Quantum-interference effects on the index of refraction in an Er^{3+}-doped yttrium aluminum garnet crystal

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(Received 7 February 2002; published 22 November 2002)

A four-level system is proposed to produce large index of refraction accompanied by vanishing absorption in the Er^{3+}-doped yttrium aluminum garnet crystal. It is found that the separation of the two absorption peaks and the maximum value of index of refraction with zero absorption can be adjusted by changing the coherent field and the incoherent pumping. It is shown that a higher index of refraction with zero absorption can be easily obtained when the coherent field is off resonance.

DOI: 10.1103/PhysRevA.66.0538XX PACS number(s): 42.65.An, 42.50.Hz, 42.50.Md

I. INTRODUCTION

Quantum coherence and interference have led to the observation of many new effects and techniques in quantum optics and atomic physics. Examples include nonabsorption (dark line) resonances [1], electromagnetically induced transparency (EIT) [2], laser without inversion (LWI) [3,4], and so on. The usual dispersion-absorption relations tell us that the absorption of the light will be large on resonance, where the index of refraction is large. Recently, however, it has been pointed that there is a way to achieve an ultrahigh index of refraction near an atomic resonance, while the absorption could be canceled [5,6]. Several recent experiments [7] have demonstrated the large dispersion of the index of refraction accompanying EIT. Index enhancement, however, allows not only for large dispersion, but also for a large refractive index itself, while maintaining a transparent medium.

In the past years, EIT in an optically dense rare-earth Pr^{3+} doped crystal Y_2SiO_5 at 5.5 K has been observed [8]. Efficient EIT in solid crystals opens potential applications such as inversionless lasers [9], high-density optical memory [10], and enhanced four-wave mixing [11]. The yttrium aluminum garnet (YAG) is an excellent optical host material compared with others, which are sensitive to moisture. The Er^{3+}-doped YAG crystals are efficient active media for lasers operating in the middle infrared band, coinciding with the telecommunications band. The theoretical study of EIT in Er^{3+}:YAG has been also reported [12].

In this paper, we investigate a four-level scheme in Er^{3+}-doped YAG crystal. It is found that, in this model, a high index of refraction with vanishing absorption can be achieved, and the index of refraction can be controlled by different coherent field and incoherent pumping. The underlying principles are atomic coherence and quantum interference. It should be emphasized that the ultrahigh index of refraction with zero absorption in solid material is more realisitic in application than in gases [13]. As we know, an instrument with high dispersion and zero absorption is important for dispersion compensation in optical communication. In Refs. [5,6], Scully and co-workers studied, how to achieve ultrahigh index of refraction with zero absorption at resonance in a three-level model. In this paper, we specially consider some cases where the coherent field is off resonance because much high index of refraction can be achieved, which has not been studied, and few interesting phenomena are discussed. Another advantage of this model is that only one incoherent pump is used.

II. EQUATIONS AND SOLUTIONS

We consider a closed four-level system shown in Fig. 1. The energy-level scheme is relevant to the Er^{3+} ions in Er^{3+}:YAG crystal, where levels $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ correspond to energy levels of Er^{3+} ions ${}^4I_{15/2}$, ${}^4I_{13/2}$, ${}^4I_{11/2}$, and ${}^4I_{9/2}$, respectively. In this system, via a fast decay from $|3\rangle$, level $|2\rangle$ is populated by an incoherent pump process Λ interacting with the transition $|1\rangle \leftrightarrow |3\rangle$. A coherent driving field E_c with Rabi frequency Ω_c drives the $|2\rangle \leftrightarrow |4\rangle$ atomic transition, and a probe field E_p with Rabi frequency Ω_p interacts with the transition labeled $|1\rangle \leftrightarrow |2\rangle$. $\Gamma_4 = \Gamma_{41} + \Gamma_{42}$

