Imaging population distribution between two coupled atomic Bose-Einstein condensates by using short laser pulses

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In this work we investigate scattering of ultrashort light pulses from two coupled neutral atomic Bose-Einstein condensates corresponding to two different ground hyperfine sublevels. Two counterpropagating σ_+ and σ_- light waves are employed to excite the atomic condensates. We find that the spectrum of scattered light is determined by the initial preparation of the atomic condensate. The spectrum is found to be a mirror image of the population distribution. The scattered light probes the population distribution between the two condensates. In particular, we find that, when the population is equally distributed between the two ground states, the quantum fluctuations in the spectrum are suppressed due to destructive quantum interference.

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INTRODUCTION

The experimental realization of Bose-Einstein condensation (BEC) of dilute atomic vapors [1–3] has generated immense interest in the properties of quantum atomic gases and their manipulation by various techniques [4,5]. Apart from the fundamental question of the possibility of achieving sufficiently low temperatures and high densities to obtain BEC, there is another question concerning detection and observation of the condensate; obviously, the latter goal may be realized by scattering light on the system of cooled atoms.

Analyzing the properties of light scattered from a sample of cold bosonic atoms can provide a means of detecting effects associated with the formation of a BEC. Proposals to utilize scattering of far-off-resonance light to probe density correlations [6,7] are an example. Studies of light scattering from degenerate atomic gases were initiated by Svistunov and Shlyapnikov [8] and by Politzer [9], who considered scattering of weak light from a low-temperature ($T\approx 0$) condensate formed from a spatially homogeneous gas. For this configuration band gaps exist in the condensate excitation spectrum, giving rise to strong reflection of resonant light from the sharp boundary of the condensate. In the context of experiments, however, this situation is inappropriate as it corresponds to the case of an infinitely large trap. A number of classic papers have since appeared dealing with light scattering from condensates confined in traps of a more realistic size and shape. A review of much of this work has been given by Lewenstein and You [10]. Lewenstein and You [11] and You *et al.* [12] have investigated the scattering of short but intense laser pulses from a trapped sample of cold bosonic atoms. They found that above the critical temperature T_c for BEC coherent scattering is weak and restricted to a narrow cone in the forward direction, while below T_c the scattering of photons occurs into a solid angle in the forward direction determined by the size of the condensate. You *et al.* [13] demonstrated that coherent scattering probes the density profile of the trapped atoms. On the experimental side, Andrews *et al.* [14] have already used dispersive light scattering to spatially image a trapped condensate. The dynamics of two BEC's that are optically coupled through a common excited state by laser-driven Raman transitions has been investigated by many authors in the context of manipulation of quantum statistics [15], measurement of the relative phase between two condensates [16], output coupling for an atom laser [17], realization of a vortex coupler for BEC [18], efficient population transfer into any desired state [19], and probing of statistical properties of BEC's with ultrashort pulses [20]. Motivated by these interesting developments in recent years of observations of properties of BEC's and their manipulation, we propose a scheme to image the population distribution between two coupled BEC's using short laser pulses.

THE MODEL

The atom in the condensate is considered as a system with internal transitions $J_g=1\rightarrow J_c=1$. Initially the atomic condensate is assumed to be prepared in one of the two ground-state sublevels $|g_-\rangle$ and $|g_+\rangle$ or in a coherent superposition of the two ground states by possible laser-cooling techniques. The two ground states differ by their internal quantum numbers.

Two counterpropagating σ_+ and σ_- light waves are employed to excite the atomic condensates (Fig. 1). The state $|g_-\rangle$ is optically coupled to the electronically excited state $|e_0\rangle$ by the driving field Ω_+ with polarization σ_+. Similarly, the driving field Ω_- with polarization σ_- couples the state $|g_+\rangle$ to the state $|e_0\rangle$. Both the laser fields are assumed to have the same frequency ω_L and time-dependent amplitude $\Omega(t)$. The Hamiltonian governing the interaction of the two light fields with N bosonic three-level Λ atoms confined in a trap takes the following second quantized form in the Fock representation:

