Quantum computing with trapped ions in an optical cavity via Raman transition

Mang Feng*

Institute for Scientific Interchange (ISI) Foundation, Villa Gualino, Viale Settimio Severo 65, I-10133 Torino, Italy
and Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China

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In the system with ions confined both in a linear trap and in a high-Q single-mode optical cavity, a quantum computing scheme is proposed by using lasers and quantized cavity field, via Raman transition. A controlled-NOT gate with reduced operations can be performed on two non-neighboring ions without the involvement of both motional states of ions and cavity mode. Experimental feasibility of achieving our scheme is discussed.

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Quantum computing with trapped ions has received much attention over the past several years since the proposal of Cirac and Zoller [1]. With current techniques, trapped ions can be cooled ultracoldly and radiated individually by lasers. The deterministic entanglement of trapped ions has been achieved and efficient detection of internal levels of the trapped ion is available [2–4]. Therefore, the ion trap has been considered to be a promising candidate of the quantum computing device. On the other hand, cavity-atom system has also been discussed intensively for the possibility of quantum computing performance due to its mathematical similarity to the trapped ion system [5]. However, to our knowledge, although there is some progress in this respect, it is still of great challenge to experimentally perform an actual quantum computing because of the sensitivity of quantum states to decoherence as well as the strict condition for preparing cold atoms. We noticed two recent proposals [6,7] performing quantum computing in combinatory system of the ion trap and the optical cavity, in which decoherence is avoided by means of large detuning between the cavity light and internal states of the trapped ions. As it involves three quantum degrees of freedom, namely, the ion's internal levels, quantum vibrational mode, and single-mode cavity field [8], such a system is of great interest in the community of quantum optics. To our knowledge, much investigation had been made for entanglement of these three quantum degrees of freedom as well as information transfer between one of them to another [9]. A recent proposal for implementing quantum computing was also based on the entanglement of three quantum degrees of freedom referred above [10]. However, in this Brief Report, we will treat the system similar to that in Refs. [6,7], in which the electromagnetic field of the ion trap is only used to fix the trapped ions, and quantum computing would be performed without the involvement of cavity mode and motional states of the ions. Besides the quantized cavity field, laser beams would be employed to radiate the ions in implementation of quantum computing, and the controlled-NOT (CNOT) gate can be carried out on two *non-neighboring* ions, mediated by the virtually excited cavity mode. Feasibility of experimentally achieving our scheme would be briefly discussed.

*Electronic address: feng@isiosf.isi.it

Consider that two identical ions A and B are fixed inside a linear trap, which itself is embedded in a high-Q single-mode optical cavity, as in Refs. [6,7]. The two ions have been cooled to Lamb-Dicke regime, confining their spatial wave packets to a region much smaller than the optical wavelength. Besides, the two ions are not required to be adjacent, but they should be separated by at least one-radiation wavelength. In each ion, three internal levels are employed, in which $|g\rangle$, $|e\rangle$, and $|r\rangle$ are, respectively, ground, metastable, and auxiliary states. Quantum computing will be performed in the subspace spanned by $|g\rangle$ and $|e\rangle$. Besides radiation from the quantized field of the cavity, the two ions are radiated by two classical lasers individually, as shown in Fig. 1, where ω_c and $\omega_{L_{A(B)}}$ are frequencies of the cavity field and laser $L_{A(B)}$, respectively. $\Delta_{A(B)}$ and $\delta_{A(B)}$ are detunings with $\Delta_{A(B)}=\omega_{er}-\omega_{L_{A(B)}}$ and $\delta_{A(B)}=\omega_{ge}+\omega_{L_{A(B)}}-\omega_c$, in which ω_{ik} is the resonance frequency between levels i and k. As $\Delta_{A(B)}$ is large enough, $|r\rangle_{A(B)}$ would not be excited. The general Hamiltonian of such a system has been described in Refs. [9,10]. However, with suitable choice of frequencies of cavity mode and laser beams, i.e., both $\Delta_{A(B)}$ and $\delta_{A(B)}$ are largely detuned from the trap frequency, the degrees of freedom of the trap can be decoupled from internal states of the ions, which leaves the system to be mainly an atom-cavity problem [6,7]. Therefore, the Hamiltonian for this Raman process in the unit of $\hbar=1$ can be written as follows:

$$H=2\omega_c a^\dagger a+\frac{\omega_{ge}}{2}\sum_{j=A}^{B}\sigma_z^j+\sum_{j=A}^{B}\Omega_j(a^\dagger\sigma_{ge}^j e^{-i\omega_{L_j}t}+\text{H.c.}), \tag{1}$$