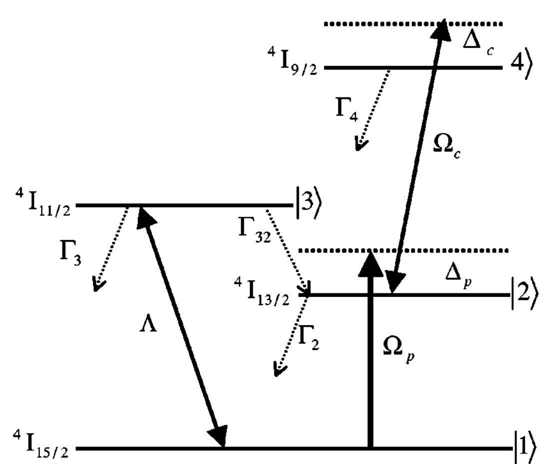

FIG. 1. Schematic diagram of a four-level system where levels $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ are corresponding to energy levels of Er^{3+} ions in Er^{3+}:YAG crystal ${}^4I_{15/2}$, ${}^4I_{13/2}$, ${}^4I_{11/2}$, and ${}^4I_{9/2}$, respectively.

$+\Gamma_{43}$ is the spontaneous decay rate of level $|4\rangle$, $\Gamma_3=\Gamma_{31}+\Gamma_{32}$ is the spontaneous decay rate of level $|3\rangle$, and $\Gamma_2=\Gamma_{21}$ is the spontaneous decay rate of level $|2\rangle$.

In the framework of the semiclassical theory, by the standard density-matrix formalism under the dipole approximation and the rotating wave approximation, the interaction Hamiltonion H_I can be represented in the interaction picture by

$$H_I=-\Delta_p|2\rangle\langle 2|-(\Delta_p+\Delta_c)|4\rangle\langle 4| \\ -(\Omega_p|2\rangle\langle 1|+\Omega_c|4\rangle\langle 2|+\text{c.c.}), \quad (1)$$

where the detunings of the probe and the coherent field are defined as $\Delta_p=\omega_p-(\omega_2-\omega_1)$, and $\Delta_c=\omega_c-(\omega_4-\omega_2)$, respectively; the Rabi frequencies Ω_c and Ω_p are defined as $\Omega_c=\mu_{42}E_c/2\hbar$ and $\Omega_p=\mu_{21}E_p/2\hbar$, respectively.

Assuming $\hbar=1$ for simplicity, the master equation of motion for the density operator in an arbitary multilevel atomic sysem in the interaction picture can be writen as

$$\frac{\partial\rho}{\partial t}=-i[H_I,\rho]+\Lambda\rho. \quad (2)$$

By expanding Eq. (2), we can easily arrive at their equations of motion

$$\dot\rho_{11}=\Lambda(\rho_{33}-\rho_{11})+\Gamma_{21}\rho_{22}+\Gamma_{31}\rho_{33}+\Gamma_{41}\rho_{44} \\ +i\Omega_p^*\rho_{21}-i\Omega_p\rho_{12},$$

$$\dot\rho_{22}=-\Gamma_{21}\rho_{22}+\Gamma_{32}\rho_{33}+\Gamma_{42}\rho_{44}+i\Omega_p\rho_{12} \\ -i\Omega_p^*\rho_{21}+i\Omega_c^*\rho_{42}-i\Omega_c\rho_{24},$$

$$\dot\rho_{33}=-\Lambda(\rho_{33}-\rho_{11})-\Gamma_{31}\rho_{33}-\Gamma_{32}\rho_{33}+\Gamma_{43}\rho_{44},$$

$$\dot\rho_{41}=-[\gamma_{41}-i(\Delta_p+\Delta_c)]\rho_{41}+i\Omega_c\rho_{21}-i\Omega_p\rho_{42},$$

$$\dot\rho_{42}=-(\gamma_{42}-i\Delta_c)\rho_{42}+i\Omega_c(\rho_{22}-\rho_{44})-i\Omega_p^*\rho_{41},$$

$$\dot\rho_{43}=-[\gamma_{43}-i(\Delta_c+\Delta_p)]\rho_{43}+i\Omega_c\rho_{23}, \quad (3)$$

$$\dot\rho_{32}=-(\gamma_{32}+i\Delta_p)\rho_{32}-i\Omega_p^*\rho_{31}-i\Omega_c\rho_{34},$$

