Spatial antibunching of photons with parametric down-conversion

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The theoretical framework behind a recent experiment by Nogueira *et al.* [Phys. Rev. Lett. **86**, 4009 (2001)] of spatial antibunching in a two-photon state generated by collinear type-II parametric down-conversion and a birefringent double slit is presented. The fourth-order quantum correlation function is evaluated and shown to violate the classical Schwarz-type inequality, ensuring that the field does not have a classical analog. We expect these results to be useful in the rapidly growing fields of quantum imaging and quantum information.

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I. INTRODUCTION

As current technology advances, more and more attention is placed upon quantum mechanics to solve future problems. Furthermore, quantum systems are capable of performing some tasks more efficiently than classical systems [1], drawing even more emphasis to quantum technologies. In particular, the fields of optical communication, optical imaging, and optical information processing have been appended by the rapidly developing fields of quantum communication [2–4], quantum imaging [5,6], and quantum information processing [1]. Thus, the study of quantum phenomena promises to be a fruitful enterprise.

For many years, researchers have studied the nonclassical behavior of light, such as squeezing [7–9] and antibunching [10–12]. However, most theoretical and experimental investigations deal with time variables only. That is, most treatments consider only one spatial mode. In a recent review article, Kolobov [13] demonstrates that many quantum phenomena also occur when considering spatial variables of the electromagnetic field. Many areas of technology stand to benefit from the possible applications provided by such quantum phenomena.

An invaluable tool in these areas of research is the generation of entangled photons using parametric down-conversion [14]. The two-photon state of light exhibits nonseparable behavior [15,16] and has been used in nearly all quantum information schemes [17].

Spatial antibunching was recently observed experimentally by Nogueira *et al.* [18] using spontaneous parametric down-conversion (SPDC). In this paper, we provide a theoretical background for the experiment reported in Ref. [18]. Section II is dedicated to the general introduction of temporal and spatial antibunching. In Sec. III we discuss the theoretical observation of spatial antibunching of photons using a two-photon entangled state produced by SPDC, as in Ref. [18]. We close with some concluding remarks in Sec. IV.

II. PHOTON BUNCHING AND ANTIBUNCHING

It is well known that any state of the electromagnetic field that has a classical analog can be described by means of a positive nonsingular Glauber-Sudarshan P distribution, which has the properties of a classical probability functional over an ensemble of coherent states. Because of this fact, the normally ordered intensity correlation function for stationary fields must obey the following inequality [19]:

$$\langle \mathcal{T}{:}\hat{I}(\mathbf{r},t)\hat{I}(\mathbf{r},t+\tau){:}\rangle \leqslant \langle{:}\hat{I}^2(\mathbf{r},t){:}\rangle, \tag{1}$$

where $\mathcal{T}{::}$ stands for time and normal ordering. Photon density operators are defined as

$$\hat{I}(\mathbf{r},t)=\hat{\boldsymbol{V}}^{\dagger}(\mathbf{r},t)\cdot\hat{\boldsymbol{V}}(\mathbf{r},t), \tag{2}$$

where

$$\hat{\boldsymbol{V}}(\mathbf{r},t)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k},\sigma}\hat{a}_{\mathbf{k},\sigma}\boldsymbol{\epsilon}_{\mathbf{k},\sigma}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)}, \tag{3}$$

$\hat{a}_{\mathbf{k},\sigma}$ is the annihilation operator for the mode with wave vector $\mathbf{k}$ and polarization σ, $\boldsymbol{\epsilon}_{\mathbf{k},\sigma}$ is the unit polarization vector, Ω is the quantization volume, and $\omega=ck$.

Expression (1) is commonly written in the shorter form,

$$G^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2,\tau)\leqslant G^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2,0), \tag{4}$$

where

$$G^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\langle \mathcal{T}{:}\hat{I}(\mathbf{r}_1,t)\hat{I}(\mathbf{r}_2,t+\tau){:}\rangle. \tag{5}$$

Since the delayed photon coincidence-detection probability $\mathcal{P}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ is proportional to $G^{(2,2)}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ [19], inequality (4) means that for the class of fields considered above, photons are detected either bunched or randomly distributed in time. Photon antibunching in time, characterized by the violation of Eq. (1), was predicted by Carmichael and Walls [10], Kimble and Mandel [11], and was first observed by Kimble, Dagenais, and Mandel in resonance fluorescence [12].