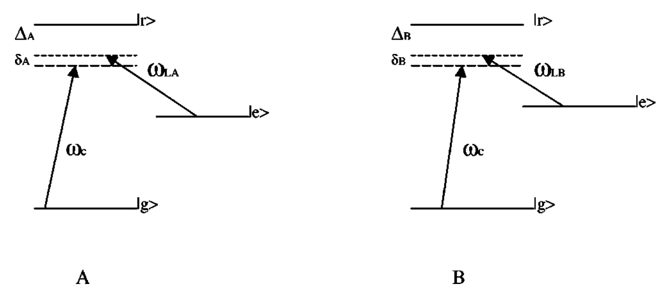

FIG. 1. Configuration of the two ions A and B, where $|g\rangle$, $|e\rangle$, and $|r\rangle$ are ground, metastable, and auxiliary internal levels, respectively. ω_c and $\omega_{L_{A(B)}}$ are frequencies of cavity and lasers, respectively. $\Delta_{A(B)}$ and $\delta_{A(B)}$ are detunings defined in the text.

where the Rabi frequency $\Omega_j = G_c^j G_L^j[1/\Delta_j + 1/(\Delta_j + \delta_j)]/2$, with G_c^j and G_L^j being the effective coupling strengths with regard to the cavity light and the laser beam, respectively. $a^\dagger$ (a) is creation (annihilation) operator of quantized cavity field, $\sigma_{ge}^j = |g\rangle_j\langle e|$ and σ_z^j is the usual Pauli operator. If we adjust the frequencies of lasers L_A and L_B to make $\delta_{A(B)}$ much smaller than ω_{ge}, but large enough compared with the cavity linewidth and $\Omega_{A(B)}$, we have the following effective Hamiltonian for the near two-photon resonance process between the ions A and B [11,12]:

$$H_{eff} = \frac{\tilde{\Omega}}{2}(\sigma_{ge}^A \sigma_{ge}^{\dagger B} + \sigma_{ge}^B \sigma_{ge}^{\dagger A}), \tag{2}$$

with

$$\left(\frac{\tilde{\Omega}}{2}\right)^2 = \left| \frac{\langle egn|H_{int}|ggn+1\rangle\langle ggn+1|H_{int}|gen\rangle}{\delta} + \frac{\langle egn|H_{int}|een-1\rangle\langle een-1|H_{int}|gen\rangle}{-\delta} \right|^2 = \left(\frac{\Omega_A\Omega_B}{\delta}\right)^2, \tag{3}$$

where H_{int} is the Hamiltonian of Eq.(1) in the rotating frame with regard to $\omega_c a^\dagger a + \omega_{ge}/2\Sigma_{j=A}^B \sigma_z^j$. $|\cdots\rangle$ is the product of internal states of ions A and B, as well as the cavity state. For simplicity, we have let $\omega_{L_A} = \omega_{L_B}$ and $\delta_A = \delta_B = \delta$. Equation (3) means that the two ions are coupled via two intermediate states $|ggn+1\rangle$ and $|een-1\rangle$. Due to the large detuning δ, which makes the two intermediate states only virtually excited, and the destructive interference between transitions along these two paths, $\tilde{\Omega}$ is independent from the cavity mode [12]. To suppress the cavity decay as much as we can, however, we will let the cavity mode in vacuum state in the remainder of this paper, so that the cavity mode is only virtually populated in the near two-photon resonance process. By means of Eq. (2), it is easy for us to obtain the time evolution of the system,

$$|ge\rangle_{AB} \rightarrow \cos\left(\frac{\tilde{\Omega}t}{2}\right)|ge\rangle_{AB} - i\sin\left(\frac{\tilde{\Omega}t}{2}\right)|eg\rangle_{AB} \tag{4}$$

and

$$|eg\rangle_{AB} \rightarrow \cos\left(\frac{\tilde{\Omega}t}{2}\right)|eg\rangle_{AB} - i\sin\left(\frac{\tilde{\Omega}t}{2}\right)|ge\rangle_{AB}. \tag{5}$$

It implies that even if ions A and B are not adjacent in the cavity, their internal states can be entangled by coupling to the same cavity mode, although the cavity mode is only virtually populated and is not involved in Eq. (2). Therefore, quantum computing can be made in the subspace spanned only by levels $|g\rangle$ and $|e\rangle$ of ions A and B, as shown later.