$$\dot\rho_{31}=-\gamma_{31}\rho_{31}-i\Omega_p\rho_{32},$$

$$\dot\rho_{21}=-(\gamma_{21}-i\Delta_p)\rho_{21}+i\Omega_p(\rho_{11}-\rho_{22})+i\Omega_c^*\rho_{41},$$

$$\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}=1,$$

$$\rho_{ij}=\rho_{ji}^*,$$

where Γ_{ij} designates the population spontaneous damping from $|i\rangle$ to $|j\rangle$, while γ_{ij} are total coherence relaxation rates between $|i\rangle$ and $|j\rangle$, given by [14]

$$\gamma_{41}=(\Gamma_{41}+\Gamma_{42}+\Gamma_{43}+\Lambda+\gamma_{41}^{dph})/2,$$

$$\gamma_{42}=(\Gamma_{41}+\Gamma_{42}+\Gamma_{43}+\Gamma_{21}+\gamma_{42}^{dph})/2,$$

$$\gamma_{43}=(\Gamma_{41}+\Gamma_{42}+\Gamma_{43}+\Gamma_{32}+\Gamma_{31}+\Lambda+\gamma_{43}^{dph})/2,$$

$$\gamma_{32}=(\Gamma_{32}+\Gamma_{31}+\Gamma_{21}+\Lambda+\gamma_{32}^{dph})/2,$$

$$\gamma_{31}=(\Gamma_{32}+\Gamma_{31}+\Lambda+\Lambda+\gamma_{31}^{dph})/2,$$

$$\gamma_{21}=(\Gamma_{21}+\Lambda+\gamma_{21}^{dph})/2, \quad (4)$$

where γ_{ij}^{dph} is the dephasing decay rate of the quantum coherence of the $|i\rangle\leftrightarrow|j\rangle$ transition. In contrast to many atomic schemes the γ_{ij}^{dph}, determined by electron-electron, interface roughness, and phonon scattering processes, are the dominant contributions to the γ_{ij} and the major obstacle to the observation of coherent effects such as EIT in solid material.

In the limit of a weak probe, under the steady-state condition, the solutions of Eq. (1) for ρ_{21} to the first order of the probe field and for other ρ_{ij} to the zero order of the probe field are

$$\rho_{22}=\frac{\Lambda\Gamma_{32}/(\Lambda+\Gamma_{31}+\Gamma_{32})}{\left[\Gamma_{21}+\{\Gamma_{41}+\Gamma_{43}(\Lambda+\Gamma_{31})/(\Lambda+\Gamma_{31}+\Gamma_{32})\}\dfrac{D2}{D1}\right]}\rho_{11},$$

$$\rho_{33}=\left[\Gamma_{21}+(\Gamma_{41}+\Gamma_{43})\frac{D2}{D1}\right]\frac{\rho_{22}}{\Gamma_{32}},$$

$$\rho_{44}=\frac{D2}{D1}\rho_{22}, \quad (5)$$

$$\rho_{42}=\frac{i\Omega_c(\rho_{22}-\rho_{44})}{\gamma_{42}-i\Delta_c},$$

and

$$\rho_{12}=\rho_{21}^*=\frac{i\Omega_p(\rho_{22}-\rho_{11})-\dfrac{i|\Omega_c|^2\Omega_p(\rho_{22}-\rho_{44})}{[\gamma_{41}+i(\Delta_p+\Delta_c)](\gamma_{42}+i\Delta_c)}}{\gamma_{21}+i\Delta_p+\dfrac{|\Omega_c|^2}{\gamma_{41}+i(\Delta_p+\Delta_c)}}, \quad (6)$$

$$P=2N\mu_{21}\rho_{12},$$

$$\text{Im}P=2N\mu_{12}\text{Im}\rho_{12}, \quad (7)$$

$$\text{Re}P=2N\mu_{12}\text{Re}\rho_{12},$$

where N is the number of atoms per unit volume, μ_{21} is the dipole matrix element between level $|1\rangle$ and level $|2\rangle$, $D1=\Gamma_{41}+\Gamma_{42}+\Gamma_{43}+[2|\Omega_c|^2\gamma_{42}/(\gamma_{42}^2+\Delta_c^2)]$, $D2=[2|\Omega_c|^2\gamma_{42}/(\gamma_{42}^2+\Delta_c^2)]$.