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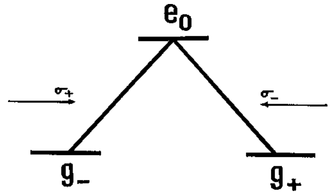

FIG. 1. The atom in the condensate is considered as a Λ system with internal transitions $J_g=1\rightarrow J_e=1$. The two ground states differ by their internal quantum numbers. Two counterpropagating σ_+ and σ_- light waves are employed to excite the atomic condensates. The state $|g_-\rangle$ is optically coupled to the electronically excited state $|e_0\rangle$ by the driving field Ω_+ with polarization σ_+. Similarly the driving field Ω_- with polarization σ_- couples the state $|g_+\rangle$ to the state $|e_0\rangle$.

$$
\begin{aligned}
H=&\hbar\sum_n\sum_{i=e,g\pm}\omega_n^i\psi_{ni}^\dagger\psi_{ni}+\hbar\int dk\, cka_{k\sigma_+}^\dagger,a_{k\sigma_+}\\
&+\hbar\int dk\, cka_{k\sigma_-}^\dagger a_{k\sigma_-}\\
&+\sum_n\int dk\,\Omega_-(k)a_{k\sigma_-}^\dagger\psi_{ng_+}^\dagger\psi_{ne}\eta_{+e}(k)\\
&+\sum_n\int dk\,\Omega_+a_{k\sigma_+}^\dagger\psi_{ng_-}^\dagger\psi_{ne}\eta_{-e}(k)+\text{H.c.},
\end{aligned}\qquad(1)
$$

where ψ_{ni} and $\psi_{ni}^\dagger$ are the annihilation and creation operators of atoms for the nth vibrational state of the center-of-mass motion of the atom in the trap, and in the states $i=g_\pm$, e. We will consider temperatures $T<T_c$ (the critical temperature) so that the Bose-Einstein condensate is assumed to be in the lowest vibrational state, i.e., $n=0$. $a_{k\sigma_\pm}$ and $a_{k\sigma_\pm}^\dagger$ denote annihilation and creation operators for photons of momentum $\vec{k}$ and polarization $\sigma_\pm$. All the annihilation and creation operators obey standard bosonic commutation relations. The coupling strengths $\Omega_\pm$ are slowly varying functions of $\vec{k}$. Finally, $\eta_{\pm,e}(\vec{k})$ are the Franck-Condon factors (i.e., matrix elements for the center-of-mass transition from the lowest state of the ground-state potential to the lowest state of the excited-state potential),

$$
\eta_{\pm,e}(\vec{k})=\langle g_\pm,0|e^{-i\vec{k}.\vec{R}}|e,0\rangle.\qquad(2)
$$

Recent self-consistent analysis has shown that, if the laser-atom interaction time is selected as short as 10 ps, the collective spontaneous emissions can be legitimately neglected. Hereafter, we will assume that the two applied light waves are pulses of the same temporal envelope with duration of the order of 10 ps and width $\gamma_L=10^{11}$ Hz. Consequently, we assume that during the interaction with the laser pulse the effects of dissipative spontaneous emission and dispersive dipole-dipole interactions are small, as compared to the effects of the coherent driving lasers. We can then rewrite the interaction terms as

$$
\hbar\frac{\Omega_\pm}{2}F\left[\gamma_L\left(t-\frac{\vec{k}_L\cdot\vec{R}}{\omega_L}\right)\right]e^{i\vec{k}_L\cdot\vec{R}-i\omega_L t}\psi_{g_\pm}^\dagger\cdot\psi_e\eta_{\pm,e}(\vec{k}_L)+\text{H.c.},\qquad(3)
$$

which corresponds to an assumption that the pulses have the forms of plane-wave packets propagating in the $\vec{k}_L$ direction with center frequency ω_L and polarization $\sigma_\pm$. $F(\gamma_L t)$ is the temporal envelope of the pulses, chosen to be real. In the trap system concerned, the characteristic length is the size of the vibrational ground-state wave function $a\approx10^{-5}$ m, while the momenta of the σ_+ and σ_- polarized photons are in the range $\hbar k_L\pm\hbar\gamma_L/c$, whose changes are negligible in comparison to $1/a\approx10^5\ \text{m}^{-1}$.

