polarization plane for the highly forbidden $M1$ transition $6s_{1/2}\rightarrow7s_{1/2}$ in cesium. The scheme of cesium energy levels is shown in Fig. 2.

A. Rotation of the polarization plane of light due to electron-nucleon interactions

The P- and T-odd electron-nucleon interactions mix s and p states of cesium. Since the amplitudes for $E1$ transitions $6s\rightarrow np$ and $7s\rightarrow n'p$ are negligibly small when $n>6$ and $n'>7$ [16], we should take into account only an admixture of $6p_{1/2}$ and $7p_{1/2}$ states. Using Eq. (14) and matrix element (38) one can represent the wave functions perturbed by P-, and T-noninvariant electron-nucleon interaction as follows:

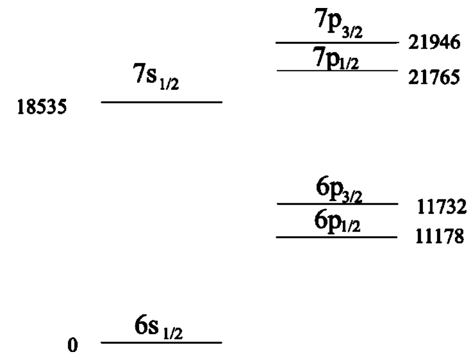

FIG. 2. Scheme of cesium energy levels. Energy of atomic levels is given in cm^{-1}.

$$|\widetilde{6s_{1/2}}\rangle=|6s_{1/2}\rangle+10^{-11}\left(2\gamma\frac{A}{N}C_s\right)(1.17|6p_{1/2}\rangle+0.34|7p_{1/2}\rangle),$$

$$|\widetilde{7s_{1/2}}\rangle=|7s_{1/2}\rangle+10^{-11}\left(2\gamma\frac{A}{N}C_s\right)(0.87|6p_{1/2}\rangle-1.33|7p_{1/2}\rangle), \quad (42)$$

where N is the number of neutrons in the atomic nucleus. Using Eq. (14) and values of radial integrals [16]

$$\rho(6s_{1/2},6p_{1/2})=-5.535,\quad \rho(7s_{1/2},6p_{1/2})=5.45,$$

$$\rho(7s_{1/2},7p_{1/2})=-12.30,$$

we obtain the reduced matrix element of P-, T-odd $E1$ transition

$$\langle 6s_{1/2}||d^{PT}||7s_{1/2}\rangle=1.27\times10^{-10}|e|a_0C_s. \quad (43)$$

The matrix element of the Stark-induced $6s_{1/2}\rightarrow7s_{1/2}$ transition in cesium is usually written as [21]

$$\langle 6s_{1/2},m'|d_i^{St}|7s_{1/2},m\rangle=\alpha E_i\delta_{mm'}+i\beta\epsilon_{ijk}E_j\langle m'|\sigma_k|m\rangle,$$

where m and m' are the magnetic quantum numbers of the ground and excited states of cesium, E_i is the electric-field strength, α and β are the scalar and vector transition polarizabilities [see also Eq. (25)]. The value of Λ^1 introduced in Eq. (28) can be expressed for cesium via the vector transition polarizability as follows: $\Lambda_1=-2\sqrt{3}\beta E$. The value of β is well known from theoretical calculations [22] as well as from experiment [23]. According to Ref. [22] it is equal to $\beta=27.0a_0^3$. Therefore,

$$\Lambda^1=-1.81\times10^{-8}|e|a_0E\ (\mathrm{V/cm}). \quad (44)$$

Suppose the temperature is $T=750$ K. Then the pressure of Cs vapor is $p=10$ kPa [24], the concentration of atoms is $N=10^{18}$ cm^{-3}, and the Doppler linewidth is $\Delta_D/\omega_0=10^{-6}$.

For transition between hyperfine levels $F_g=4\rightarrow F_e=4$ coefficient K^2 in formula (28) is maximal ($K^2=15/8$).

Suppose detuning $\Delta\sim\Delta_D$, then $v=\Gamma/2\Delta_D\simeq 0.1$ and $f\approx 1$, $g\approx 0.7$. Absorption length in longitudinal electric field $E=10^4$ V/cm is equal to $L_{abs}=7$ m.

As a result the angle of *P*-, *T*-noninvariant rotation of the polarization plane is

$$|\phi|=1.0\times 10^{-13}C_s lE.$$

The optimal signal to noise ratio is achieved when $l=2L_{abs}$ [15]. The best limit on the parameters of electron-nucleon interaction $C_s<4\times 10^{-7}$ was set in Ref. [5]. The corresponding limit to the angle of rotation of the polarization plane is $|\phi|<0.5\times 10^{-12}$ rad.

B. Rotation of the polarization plane of light due to cesium EDM

Using wave functions (42) we can obtain the EDM in the $6s_{1/2}$ and $7s_{1/2}$ states of Cs,

$$d_{6s_{1/2}}=-1.35\times 10^{-10}C_s|e|a_0,$$

$$d_{7s_{1/2}}=-4.39\times 10^{-10}C_s|e|a_0.$$

As a result, expression (31) yields the angle of polarization plane rotation due to level splitting in the electric field,

$$|\phi_1|=1.4\times 10^{-24}lC_sE_z^3\ (\text{V/cm})<8\times 10^{-16}$$

and the angle of rotation due to hyperfine levels mixing

$$|\phi_2|=2.1\times 10^{-24}lC_sE_z^3\ (\text{V/cm})<1.2\times 10^{-15}.$$

[We assume here that for detuning $\Delta\sim\Delta_D$ functions $g(u,v)\simeq 0.7$, $\partial g(u,v)/\partial u\simeq 1.1$.] These angles are three orders of magnitude lower than the angle of polarization plane rotation arising from the interference of Stark-induced and *P*-, *T*-noninvariant transition amplitudes.