In the space domain, the concept analogous to stationarity is homogeneity. For a homogeneous field, the expectation value of any quantity that is a function of position is invariant under translation of the origin [19]. In particular, on a plane surface normal to the propagation direction,

$$G^{(2,2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\tau)=G^{(2,2)}(\boldsymbol{\delta},\tau) \tag{6}$$

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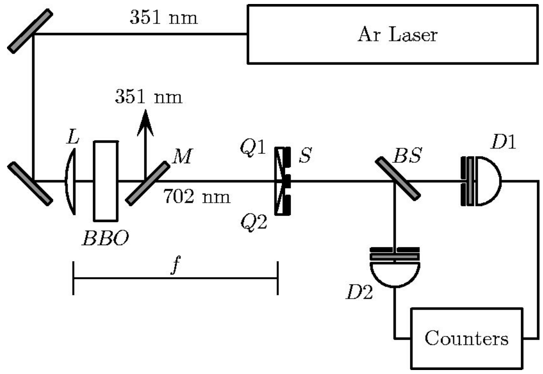

FIG. 1. Schematic diagram of spatial antibunching setup. An Ar laser pumps a BBO crystal, generating correlated photons. The down-converted photons are incident on the birefringent double slit S and then the beam splitter BS. The pump beam is focused on the double slit. Single and coincidence counts are registered with detectors D_1 and D_2.

and

$$\langle :I^n(\boldsymbol{\rho}+\boldsymbol{\delta},t+\tau):\rangle = \langle :I^n(\boldsymbol{\rho},t):\rangle, \tag{7}$$

where $\boldsymbol{\rho}$ is the transverse position vector, $\boldsymbol{\delta}=\boldsymbol{\rho}_1-\boldsymbol{\rho}_2$, and $n=1,2,\ldots$

For homogeneous and stationary fields described by positive nonsingular P distributions, the Schwarz inequality implies that

$$\langle \mathcal{T}:\hat{I}(\boldsymbol{\rho},t)\hat{I}(\boldsymbol{\rho}+\boldsymbol{\delta},t+\tau):\rangle \leqslant \langle :\hat{I}^2(\boldsymbol{\rho},t):\rangle, \tag{8}$$

that is,

$$G^{(2,2)}(\boldsymbol{\delta},\tau) \leqslant G^{(2,2)}(\mathbf{0},0). \tag{9}$$

Analogously to what was concluded from inequality (4), for fields that admit classical stochastic models, inequality (9) implies that photons are detected either spatially bunched or randomly spaced in a transverse detection screen. Violation of Eq. (9) implicates the possibility of quantum fields exhibiting spatial antibunching. Spatial antibunching of photons has been predicted by some authors [13,20–23].

III. SPATIAL ANTIBUNCHING WITH DOWN-CONVERSION

In this section, we show that a field that violates inequality (9) can be generated by means of spontaneous parametric down-conversion. The experimental setup, we are considering is shown in Fig. 1. A nonlinear birefringent crystal is used to generate collinear entangled photon pairs. The down-converted photons are then incident on a birefringent double slit (see Sec. III A) and coincidences are detected by detectors D_1 and D_2. The pump beam is focused on the center of the plane of the double slit, between the two slits. Interference filters are used such that the monochromatic approximation is valid.

The following discussion refers to the basic geometry illustrated in Fig. 2, where a thin crystal is separated from an aperture plane by a distance s and the aperture plane is separated from a detection plane by a distance z.