We may thus safely set $\vec{k}\approx\vec{k}_L$ inside $\eta_{\pm,e}(\vec{k}_L)$. Using Eq. (3), the Hamiltonian (1) becomes

$$
\begin{aligned}
H=&\hbar\sum_{i=e,g_\pm}\omega^i\psi_i^\dagger\psi_i\\
&+\hbar\sum_i\frac{\Omega_i}{2}F(\gamma_L t)\{e^{i\omega_L t}\psi_i^\dagger\psi_e\eta_{i,e}(\vec{k}_L)+\text{H.c.}\}.
\end{aligned}\qquad(4)
$$

The Heisenberg equation for ψ_i that follows from the Hamiltonian (4) now becomes linear. Thus, at resonance, $\omega_L\approx\omega_0=\omega^e-\omega^g$ (we have taken the two ground states to be degenerate so that $\omega^{g+}\approx\omega^{g-}=\omega^g$), and in the rotating frame in which $\psi_{g_\pm}\rightarrow e^{-i\omega^{g}t}\psi_{g_\pm}$, $\psi_e\rightarrow e^{-i(\omega^g+\omega_L)t}\psi_e$, they can be written as

$$
\frac{\partial\psi_e(\vec{k},t)}{\partial t}=-i\left\{\frac{\Omega_1^*(\vec{k},t)\psi_{g_-}(\vec{k},t)}{2}+\frac{\Omega_2^*(\vec{k},t)\psi_{g_+}(\vec{k},t)}{2}\right\},\qquad(5)
$$

$$
\frac{\partial\psi_{g_-}(\vec{k},t)}{\partial t}=-i\frac{\Omega_1(\vec{k},t)\psi_e(\vec{k},t)}{2},\qquad(6)
$$

$$
\frac{\partial\psi_{g_+}(\vec{k},t)}{\partial t}=-i\frac{\Omega_2(\vec{k},t)\psi_e(\vec{k},t)}{2},\qquad(7)
$$

where

$$
\begin{aligned}
\frac{\Omega_1(\vec{k},t)}{2}&=\frac{\Omega_+F(\gamma_L t)\eta_{-,e}(\vec{k}_L)}{2},\\
\frac{\Omega_2(\vec{k},t)}{2}&=\frac{\Omega_-F(\gamma_L t)\eta_{+,e}(\vec{k}_L)}{2}.
\end{aligned}\qquad(8)
$$

Equations (5)–(7) may be easily solved analytically for any pulse envelope and appropriate initial condition. Initially, the atomic condensate is assumed to be prepared in the ground-state sublevel $|g_-\rangle$. With the pulse area defined as

$$A(t)=\frac{\Omega_{\pm}}{2}\int_{-\infty}^{t}F(\gamma_L t)dt' \tag{9}$$

and the initial conditions

$$\psi_{g_-}(t=-\infty)=\sqrt{N_c}, \quad \psi_{g_+}(t=-\infty)=0,$$

$$\psi_e(t=-\infty)=0, \tag{10}$$

the solutions are

$$\psi_e(\vec{k}_L,t)=-i\,\eta^*_{-,e}(\vec{k}_L)\sqrt{N_c/2}\sin[A(t)], \tag{11}$$

$$\psi_{g_-}(\vec{k}_L,t)=\frac{\sqrt{N_c}}{2}\{\cos[A(t)]+1\}, \tag{12}$$

$$\psi_{g_+}(\vec{k}_L,t)=\frac{\sqrt{N_c}}{2}\{\cos[A(t)]-1\}, \tag{13}$$

where N_c is the total number of condensate atoms.