III. ANALYSIS OF THE SOLUTIONS

The indexes of refraction and absorption are governed by the real and imaginary parts of the complex polarization, i.e., by ρ_{12}, according to Eq. (7). We have written the steady-state solution of ρ_{12} as shown in Eq. (6), from which we see that ImP and ReP depend on detunings, coherence relaxation rates, and the incoherent pumping rate Λ as well as on the Rabi frequency Ω_c. The absorption of the probe

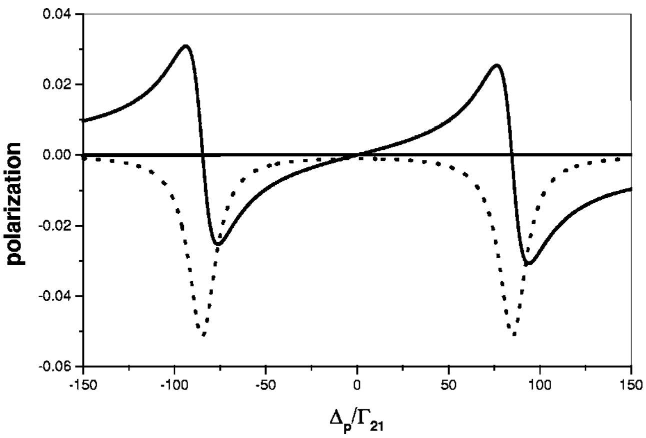

FIG. 2. Plots of the dimensionless absorptive ImP (dotted) and dispersive ReP(solid) parts of polarization versus probe detuning Δ_p/Γ_{21} at incoherent pumping rate $\Lambda=0.0$ and the Rabi frequency of the resonant coherent field $\Omega_c=85.0$. At $\Delta_p=0$, both ImP and ReP are equal to zero. The other parameters used are $\Gamma_{21}=239.1\ \mathrm{s}^{-1}$, $\Gamma_{31}=0.80\Gamma_{21}$, $\Gamma_{32}=10.0\Gamma_{21}$, $\Gamma_{41}=0.86\Gamma_{21}$, $\Gamma_{42}=0.29\Gamma_{21}$, $\Gamma_{43}=0.04\Gamma_{21}$, $\Omega_p=1.0\Gamma_{21}$.

$[\propto \mathrm{Im}(\rho_{12})]$ is contributed by two terms, the population difference term $[\propto(\rho_{22}-\rho_{11})]$, and the atomic coherence term $[\propto -|\Omega_c|^2(\rho_{22}-\rho_{44})]$. Without the incoherent pump, the system is commonly known as an EIT ladder type. In such case, the absorption of the probe can be reduced, and the index of refraction is zero but has a large slope where absorption vanishes, as shown in Fig. 2. With the increasing incoherent pumping rate, the population difference $(\rho_{22}-\rho_{11})$ will get larger and larger because of the fast decay from level $|3\rangle$ to metastable state $|2\rangle$ in which the population can be kept. When the contribution from population difference $(\rho_{22}-\rho_{11})$ equals to that of atomic coherence $[\propto -|\Omega_c|^2(\rho_{22}-\rho_{44})]$, the high dispersion with zero absorption can be obtained in the steady-state condition.

In the following, by numerical calculation, we discuss how to combinate the incoherent pump and the coherent field to realize the enhancement of the index of refraction accompanied by vanishing absorption.