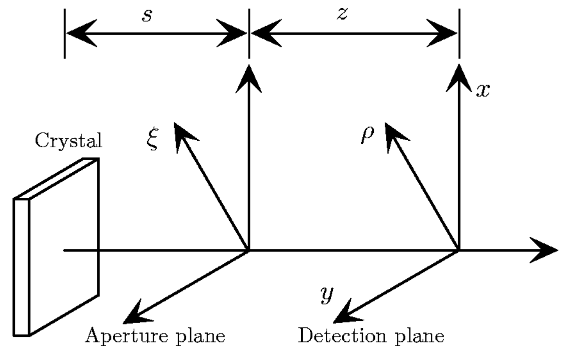

FIG. 2. Illustration of the geometry. s is the crystal-aperture distance and z is the aperture-detector distance.

Using a treatment based on Ref. [24], in the paraxial and monochromatic approximations, collinear SPDC generates a quantum state of the form [25]

$$|\psi\rangle_{\mathrm{SPDC}} = C_1|\mathrm{vac}\rangle + C_2|\psi\rangle, \tag{10}$$

with

$$|\psi\rangle = \int\int_D d\mathbf{q}_1 d\mathbf{q}_2 \Phi(\mathbf{q}_1,\mathbf{q}_2)|\mathbf{q}_1,\sigma_1\rangle|\mathbf{q}_2,\sigma_2\rangle. \tag{11}$$

The coefficients C_1 and C_2 are such that $|C_2| \ll |C_1|$. C_2 depends on the crystal length, the nonlinearity coefficient and the magnitude of the pump field, among other factors. The kets $|\mathbf{q}_j,\sigma_j\rangle$ represent Fock states labeled by the transverse component $\mathbf{q}_j$ of the wave vector $\mathbf{k}_j$ and the polarization σ_j of the down-converted photon $j=1,2$. In this paper, we consider type-II phase matching, in which case $\sigma_1=e$ and $\sigma_2=o$, where e (o) stands for extraordinary (ordinary) polarization. $|\psi\rangle$ is the two-photon component of the total quantum state. The function $\Phi(\mathbf{q}_1,\mathbf{q}_2)$, which can be regarded as the normalized angular spectrum of the two-photon field [25], is given by

$$\Phi(\mathbf{q}_1,\mathbf{q}_2) = \left(\frac{1}{\pi}\right)\sqrt{\frac{L}{K}}v(\mathbf{q}_1+\mathbf{q}_2)\,\mathrm{sinc}\left(\frac{L|\mathbf{q}_1-\mathbf{q}_2|^2}{4K}\right), \tag{12}$$

where $v(\mathbf{q})$ is the normalized angular spectrum of the pump beam, L is the length of the nonlinear crystal in the z direction, and K is the magnitude of the pump field wave vector. The integration domain D is, in principle, defined by the conditions $q_1^2 \leqslant k_1^2$ and $q_2^2 \leqslant k_2^2$. However, in most experimental conditions, the domain in which $\Phi(\mathbf{q}_1,\mathbf{q}_2)$ is appreciable is much smaller than that. The state written above is not to be considered as a general expression for the SPDC process. Its validity is determined by experimental conditions, especially by the detection apparatus. As long as the monochromatic and paraxial approximations are valid, the results predicted by expression (11) are in excellent agreement with experience. Monochromatic approximation is guaranteed by the presence of narrow-band interference filters in the detection apertures, whereas paraxial approximation is guaranteed by keeping transverse detection regions much smaller than their distance from the crystal.

We consider for now that the down-converted fields are incident on some sort of aperture, so as to produce fourth-order interference in the absence of second-order interference. The reason for such a requirement is the following: Spatial photon antibunching is a fourth-order effect in a homogeneous field, that is to say, in a field that [according to Eq. (7)], does not show intensity patterns. With this scheme, we are seeking for a fourth-order interference pattern that depends only on x_1-x_2, the relative position of detectors. Furthermore, this fourth-order interference pattern must have a minimum when $x_1=x_2$, in order to produce antibunching. Fourth-order spatial interference in the absence of second order can be achieved in spontaneous parametric down-conversion by means of a double slit, whose slit separation is much greater than the transverse coherence length of the down-converted field, as reported by Fonseca *et al.* [26]. However, in Ref. [26], the fourth-order correlation function, which is proportional to the coincidence rate, depends on x_1+x_2 instead of x_1-x_2. In order to achieve a minimum of coincidences when $x_1=x_2$, we have to introduce a phase difference of π between the two possibilities (photon 1 through slit 1, photon 2 through slit 2) and (photon 1 through slit 2, photon 2 through slit 1). In our experiment, the phase difference was introduced by means of birefringent elements placed in front of each slit, as described later. After the aperture, the two-photon state can be written as