In deriving Eqs. (11)–(13), we have made use of the following relation:

$$\eta_{+,e}(\vec{k}_L)\,\eta_{-,e}(\vec{k}_L)=\langle +,0|e^{-\vec{k}_L\cdot\vec{R}}|0,e\rangle\langle e,0|e^{i\vec{k}_L\cdot\vec{R}}|0,-\rangle$$
$$=e^{-(\vec{k}_L-\vec{k}_L)^2a^2/2}=1, \tag{14}$$

where a is the dimension of the trap (related to the size of the ground-state wave function). We have considered the dimensions of the traps for $|g_+\rangle$ and $|g_-\rangle$ condensates to be the same.

SPECTRUM OF SCATTERED LIGHT

We can now calculate the spectrum of scattered photons with polarizations $\sigma_{\pm}$. Thus, using the Hamiltonian (1), we derive the Heisenberg equations for the photon annihilation operators $a_{k\sigma_+}$ and $a_{k\sigma_-}$:

$$\dot{a}_{k\sigma_{\pm}}=-icka_{k\sigma_{\pm}}-i\Omega_{\pm}(\vec{k})\psi^+_{g_{\mp}}(\vec{k}_L,t)\psi_e(\vec{k}_L,t)\,\eta_{\mp,e}(\vec{k}). \tag{15}$$

Formally integrating Eq. (15), together with the initial conditions (10), we obtain

$$a_{k\sigma_-}(\vec{k},t)=e^{-ickt}a_{k\sigma_-}(-\infty)-\frac{\Omega_-N_c}{2\sqrt{2}}\,\eta_{+,e}(\vec{k})\,\eta^*_{-,e}(\vec{k}_L)$$
$$\times\int_{-\infty}^{t}dt'\,e^{-ick(t-t')}\sin[A(t')]$$
$$\times\{\cos[A(t')]-1\}, \tag{16}$$

$$a_{k\sigma_+}(\vec{k},t)=e^{-ickt}a_{k\sigma_+}(-\infty)-\frac{\Omega_+N_c}{2\sqrt{2}}\,\eta_{-,e}(\vec{k})\,\eta^*_{-,e}(\vec{k}_L)$$
$$\times\int_{-\infty}^{t}dt'\,e^{-ick(t-t')}\sin[A(t')]$$
$$\times\{\cos[A(t')]+1\}. \tag{17}$$

The Franck-Condon factors appearing in Eqs. (16) and (17) are evaluated as

$$\eta_{+,e}(\vec{k})\,\eta^*_{-,e}(\vec{k}_L)=\langle +,0|e^{-\vec{k}\cdot\vec{R}}|e,0\rangle\langle e,0|e^{i\vec{k}_L\cdot\vec{R}}|-,0\rangle$$
$$=\langle +,0|e^{-i\vec{k}_{\sigma_-}-\vec{k}_{L\sigma_+}\cdot\vec{R}}|-,0\rangle=e^{-\delta k_1^2a^2/2}, \tag{18}$$

$$\eta_{-,e}(\vec{k})\,\eta^*_{-,e}(\vec{k}_L)=e^{-\delta k_2^2a^2/2}, \tag{19}$$

where $\delta k_1=(\vec{k}_{\sigma_-}-\vec{k}_{L\sigma_+})$ and $\delta k_2=(\vec{k}_{\sigma_+}-\vec{k}_{L\sigma_+})$.

The total spectrum of scattered $\sigma_{\pm}$ photons is defined as

$$C_{\sigma_{\pm}}(\vec{k})=\lim_{t\to\infty}\{\langle a^+_{k\sigma_{\pm}}(t)a_{k\sigma_{\pm}}(t)\rangle+\langle a^+_{k\sigma_{\pm}}(t)a_{k\sigma_{\mp}}(t)\rangle\}. \tag{20}$$