Based on Refs. [15,16], we can get the population spontaneous emission probabilities Γ_{ij} of the Er^{3+} ions in Er^{3+}:YAG crystals containing 0.52 at.% concentrations of Er^{3+} ion at room temperature. So it is reasonable that we always chose the parameters as $\Gamma_{31}=0.80\Gamma_{21}$, $\Gamma_{32}=10.0\Gamma_{21}$, $\Gamma_{41}=0.86\Gamma_{21}$, $\Gamma_{42}=0.29\Gamma_{21}$, and $\Gamma_{43}=0.04\Gamma_{21}$ in the following. In Ref. [17], we have found the dephasing time of Er^{3+}:YAG crystal with an Er^{3+} concentration of 0.1%, $T_2=75\ \mu s$ on the ${}^4I_{15/2}\leftrightarrow{}^4I_{13/2}$ transition of Er^{3+} at 1526.97 nm, the homogeneous linewidth $\Gamma_h=4.286$ kHz. So it is reasonable for us to estimate the dephasing decay rate as $\gamma_{21}^{dph}=\gamma_{31}^{dph}=\gamma_{32}^{dph}=\gamma_{41}^{dph}=\gamma_{42}^{dph}=\gamma_{43}^{dph}=15\Gamma_{21}$. In this paper, all the parameters have been scaled by $\Gamma_{21}=239.1\ \mathrm{s}^{-1}$.

To observe how atomic coherence and interference effects lead to complete absorption cancellation and an ultrahigh index of refraction, we depict ImP and ReP of the probe field versus the detuning of the probe in Figs. 2–6 whether

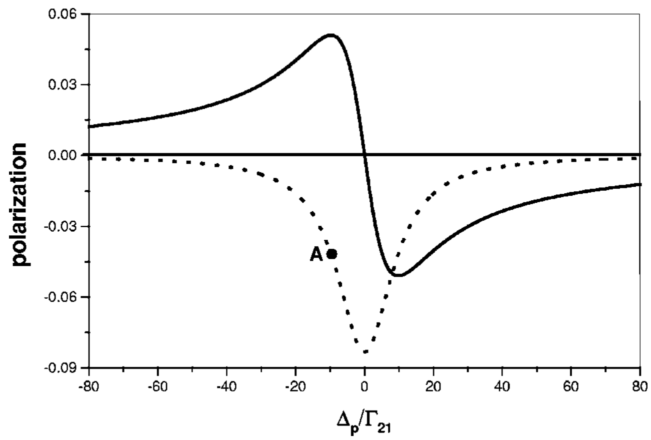

FIG. 3. Same as Fig. 2. $\Lambda=0.0$, $\Omega_c=0.0$, $\Delta_c=0.0$, at point A have nonzero ImP and nonzero ReP. The other parameters used are same as those in Fig. 2.

the incoherent pumping and the coherent field are in action or not, where the polarization is in the unit of $2N\mu_{21}$ and the detuning of the probe field is in the unit of $\Gamma_{21}=239.1\ \mathrm{s}^{-1}$. When $\Lambda=0.0$ and $\Omega_c=0.0$, it can be seen immediately in Fig. 3 that a high refractive index is always accompanied by nonzero absorption at point A. The situation is completely different, however, if multilevel schemes are considered in which atomic coherence is established or quantum-interference effects occur. When a resonant coherent field $\Omega_c=85.0$ is exerted between states $|2\rangle$ and $|4\rangle$ only, it was found that both ImP and ReP vanish [5] at the same detuning, i.e., at the point $\Delta_p=0$ in Fig. 2. The main difference between the present model and that of Ref. [5] is that one incoherent pumping is used only. The incoherent pumping and the coherent driving field are two crucial ingredients necessary for having zero ImP and large ReP. If there is no incoherent pumping, both ImP and ReP vanish at $\Delta_p=0$ as shown in Fig. 2, in agreement with the results of Refs. [5,18]. When the incoherent pumping rate $\Lambda=2.50$, and the Rabi frequency of the resonant coherent field $\Omega_c=20.0$, as shown in Fig. 4(a), it immediately becomes obvious that there is a test-field frequency that experiences both vanishing absorption and a high index of refraction at point B. When the incoherent pumping rate increase to $\Lambda=2.75$, as shown in Fig. 4(b), the system shows gain because the contribution from the incoherent pump can be larger than that of atomic coherence. So with a fixed Rabi frequency, only in a proper region of the incoherent pumping rate Λ, can the test field experience both zero absorption and a high index of refraction. If we fix the incoherent pumping $\Lambda=2.50$, with the increasing of Rabi frequency Ω_c, as shown in Fig. 4(c), the separation of the two gain peaks increases, but at the same time, the value of ReP at the point E with vanishing absorption decreases in comparison with Fig. 4(a).