C. Rotation of the polarization plane of light due to an electron EDM

If the *P*-, *T*-noninvariant interaction in the atom is induced by the interaction of an electron EDM with the electric field of the nucleus, then using Eqs. (14) and (41) one can represent the wave functions of $6s$ and $7s$ states of cesium as follows:

$$|\widetilde{6}s_{1/2}\rangle=|6s_{1/2}\rangle-(35|6p_{1/2}\rangle+10.5|7p_{1/2}\rangle)d_e/(ea_0),$$

$$|\widetilde{7}s_{1/2}\rangle=|7s_{1/2}\rangle+(27.7|6p_{1/2}\rangle-36.2|7p_{1/2}\rangle d_e/(ea_0). \qquad (45)$$

Using Eq. (45) we can obtain the reduced matrix element of electric dipole transitions between the $\widetilde{6}s_{1/2}$ and $\widetilde{7}s_{1/2}$ states,

$$\langle 6s_{1/2}||d^{PT}||7s_{1/2}\rangle=-72d_e, \qquad (46)$$

and the electric dipole moment of cesium in the ground state $d_{6s_{1/2}}$ and excited state $d_{7s_{1/2}}$,

$$d_{6s_{1/2}}=131d_e$$

$$d_{7s_{1/2}}=400d_e, \qquad (47)$$

where d_e is the electron EDM.

As we mentioned before, two effects induce *T*-noninvariant rotation of the polarization plane in the electric field. The first of them is the interference of the *P*-, *T*-odd and Stark-induced transition amplitudes and the second is the interaction of the atomic EDM with the electric field. Substituting Eq. (46) to Eq. (28) one can obtain the angle of rotation arising from the interference of amplitudes $|\phi|<0.6\times 10^{-12}$ in the same experimental conditions as before.

Rotation caused by the atomic EDM is a sum of two contributions. Using the first term in Eqs. (31) and (47) one can obtain the angle of rotation induced by the splitting of magnetic sublevels in the electric field $|\phi_1|<1.3\times 10^{-15}$. The mix of hyperfine components [second term in Eq. (31)] gives the contribution $|\phi_2|<2\times 10^{-15}$.

For estimates we use the experimental limit on electron EDM from Ref. [5] $|d_e|<4\times 10^{-27}|e|$ cm. We should note that estimates of the rotation angle for electron-nucleon *T*-noninvariant weak interactions and electron EDM give close values.

VIII. CONCLUSION

In the present paper we have considered the phenomenon of rotation of the polarization plane of light in gas placed in an electric field. Calculations of the angle of polarization plane rotation are performed for $6S_{1/2}\rightarrow 7S_{1/2}$ transition in atomic cesium. Two mechanisms of the effect are considered. They are as follows.

(1) Light polarization plane rotation caused by the pseudo-Zeeman splitting of atomic levels in atoms with nonzero EDM in an electric field [12,13,14].

(2) Light polarization plane rotation due to the interference of *P*-, *T*-odd and Stark-induced transition amplitudes [8–10].

Both of them can be induced by *P*-, *T*-noninvariant interaction between electrons and the atomic nucleus and by the interaction of the electron EDM with the electric field inside the atom.

For the highly forbidden $M1$ transition $6S_{1/2}\rightarrow 7S_{1/2}$ in cesium we can expect the angle of polarization plane rotation per absorption length due to *P*-, *T*-odd atomic polarizability β_{mix}^{PT} about $|\phi|<10^{-12}$. The rotation induced by atomic EDM for this transition is three orders of magnitude smaller.

The angle of polarization plane rotation can be significantly greater for other atoms, for example, rare-earth elements, where additional amplification arises from close levels of opposite parity.

The simplest experimental scheme to observe the pseudo-Faraday rotation of the polarization plane of light in the electric field includes a cell with atomic gas placed in the electric field and a sensitive polarimeter. In the case of large absorption length one can place this cell in a resonator or delay line optical cavity to reduce the size of the experimental setup (see Ref. [25]).

Several schemes are proposed to increase the sensitivity of measurements. One of them is based on the nonlinear magneto-optic effect [13,14]. Since the change of the rotation angle with the change of the applied field in this case is several orders of magnitude larger than in the traditional scheme, sensitivity of this kind of experiment can be very high. The authors of Ref. [13] expect to achieve the sensitivity for the cesium EDM $|d_{Cs}|<10^{-26}|e|$ cm. The corresponding limit for the electron EDM is $|d_e|<10^{-28}|e|$ cm.

The method of measurement of polarization plane rotation proposed in Refs. [8–10] can probably provide even higher sensitivity. This method is based on the observation of the evolution of the polarization of light in a cell with atomic vapor and amplifying media placed in a resonator. According to Refs. [8–10] the compensation for the absorption of light in a cell allows one to increase the observed angle of polarization plane rotation and, according to estimates [9], allows one to increase the sensitivity for the electron EDM up to $|d_e|<10^{-30}|e|$ cm.

Therefore, we can hope that the experimental measurement of the described phenomenon can provide sensitivity for parameters of *P*-, *T*-noninvariant interactions between the electron and nucleus and the electron EDM, comparable to or even higher than current atomic EDM experiments.

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