In Eq. (20), the first term is the contribution due to creation of a $\sigma_{\pm}$ photon with the simultaneous destruction of a $\sigma_{\pm}$ photon incident on the $|g_{\mp}\rangle$ condensate, while the second term is the contribution originating from the creation of a $\sigma_{\pm}$ photon with the simultaneous destruction of a $\sigma_{\mp}$ photon incident on the $|g_{\pm}\rangle$ condensate. The total spectrum from Eqs. (16), (17), and (20) takes the form

$$C_{\sigma_{\mp}}(\vec{k})=\frac{|\Omega|^2N_c^2}{8}\Bigg\{e^{-\delta k_{1,2}^2a^2}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2e^{-i[ck-\omega_L]t_1}e^{i[ck-\omega_L]t_2}\sin[A(t_1)]\{\cos[A(t_1)]\mp 1\}$$
$$\times\sin[A(t_2)]\Bigg\{\cos[A(t_2)\mp 1\}e^{-[\delta k_1^2+\delta k_2^2]a^2/2}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2e^{-i[ck-\omega_L]t_1}e^{i[ck-\omega_L]t_2}\sin[A(t_1)]$$
$$\times\{\cos[A(t_1)]\mp 1\}\sin[A(t_2)]\{\cos[A(t_2)\pm 1\}\}, \tag{21}$$

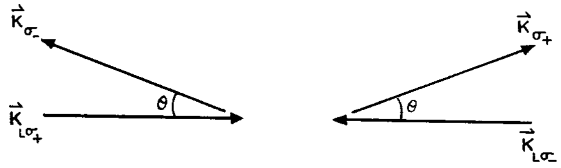

FIG. 2. Relative orientation between the incident photon with wave vector $\vec{k}_L$ and scattered photon with wave vector $\vec{k}$.

where we have taken $|\Omega_+|=|\Omega_-|=|\Omega|$. We will consider an important special case of the hyperbolic secant pulse of area 2π:

$$F(t)=\mathrm{sech}(\gamma_L t), \tag{22}$$

where γ_L is the Fourier width of the pulse. The final analytic expression for $C_{\sigma_\mp}(\vec{k})$ takes the following form:

$$C_{\sigma_\mp}(\vec{k})=\frac{|\Omega|^2N_c^2\pi^2e^{-2k_L^2a^2}}{8\gamma_L^2}\times\frac{\{e^{2k_L^2\cos\theta a^2}(x^2+1)+[(x-1)^2+4x^2]^{1/2}\}}{\cosh^2(\pi x/2)}, \tag{23}$$

where we have taken $|k|/|k_L|=1$, $x=(\omega-\omega_L)/\gamma_L$, and θ is the angle between the incident photon with wave vector $\vec{k}_L$ and the scattered photon with wave vector $\vec{k}$ as shown in Fig. 2. The influence of the initial state of the Bose-Einstein condensate on the spectrum is analyzed by changing the initial condition.

We now recalculate the spectrum for the following initial condition:

$$\psi_{g_-}(t=-\infty)=\sqrt{N_c/2},\quad \psi_{g_+}(t=-\infty)=\sqrt{N_c/2},$$

$$\psi_e(t=-\infty)=0. \tag{24}$$

Since the two hyperfine ground states are taken to be degenerate, the phase difference between the two condensates is zero. The selection of a nonzero phase implies the breaking of the degeneracy of the ground state of the system. Even though we start initially from number states without any initial phase, the relative phase between the two condensates subsequently starts accumulating and evolves at a rate proportional to the local difference in chemical potential between the two condensates, which in general is a function of time. Due to the ultrashort duration of the laser pulse this accumulated relative phase is negligible and can safely be neglected. With the initial condition (24), the solutions of Eqs. (5)–(7) are

$$\psi_e(\vec{k}_L,t)=-i\{\eta^*_{-,e}(\vec{k}_L)+\eta^*_{+,e}(\vec{k}_L)\}\frac{\sqrt{N_c}}{2\sqrt{2}}\sin[A(t)], \tag{25}$$

$$\psi_{g_-}(\vec{k}_Lt)=\psi_{g_+}(\vec{k}_L,t)=\sqrt{N_c/2}\cos[A(t)]. \tag{26}$$