From the above disscussion, it is found that the present system provides a high refractive index accompanied by perfect transparency only for a very narrow frequency region. But in Fig. 5, compared with curve m [which is the same as Fig. 4(a)], a high index of refraction with low or zero absorption can be achieved in a much broader spectral region, as

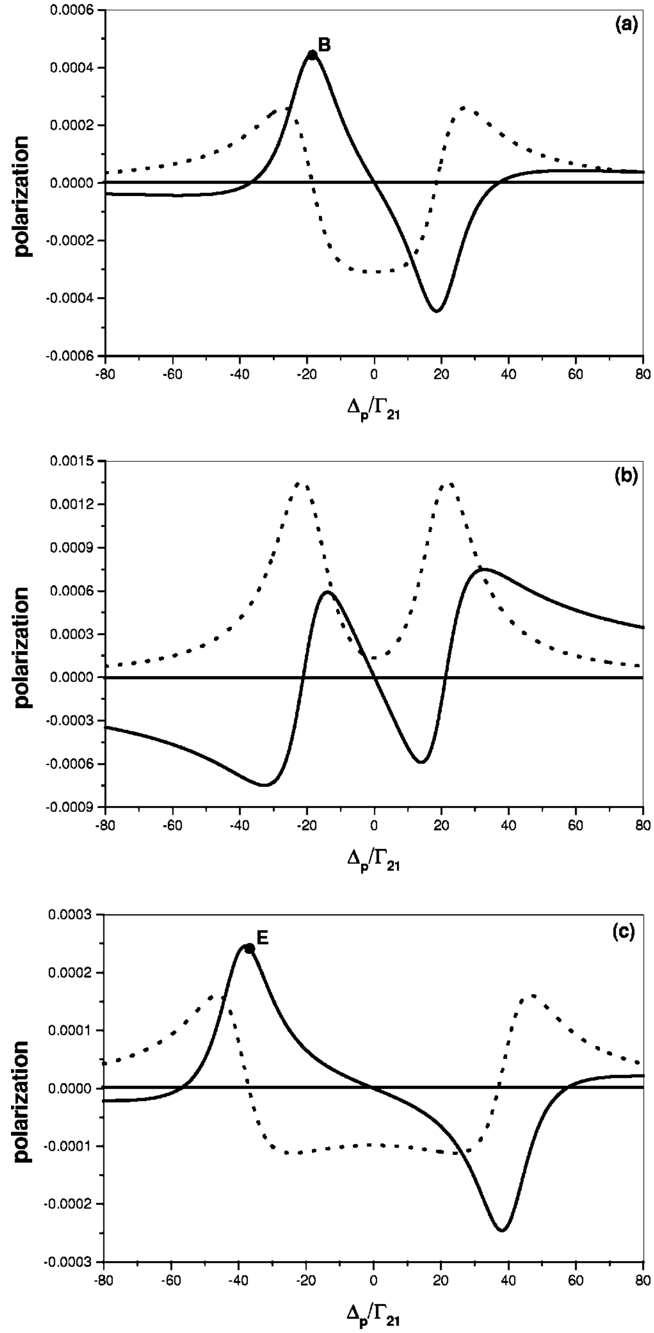

FIG. 4. Same as Fig. 2. (a) $\Lambda=2.50$, $\Omega_c=20.0$, $\Delta_c=0.0$, at point B have zero ImP and nonzero ReP. (b) $\Lambda=2.75$, $\Omega_c=20.0$, $\Delta_c=0.0$. (c) $\Lambda=2.50$, $\Omega_c=40.0$, $\Delta_c=0.0$, at point E have zero ImP and nonzero ReP. The other parameters used are same as those in Fig. 2.

shown by curve n. The necessary conditions for this situation include the large enough incoherent pumping rate and the large enough Rabi frequency of the coherent field. The large incoherent pumping rate makes it possible to achieve gain in the whole spectral region, and the strong coherent field makes it possible to seperate two gain peaks away from each other.