$$|\psi\rangle=M\sum_{\sigma_1',\sigma_2'}\int\int\int\int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_1' d\mathbf{q}_2' \Phi_A(\mathbf{q}_1,\mathbf{q}_2)\times T_{\sigma_1\sigma_1'}(\mathbf{q}_1'-\mathbf{q}_1)T_{\sigma_2\sigma_2'}(\mathbf{q}_2'-\mathbf{q}_2)|\mathbf{q}_1',\sigma_1'\rangle|\mathbf{q}_2',\sigma_2'\rangle, \tag{13}$$

where M is a normalization constant, $\Phi_A(\mathbf{q}_1,\mathbf{q}_2)$ is the angular spectrum of the biphoton field on the aperture plane, that is,

$$\Phi_A(\mathbf{q}_1,\mathbf{q}_2)=\left(\frac{1}{\pi}\right)\sqrt{\frac{L}{K}}v(\mathbf{q}_1+\mathbf{q}_2)\mathrm{sinc}\left(\frac{L}{4K}|\mathbf{q}_1-\mathbf{q}_2|^2\right)\times\exp\left[is\left(k_1+k_2-\frac{q_1^2}{2k_1}-\frac{q_2^2}{2k_2}\right)\right]. \tag{14}$$

$T_{\sigma\sigma'}(\mathbf{q})$ is the transfer function of the aperture, linking the incident field with transverse wave vector $\mathbf{q}$ and polarization σ with the scattered field with transverse wave vector $\mathbf{q}'$ and polarization σ'. $T_{\sigma\sigma'}(\mathbf{q})$ is given by the Fourier transform of the aperture function $A_{\sigma\sigma'}(\boldsymbol{\xi})$.

Since, we are working with collinear SPDC, with $k_1=k_2=\frac{1}{2}K$, Φ_A is written as

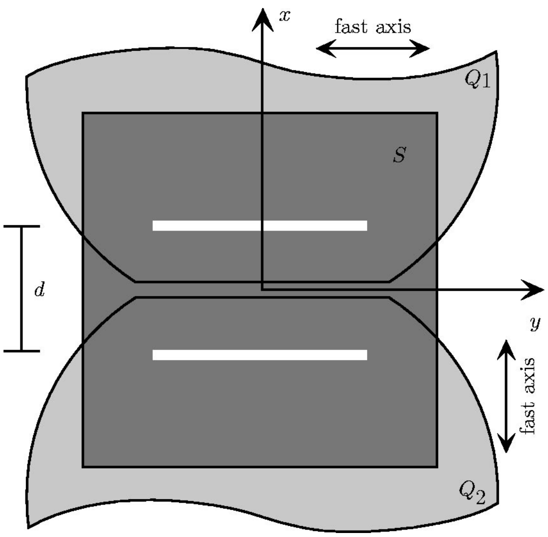

FIG. 3. The birefringent double slit. The quarter wave plates Q_1 and Q_2 are aligned with orthogonal fast axes. S is a double slit with slit separation $2b$.

$$\Phi_A(\mathbf{q}_1,\mathbf{q}_2)=\left(\frac{1}{\pi}\right)\sqrt{\frac{L}{K}}v(\mathbf{q}_1+\mathbf{q}_2)\mathrm{sinc}\left(\frac{L}{4K}|\mathbf{q}_1-\mathbf{q}_2|^2\right)\times\exp\left[\frac{-is}{2K}(|\mathbf{q}_1+\mathbf{q}_2|^2+|\mathbf{q}_1-\mathbf{q}_2|^2)\right], \tag{15}$$

where the irrelevant phase factor e^{iKs} is omitted.