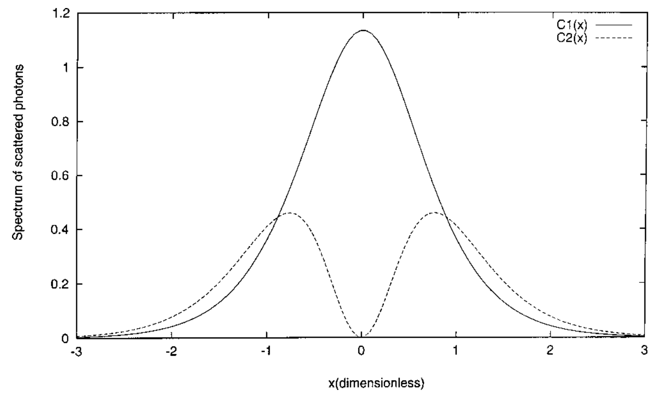

FIG. 3. The frequency spectrum for photons emitted at an angle $\cos\theta=1/k_L^2a^2$ calculated from Eqs. (23) (shown by curve C1) and (28) (shown by curve C2) for $k_La\approx1$.

The photon annihilation operators $a_{k\sigma_+}$ and $a_{k\sigma_-}$ are then given by

$$a_{k\sigma_\pm}(\vec{k},t)=e^{-ickt}a_{k\sigma_\pm}(-\infty)-\frac{|\Omega|N_c}{4}\eta_{\mp,e}(\vec{k})\times\{\eta^*_{-,e}(\vec{k}_L)+\eta^*_{+,e}(\vec{k}_L)\}\times\int_{-\infty}^{t}dt'e^{-ick(t-t')}\sin[A(t')]\cos[A(t')]. \tag{27}$$

The final analytic expression for the spectrum corresponding to the initial condition (24) is derived as

$$C_{\sigma_\mp}(\vec{k})=\frac{|\Omega|^2N_c^2\pi^2e^{-2k_L^2a^2}}{8\gamma_L^2}\frac{8x^2\cosh^2[k_L^2a^2\cos\theta]}{\cosh^2(\pi x/2)}. \tag{28}$$

DISCUSSION OF THE SPECTRUM

We are now in a position to discuss the angular and spectral dependence of the scattering spectrum as given by Eqs. (23) and (28). The spectra of scattered photons with polarization σ_+ and σ_- show similar angular and spectral dependence. The frequency spectrum for photons emitted at an angle $\cos\theta=1/k_L^2a^2$ calculated from Eqs. (23) and (28) for $k_La\approx1$ is shown in Fig. 3, curves C1 and C2. Clearly, we find two completely different population distributions. In general, the total spectrum of scattered photons consists of coherent and incoherent parts. The coherent part is, as in the single-atom case, proportional to the modulus squared of the Fourier transform of the mean atomic polarization. The incoherent part originates from the quantum fluctuations of the atomic polarization.

A nonzero value of the scattered intensity in the frequency spectrum at resonance is a signature of quantum fluctuations. Clearly, when the two ground states are equally populated, the quantum fluctuations are absent as shown in Fig. 3, curve C2, which could be due to destructive quantum interference between the two channels $|g_+\rangle\rightarrow|e\rangle$ and $|g_-\rangle\rightarrow|e\rangle$. On the

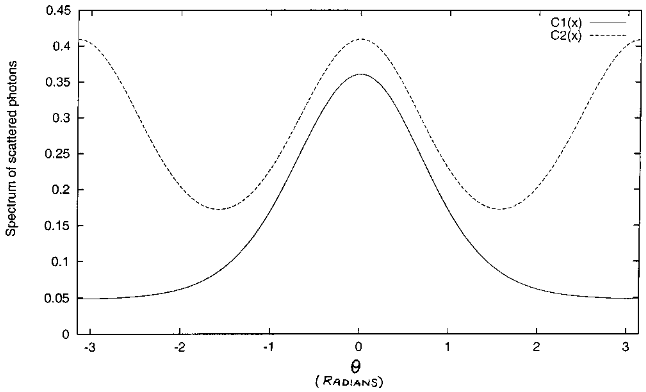

FIG. 4. The angular dependence of the scattered spectrum for $(\omega-\omega_L)/\gamma_L=1$ and $k_La\approx1$. The case when the $|g_+\rangle$ state is completely empty is shown as curve C1. The case when the two ground states are equally populated is shown as curve C2.

