Multipartite entanglement for entanglement teleportation

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A scheme for entanglement teleportation is proposed to incorporate multipartite entanglement of four qubits as a quantum channel. Based on the invariance of entanglement teleportation under an arbitrary two-qubit unitary transformation, we derive relations for the separabilities of joint measurements at a sending station and of unitary operations at a receiving station. From the relations of the separabilities it is found that an inseparable quantum channel always leads to total teleportation of entanglement with an inseparable joint measurement and/or a nonlocal unitary operation.

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I. INTRODUCTION

Quantum teleportation is one of the most striking features emerging from the quantum entanglement that is inherent in quantum mechanics [1]. Entangled systems divided into two parts enable transfer of the quantum information of an unknown quantum state to a remote place while the original state is destroyed. No information about the unknown state is ever revealed during the teleportation process. Quantum teleportation has been especially considered in single-body systems of two-level, *N*-dimensional, and continuous variable states [1–3].

Entanglement teleportation transfers the entanglement initially imposed on an unknown multipartite state to a multipartite state at a remote place [4]. The entanglement is transferred onto a composite system of subsystems which have never directly interacted. In this sense, entanglement teleportation is similar to entanglement swapping [5]. However, entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure (the entangled state itself). Entanglement teleportation of two qubits has recently been studied for pure and noisy quantum channels [4,6]. It is closely related to quantum computation as two-qubit teleportation together with one-qubit unitary operations are sufficient to implement the universal gates required for quantum computation [7].

In earlier protocols for two-qubit teleportation, separate Einstein-Podolsky-Rosen (EPR) pairs are utilized for the quantum channel so that the joint measurement is decomposable into two independent Bell-state measurements and the unitary operation into two local one-qubit operations. This implies that entanglement teleportation can be implemented by a series of single-qubit teleportations [1,4] which we call "series teleportation of entanglement." It is desirable to ask the following questions: Is a quantum channel restricted only to EPR entanglement, and, if not, what other types of entanglement are possible, and what role do they play in entanglement teleportation? These questions have been addressed in part by employing Greenberger-Horne-Zeilinger (GHZ) entanglement [8] of three and four qubits as a quantum channel [9,10]. However, the investigations have been restricted thus far to partially unknown entangled states such as $a|01\rangle+b|10\rangle$ and do not cover all possible states of a two-qubit system.

In this paper we consider entanglement teleportation of completely unknown entangled states such as

$$a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle \tag{1}$$

where *a*, *b*, *c*, and *d* are complex numbers and $\{|ij\rangle\}$ is an orthonormal basis set. The present scheme is formulated so as to employ multipartite entanglement of four qubits as a quantum channel; the composite system of four qubits may have various types of entanglement, for example, two EPR pairs, four GHZ triads, etc. We show that entanglement teleportation has invariance under arbitrary two-qubit unitary transformation and variant protocols are available. Using the invariance of entanglement teleportation, we derive relations of separabilities for joint measurements at a sending station and for unitary operations at a receiving station. Due to the relations of the separabilities, we show that an inseparable quantum channel always leads to a "total teleportation of entanglement," which employs an inseparable joint measurement and/or a nonlocal unitary operation, as opposed to a series teleportation of entanglement.

II. TWO-QUBIT TELEPORTATION

In the original proposal [1], quantum teleportation utilizes an EPR pair as a quantum channel which is shared by a sender Alice and a receiver Bob. After she receives a particle in an unknown state and one of the entangled pair, Alice performs a joint measurement on their composite state. She transmits the outcome to Bob through a classical channel. Bob applies a corresponding unitary operation on his particle of the entangled pair, which is chosen in accordance with the

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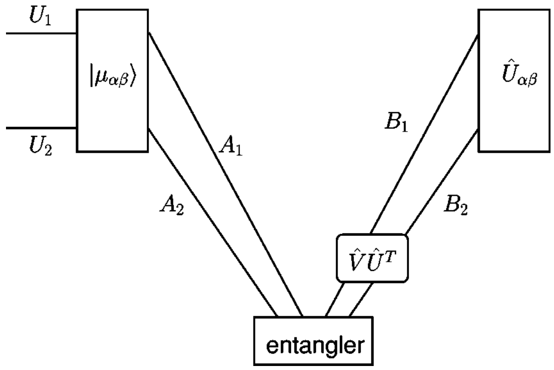