Using the orthonormal properties of the Fock states, we can define

$$\boldsymbol{\Psi}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle\mathrm{vac}|\hat{\boldsymbol{V}}(\boldsymbol{\rho}_2)\otimes\hat{\boldsymbol{V}}(\boldsymbol{\rho}_1)|\psi\rangle \tag{16}$$

as the two-photon coincidence-detection amplitude, where

$$\hat{\boldsymbol{V}}(\boldsymbol{\rho})=e^{ikz}\sum_{\sigma}\int d\mathbf{q}\hat{a}_{\sigma}(\mathbf{q})\boldsymbol{\epsilon}_{\sigma}e^{i[\mathbf{q}\cdot\boldsymbol{\rho}-(q^2/2k)z]} \tag{17}$$

is the monochromatic form of Eq. (3) in the paraxial approximation and z is the distance between the aperture plane and the detection plane, as shown in Fig. 2. It is assumed that the polarization vector $\boldsymbol{\epsilon}$ is independent of $\mathbf{q}$. The two-photon coincidence-detection probability for stationary fields is proportional to the fourth-order correlation function with $\tau=0$:

$$\mathcal{P}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\propto G^{(2,2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,0)=||\boldsymbol{\Psi}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)||^2. \tag{18}$$

A. The birefringent double slit

The birefringent double slit consists of two quarter-wave plates mounted in front of a typical double slit, such that each wave plate covers only one slit and their fast axes are orthogonal to one another, as shown in Fig. 3. The slits are separated by a distance *d*. With the plate-slit aperture oriented, such that the slits and one fast axis are parallel to the *e* (*y*) direction and the other fast axis parallel to the *o* (*x*) direction, we can approximate the field-aperture functions by

$$A_{oo}(\boldsymbol{\xi}) = -i\,\delta(\xi_x - d/2) + \delta(\xi_x + d/2),$$
$$A_{ee}(\boldsymbol{\xi}) = \delta(\xi_x - d/2) - i\,\delta(\xi_x + d/2),$$
$$A_{eo}(\boldsymbol{\xi}) = 0,$$
$$A_{oe}(\boldsymbol{\xi}) = 0, \tag{19}$$

where ξ_x is the x component of $\boldsymbol{\xi}$. The plate-slit apertures provide a controlled phase factor, that is, no phase will be added to a field with polarization parallel to the direction of the fast axis of the wave plate, while a field with perpendicular polarization will be modified by a phase factor of exp $(-i\pi/2)$. Thus, the phase factor depends on the polarization of the field as well as through which slit the field "passes."

B. The coincidence-detection probability amplitude

Combining Eqs. (13)–(17) and (19), we arrive at the following expression for coincidence-detection amplitude in the Fraunhofer approximation:

$$\boldsymbol{\Psi} = \Psi_{eo}[\boldsymbol{\epsilon}_e \otimes \boldsymbol{\epsilon}_o] + \Psi_{oe}[\boldsymbol{\epsilon}_o \otimes \boldsymbol{\epsilon}_e], \tag{20}$$

where

$$\Psi_{\sigma_1\sigma_2} \propto \int\int d\mathbf{q}_1 d\mathbf{q}_2 \Phi_A(\mathbf{q}_1,\mathbf{q}_2)\left\{\cos\left[\frac{d}{2}(q_{1x}+q_{2x}) - \frac{kd}{2z}(x_1 + x_2)\right] \pm \sin\left[\frac{d}{2}(q_{1x}-q_{2x}) - \frac{kd}{2z}(x_1-x_2)\right]\right\}, \tag{21}$$

where the "+" holds for Ψ_{eo} and the "−" holds for Ψ_{oe}.