other hand, unequal population distribution between the two ground states gives rise to quantum fluctuations, which is evident from Fig. 3, curve C1. The complete absence of the central dip at resonance [i.e., at $(\omega-\omega_L)/\gamma_L=0$] indicates the fact that the number of incoherently scattered photons is maximum for this particular case. Note that at resonance only incoherent scattering dominates. With increasing value of $(\omega-\omega_L)/\gamma_L$, the coherent scattering begins to contribute to the total spectrum. Both the coherent and incoherent parts vanish for large values of $(\omega-\omega_L)/\gamma_L$. Hence the central dip may be used as a means to probe the relative distribution of the condensates in the two hyperfine levels. The angular dependence of the scattered spectrum for $(\omega-\omega_L)/\gamma_L=1$ and $k_La\approx1$ is presented in Fig. 4. Significant difference in the angular dependence is noticed for the two cases under consideration. For the case when the $|g_+\rangle$ state is completely empty, scattering takes place predominantly in the forward direction (curve C1). When the two ground states are equally populated, the angular distribution changes dramatically. The spectrum now indicates scattering not only in the forward direction but also in the backward direction, with a minimum at $\theta=\pi/2$ (curve C2). This case has an advantage in the sense that the difficulty associated with the detection of photons scattered in the forward direction (as they cannot be distinguished from the laser photons) is now removed. To measure the spectrum corresponding to a given polarization, we would need to select from the scattered light the required polarization. If the detector is insensitive to polarization of scattered light, the spectrum has contributions from both the polarizations. By moving the detector around $\theta=\pi/2$, we can distinguish between the two spectra corresponding to Eqs. (23) and (28), since only that due to Eq. (28) will show a dip around $\theta=\pi/2$.

In 1996, Andrews *et al.* reported the direct, nondestructive observation of a Bose condensate [14]. Dispersive light scattering was used to observe the separation between the condensed and normal components of sodium atoms in the $F=1$, $m_F=-1$ hyperfine state at a temperature of 1.5 μK. They found the incident photons to be elastically scattered in the forward direction by an angle $\lambda/4R$ (R is the radius of the Bose gas), which was typically 0.02 rad. An increase in the density of the Bose gas was accompanied by an increase of the scattering angle. Andrews *et al.* established that short light pulses could provide good temporal resolution.

More recently, Inouye *et al.* studied Rayleigh scattering off a BE condensate [21]. An elongated condensate was illuminated with a single off-resonant laser beam. Collective scattering led to photon scattered predominantly along the axial direction. The angular pattern of the scattered light along the axial direction changed with the pulse width. Decreasing the pulse duration increased the scattering angle. In actual experiments the angular distribution of the scattered light would become anisotropic because of the elongated shape of the Bose cloud. Unequal confining potentials for the two hyperfine states would result in a relative phase between the two condensates.

CONCLUSIONS

In this work we have investigated theoretically the possibility of using the scattered spectrum of intense short laser pulses off two coupled BEC's as an imaging technique to probe the population distribution between the two condensates. A nonzero value of the scattered intensity in the frequency spectrum at resonance is a signature of quantum fluctuations, which we find arises only when population is unequally distributed between the two ground states. This peak at resonance slowly transforms to a dip as the population difference between the two ground states decreases. When the population is equally distributed, the quantum fluctuation completely disappears which is traced back to destructive quantum interference. Equal population distribution is also reflected in the angular dependence of the scattered photons, where we find that scattering takes place both in the forward as well as the backward directions. Initially, if one of the ground states is completely empty, photons are scattered only in the forward direction. By placing the detector at the appropriate point, we find that it is possible to know the population distribution at that time by simply looking at the angular dependence of the spectrum.

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