Next, we consider the case where the coherent field is not at resonance with the transition $|2\rangle \leftrightarrow |4\rangle$. It is found that one

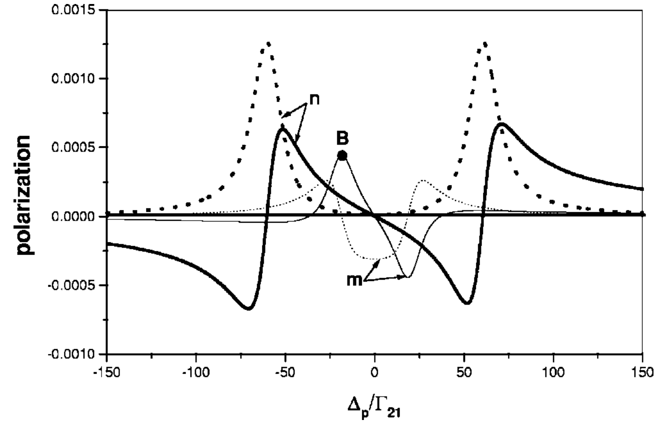

FIG. 5. Same as Fig. 2. The curves m and n correspond to (1) $\Lambda=2.50$, $\Omega_c=20.0$, and $\Delta_c=0.0$, at point B have zero ImP and nonzero ReP; (2) $\Lambda=2.75$, $\Omega_c=60.0$, and $\Delta_c=0.0$, respectively. The other parameters used are same as those in Fig. 2.

of the two symmetric gain peaks becomes higher than before, while the other one changes from positive to negative. Simultaneously, a higher index of refraction with vanishing absorption is obtained. In Fig. 6, with $\Lambda=2.50$ and $\Omega_c=20.0$, we find that, when the detuning of the coherent field Δ_c is changed from 0 to -10.0, the index of refraction with zero absorption at point G in curve n becomes much higer than that at point B in curve m. So the high index of refraction with zero absorption can be modified by the incoherent pumping rate Λ, the detuning Δ_c and the Rabi frequency Ω_c of the coherent field.

IV. CONCLUSIONS

In the present paper, a four-level system is constructed in order to enhance the index of refraction in a Er^{3+}-doped YAG crystal accompanied by vanishing or low absorption. It is found that the high index of refraction with zero absorption can be provided by adjusting the incoherent pumping and the coherent field. Furthermore, both the separation of

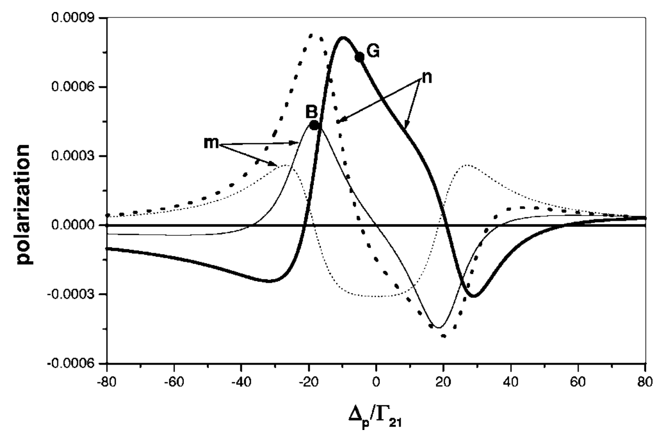

FIG. 6. Same as Fig. 2. The curves m and n correspond to (1) $\Lambda=2.50$, $\Omega_c=20.0$, and $\Delta_c=0.0$; (2) $\Lambda=2.50$, $\Omega_c=20.0$, and $\Delta_c=-10.0$, respectively. The other parameters used are same as those in Fig. 2.

the two absorption peaks and the maximum value of the index of refraction with zero absorption can be modified by the incoherent pump and the coherent field. In particular, it should be pointed that a higher index of refraction with zero absorption is easily achieved when the coherent field is off resonance.

ACKNOWLEDGMENTS

The authors would like to thank the NNSF of China, the Doctoral Program Foundation of Institution of Higher Education of China, and Ministry of Education of China for support.

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