We assume that the pump field is a Gaussian beam, whose waist is located on the aperture plane:

$$u_A(\boldsymbol{\xi}) \propto e^{-\xi^2/w_0^2}. \tag{22}$$

Its angular spectrum is

$$v_A(\mathbf{q}) = v(\mathbf{q})\exp\left[is\left(K - \frac{q^2}{2K}\right)\right] \propto e^{-w_0^2 q^2/4}, \tag{23}$$

where w_0 is the radius of the beam waist. Using Eqs. (15) and (23) in Eq. (21), it is straightforward to show that

$$\Psi_{\sigma_1\sigma_2} \propto e^{-(d/2w_0)^2}\cos\left[\frac{kd}{2z}(x_1+x_2)\right] \mp \sin\left[\frac{kd}{2z}(x_1-x_2)\right]. \tag{24}$$

It is interesting to note that the length L of the nonlinear crystal enters in the coincidence-detection amplitude only as a multiplicative constant. It is clear from expression (24) above that the fulfillment of the homogeneity condition (7) for $n=2$ in the x direction depends on the factor $e^{-(d/2w_0)^2}$. If $w_0 \ll d$, the dependence on x_1+x_2 disappears and transverse field on the detection plane can be considered as homogeneous. This is the reason why the pump beam must be focused on the center of the double slit. In this case,

$$\Psi_{eo}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = -\Psi_{oe}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \propto \sin\left[\frac{kd}{2z}(x_1-x_2)\right]. \tag{25}$$

Thus, the coincidence-detection probability is

$$\mathcal{P}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \propto 1 - \cos\left[\frac{kd}{z}(x_1-x_2)\right]. \tag{26}$$

When $x_1 = x_2$, the coincidence count rate is zero and increases with $x_1 - x_2$ until $(x_1-x_2)kd/z = \pm\pi/2$. Therefore, the fourth-order correlation function $G^{(2,2)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,t)$, which is proportional to the coincidence-detection probability $\mathcal{P}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$, does not have a maximum at $x_1 = x_2$. This contradicts Eq. (9), thus characterizing spatial antibunching of photons.

IV. DISCUSSION AND CONCLUSION

We have shown the theoretical background behind the spatial antibunching of photons using parametric down-conversion. It may be instructive for the reader to compare the experiment analyzed here with its classical counterpart. In this context, the single count detection rate $R_{cl}(x)$ should be proportional to the classical average intensity $\langle I(x)\rangle$, whereas the coincidence count rate $C_{cl}(x_1,x_2)$ should be proportional to the intensity-intensity (or the fourth-order) correlation function $\langle I(x_1)I(x_2)\rangle$. The single count detection rate of down-converted light in the presence of a double slit has been studied in previous works [26–28]. In Ref. [27], it was demonstrated that in terms of its single count rate, SPDC behaves like a classical Schell-model extended light source. In our experiment, the transverse coherence length being shorter than the slits separation and shorter than the slits width themselves, the single count rate is given by the classical expression for incoherent illumination, which can be approximated by a Gaussian,

$$R_{cl}(x) \propto e^{-x^2/2\sigma^2}, \tag{27}$$

where $\sigma = z/ka$ and a is the width of the slits. Since the transverse detection range $x_{max} - x_{min}$ is much shorter than the width of this Gaussian profile for the slit-detectors distance considered, the single count rate is fairly constant over the detection range [18]. By another side, the coincidence-detection rate due to a classical source is totally different from that observed with a down-converted light. Perhaps, the best classical model for type-II SPDC is a superposition of two extended light sources orthogonally polarized and correlated in intensity. After the light is diffracted by the birefringent double slit, the calculation of the classical fourth-order correlation function is quite similar to the case of the Brown-Twiss intensity interferometer [29]. Classical intensity interferometry is known to be insensitive to phase. Therefore, the birefringent elements have no effect on the predicted fourth-order correlation, that is,

$$C_{cl}(x_1,x_2) \propto 1 + v\,\cos\left[\frac{kd}{z}(x_1-x_2)\right]. \tag{28}$$

The visibility v is in the range $0 \leqslant v \leqslant \frac{1}{2}$ and depends on the statistics of the source. It is clear from expression (28) above that $C_{cl}(x_1, x_2)$ predicts spatial bunching, as expected from any classical light source. In view of the above analysis, the results presented here describe an entirely quantum fourth-order interference effect, with no classical analog [30]. In addition to rendering further interest in the study of nonclassical states of light, spatial antibunching promises to be a useful tool in quantum imaging and quantum information technologies.