It is known that some single qubit operations, together with a nontrivial two-qubits gate constitute a universal set of quantum computing gates [13]. The single-qubit operation can be generally written as [1]

$$\hat{V}_j^k(\phi) = \begin{pmatrix} \cos\left(\frac{k\pi}{2}\right) & -ie^{i\phi}\sin\left(\frac{k\pi}{2}\right) \\ -ie^{-i\phi}\sin\left(\frac{k\pi}{2}\right) & \cos\left(\frac{k\pi}{2}\right) \end{pmatrix}, \tag{6}$$

where $j = L_A$ and L_B, k is the parameter proportional to the wave vector of the laser j, and ϕ is the phase of the laser j. $\hat{V}_j^k(\phi)$ can be easily achieved by laser pulses in the current experiment of trapped ions. So our following discussion would be focused on the controlled-NOT gate with ions A and B, performed as follows:

(1) A laser applied on B with the pulse of $V_{L_B}^{1/2}(3\pi/2)$ resonant with ω_{eg}, which makes $|e\rangle_B \rightarrow 1/\sqrt{2}(|e\rangle_B - |g\rangle_B)$ and $|g\rangle_B \rightarrow 1/\sqrt{2}(|g\rangle_B + |e\rangle_B)$.

(2) A laser applied on B with the pulse of $V_{L_B}^1(3\pi/2)$ resonant with ω_{er}, which makes $|e\rangle_B \rightarrow |r\rangle_B$.

(3) "2π" pulse of lasers L_A and L_B applied on A and B, respectively, which makes $|eg\rangle_{AB} \rightarrow -|eg\rangle_{AB}$ and $|ge\rangle_{AB} \rightarrow -|ge\rangle_{AB}$.

(4) A laser applied on B with the pulse of $V_{L_B}^1(\pi/2)$ resonant with ω_{er}, which makes $|r\rangle_B \rightarrow |e\rangle_B$.

(5) A laser applied on B with the pulse of $V_{L_B}^{1/2}(\pi/2)$ resonant with ω_{eg}, which makes $|e\rangle_B \rightarrow 1/\sqrt{2}(|e\rangle_B + |g\rangle_B)$ and $|g\rangle_B \rightarrow 1/\sqrt{2}(|g\rangle_B - |e\rangle_B)$.

It is easily verified that the operation sequences performed above would produce a controlled-NOT gate with ions A and B being the control and target qubits respectively, i.e., $|ee\rangle_{AB} \rightarrow |eg\rangle_{AB}$, $|eg\rangle_{AB} \rightarrow |ee\rangle_{AB}$, $|ge\rangle_{AB} \rightarrow |ge\rangle_{AB}$, and $|gg\rangle_{AB} \rightarrow |gg\rangle_{AB}$. If we replace $V_{L_B}^k(\phi)$ with $V_{L_A}^k(\phi)$ in the above steps, we would obtain a controlled-NOT gate with ions A and B being the target and control qubits, respectively.

To our knowledge, the result similar to Eq. (2) has been presented previously in the ion-trap system [12] and cavity-atom one [14]. As only internal degrees of freedom of the atoms are involved, quantum computing based on Eq. (2)—type result is called "hot quantum computing." However, the difference of our scheme from Ref. [12] is that, in our scheme, the electromagnetic field of the trap is only used to fix the ions, and the virtually populated cavity mode plays the role of data bus. Comparing with Ref. [14], in which qubits are always flying, the qubits in our scheme are easier to be controlled. Moreover, there are some similarities between our scheme and Ref. [7]. Both of them are performed via Raman transition with cavity modes being excluded from the computational subspace. However in our scheme, the exclusion of cavity mode from the computational subspace is resulted from the destructive interference of two transition paths, besides the large detuning. As a result, there are only three levels involved in our scheme, less than the number of levels in Ref. [7]. As involvement of more levels would increase the sensitivity of the controlled-NOT operation to the fluctuation of external magnetic fields, and decrease the speed of quantum computing [15], our scheme is simpler and more efficient than that in Ref. [7]. We also noticed the analogy between our scheme and a proposal of semiconductor quantum computing [16], in which quantum dots located in a cavity under Raman process could have the same result of Eq. (2). But our controlled-NOT gate implementation is more practical than that in Ref. [16]. The performance of controlled-NOT gate in Ref. [16] includes seven single-qubit and two two-qubit operations, whereas there are only four operations of single qubit and one of two qubit in the our scheme. As some uncontrollable factors existing in current experiments may yield errors in quantum computing implementation, the less the operation, the more accurate the result of quantum computing. The other difference is that our scheme is under the reach of current or foreseeable future technique. In contrast, it is very hard to achieve Ref. [16] experimentally. The difficulty lies in many aspects, such as individual addressing of quantum dots with lights, coherent manipulation of quantum dots, and efficient readout of the final result.

We noticed that an experiment has been made recently by using a single Calcium ion in the ion-trap-cavity system [17]. Although what has been done in that work is different from our purpose, the experimental data in it is helpful for us to discuss the experimental feasibility of achieving our scheme. Suppose that the desired operations are performed ideally and there is no detection inefficiency. Then the main detrimental effect on our scheme is the spontaneous emission from both $|e\rangle$ and $|r\rangle$. To avoid this kind of effect, let us first estimate the operation time of the above controlled-NOT gate. Since single-qubit operation is much faster than the two-qubits one, we only need to calculate the time of two qubits implementation. If $\Omega_A \approx \Omega_B = 2\times 10^4$ Hz [17] and $\delta = 10^5$ Hz, the operation time of our CNOT is of the order of milliseconds, which implies that both $|e\rangle_{A(B)}$ and $|r\rangle_{A(B)}$ should be metastable levels so that our controlled-NOT operation can be finished before spontaneous emission takes place [18]. Moreover, we should also pay attention to the lifetime of the cavity mode, although the cavity has almost no population throughout the controlled-NOT implementation. As the cavity mode plays the role of data bus, any unpredictable decay taking place during the gate performance would probably affect our scheme. The cavity decay rate in the current experiment is $2\pi\times 102$ kHz [17]. If we adopt this data [19] and suppose that only 4% $(=\Omega^2_{A(B)}/\delta^2)$ cavity mode is excited in our gate implementation, then the decay time of the cavity is 4×10^{-5} sec, which is shorter than the controlled-NOT gating time. So it is necessary to improve the current-cavity quality for achieving our proposed scheme. If the current decay rate of the cavity mode can be reduced by four degrees of magnitude, the controlled-NOT gate proposed here can be performed coherently for hundreds of times. Furthermore, we should notice that, to avoid any possible detrimental effect on our controlled-NOT gate due to the evolution between $|r\rangle_{A(B)}$ and $|g\rangle_{A(B)}$ in the implementation of steps 2, 3 and 4, ω_{gr} should be much larger than both ω_{er} and ω_{ge}, because the large detuning coupling between the atomic ion and the cavity mode would yield evolutions associated to the Rabi frequency $(G_c^j)^2/\Delta$ [14]. In our scheme, however, we assume $G_c^j \ll G_L^j \approx \Delta$. It implies that the possible undesired evolution is of the probability smaller than $\cos^2(G_c^j t/\Delta)$, which can be neglected in the case of $G_c^j/\Delta < 10^{-4}$.

Our scheme can be used in the decoherence-free quantum computing [20] for suppressing the collective dephasing with the pair states $|eg\rangle$ and $|ge\rangle$. If ions A and B are considered to be neighboring, we cannot only use the ion pairs for safe storage of quantum information [20], but implement a robust Grover search by means of Eqs. (4) and (5) [21]. Finally, our scheme is also applicable to a recently proposed model of encoded quantum computing (EQC) [22]. While for the system under consideration, we may perform the single-qubit rotation very easily. So the quantum gate scheme proposed here is more suitable than EQC for quantum computing in the ion-trap-cavity system.

In conclusion, we have reported a scheme for performing quantum computing in the system with trapped ions placed in a high-Q single-mode optical cavity. As degrees of freedom of the trap are decoupled from our model, decoherence due to heating has no effect on our scheme. Moreover, due to suitably adjusted detuning, the cavity mode is only virtually excited during the gate operation. So cavity decay can be effectively suppressed. Comparing with former similar schemes, our proposal also enjoys advantages of reduced operations of quantum gate implementation on non-neighboring qubits. Theoretically, this model can be generalized to the case of many ions trapped in the same high-Q optical cavity. However, there exist some technical difficulties for the scalability of this scheme. For example, the increase of the cavity size along with the increase of trapped ions will decrease the atom-cavity coupling, and it is also very difficult for confinement of many ions to avoid the mismatch of the ion spacings and cavity mode standing-wave pattern. Nevertheless, our scheme is very simple and interesting. It is applicable to the small-scale quantum computing with current ion-trap-cavity technique.

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