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[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University, Cambridge, 2000).
[2] G.A. Barbosa, Phys. Rev. A **58**, 3332 (1998).
[3] S.J. van Enk, H.J. Kimble, J.I. Cirac, and P. Zoller, Phys. Rev. A **59**, 2659 (1999).
[4] S.F. Pereira, Z.Y. Ou, and H.J. Kimble, Phys. Rev. A **62**, 042311 (2000).
[5] A. Gatti, E. Brambilla, L. Lugiato, and M.I. Kolobov, Phys. Rev. Lett. **83**, 1763 (1999).
[6] A.F. Abouraddy, B.E.A. Saleh, A.V. Sergienko, and M.C. Teich, Phys. Rev. Lett. **87**, 123602 (2001).
[7] D. Stoler, Phys. Rev. D **1**, 3217 (1970).
[8] D. Stoler, Phys. Rev. D **4**, 1925 (1971).
[9] R. Slusher, L. Hollberg, B. Yurke, J. Mertz, and J. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[10] H.J. Carmichael and F.F. Walls, J. Phys. B **9**, L43 (1976).
[11] H.J. Kimble and L. Mandel, Phys. Rev. A **13**, 2123 (1976).
[12] H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[13] M.I. Kolobov, Rev. Mod. Phys. **71**, 1539 (1999).
[14] D. Burnham and D. Weinberg, Phys. Rev. Lett. **25**, 84 (1970).
[15] E.J.S. Fonseca, C.H. Monken, and S. Pádua, Phys. Rev. Lett. **82**, 2868 (1999).
[16] E. Fonseca, Z. Paulini, P. Nussenzveig, C. Monken, and S. Pádua, Phys. Rev. A **63**, 043819 (2001).
[17] See M. A. Nielsen and I. L. Chuang, in *Quantum Computation and Quantum Information* (Ref. [1]); A. Cabello, e-print quant-ph/0012089.
[18] W.A.T. Nogueira, S.P. Walborn, S. Pádua, and C.H. Monken, Phys. Rev. Lett. **86**, 4009 (2001).
[19] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
[20] M.L. Berre-Rousseau, E. Ressayre, and A. Tallet, Phys. Rev. Lett. **43**, 1314 (1979).
[21] D.N. Klyshko, Zh. Eksp. Teor. Fiz. **83**, 1313 (1982) [Sov. Phys. JETP **56**, 753 (1982)].
[22] Z. Bialynicka-Birula, I. Bialynicki-Birula, and G.M. Salamone, Phys. Rev. A **43**, 3696 (1991).
[23] M.I. Kolobov and I. Sokolov, Europhys. Lett. **15**, 271 (1991).
[24] C.K. Hong and L. Mandel, Phys. Rev. A **31**, 2409 (1985).
[25] C.H. Monken, P.H.S. Ribeiro, and S. Pádua, Phys. Rev. A **57**, 3123 (1998).
[26] E.J.S. Fonseca, C.H. Monken, S. Pádua, and G.A. Barbosa, Phys. Rev. A **59**, 1608 (1999).
[27] P.H.S. Ribeiro, C.H. Monken, and G.A. Barbosa, Appl. Opt. **33**, 352 (1994).
[28] P.H. Souto Ribeiro, S. Pádua, and C.H. Monken, Phys. Rev. A **60**, 5074 (1999).
[29] R. Hanbury Brown, *The Intensity Interferometer* (Taylor & Francis, London, 1974).
[30] A.V. Belinsky and D.N. Klyshko, Phys. Lett. A **166**, 303 (1992).