FIG. 1. A schematic drawing of a total teleportation of the two-qubit unknown state (2) via an inseparable quantum channel of four qubits A_1, A_2, B_1, and B_2. The inseparable quantum channel is in the state given by Eq. (4) as the two EPR pairs generated in the entangler are transformed by the nonlocal unitary operator $\hat{V}\hat{U}^T$ in the two-qubit gate. Suppose Alice obtains an outcome (α,β) in her joint measurement represented by the basis set $\{|\mu_{\alpha\beta}\rangle\}$. After receiving the four-bit classical message (α,β), Bob applies the corresponding unitary operator $\hat{U}_{\alpha\beta}$.

outcome of the joint measurement. The final state of Bob's particle is completely equivalent to the original unknown state if the quantum channel is maximally entangled.

A completely unknown state of two qubits U_1 and U_2 to teleport can be represented by

$$|\phi_u\rangle_U=\sum_{i,j=0}^{1} c_{ij}|i,j\rangle_U, \tag{2}$$

where the subscript U denotes the composite system of two qubits U_1 and U_2, c_{ij} is a complex number, and $\{|i,j\rangle_U\}$ is an orthonormal basis set; $|i,j\rangle_U=|i\rangle_{U_1}\otimes|j\rangle_{U_2}$ with the basis set $\{|0\rangle_q,|1\rangle_q\}$ of qubit q. Note that the unknown state in Eq. (2) is entangled unless the coefficient matrix c_{ij} is decomposable such that $c_{ij}=d_i e_j$ for some complex vectors d_i and e_j.

We consider a quantum channel of four qubits which are divided into two parts, i.e., two qubits are sent to Alice and the others to Bob as shown in Fig. 1. Alice's two qubits A_1 and A_2 are denoted by A and Bob's two qubits B_1 and B_2 by B. A perfect teleportation requires that two parts of A and B be in a maximally entangled pure state, that is, the state $|\phi_c\rangle_{AB}$ of the quantum channel satisfies the relation

$$\mathrm{Tr}_{B(A)}(|\phi_c\rangle_{AB}\langle\phi_c|)=\frac{1}{4}\mathbb{1}_{A(B)}, \tag{3}$$

where Tr_i is a partial trace over subsystem i and $\mathbb{1}_i$ is an identity operator of part i. The channel state $|\phi_c\rangle$ can be written by Schmidt decomposition as

$$|\phi_c\rangle_{AB}=\frac{1}{2}\sum_{i,j=0}^{1}|\psi_{ij}\rangle_A\otimes|\varphi_{ij}\rangle_B, \tag{4}$$

where $|\psi_{ij}\rangle_A=\hat{U}|i,j\rangle_A$, $|\varphi_{ij}\rangle_B=\hat{V}|i,j\rangle_B$, and $\hat{U}$ and $\hat{V}$ are two-qubit unitary operators. Note that $|\psi_{ij}\rangle$ is an entangled state of Alice's two qubits if the unitary operator $\hat{U}$ is nonlocal. Similarly, $|\varphi_{ij}\rangle$ is an entangled state of Bob's two qubits if $\hat{V}$ is nonlocal.

The channel state $|\phi_c\rangle_{AB}$ in Eq. (4) can be represented in the more convenient form of

$$|\phi_c\rangle_{AB}=(\mathbb{1}_A\otimes\hat{V}\hat{U}^T)|\overline{\phi_c}\rangle_{AB}, \tag{5}$$

where

$$|\overline{\phi_c}\rangle_{AB}=\frac{1}{2}\sum_{i,j=0}^{1}|i,j\rangle_A\otimes|i,j\rangle_B. \tag{6}$$

The state $|\overline{\phi_c}\rangle$ is also a maximally entangled state of the two parts A and B. On the other hand, it is separable into (A_1,B_1) and (A_2,B_2) such that $|\overline{\phi_c}\rangle_{AB}=|\mathrm{EPR}\rangle_{A_1B_1}\otimes|\mathrm{EPR}\rangle_{A_2B_2}$ where $|\mathrm{EPR}\rangle=\Sigma_i|i,i\rangle/\sqrt{2}$. The state $|\overline{\phi_c}\rangle$ has been used as a quantum channel for entanglement teleportation [4,6].

A pure generalized GHZ state of four qubits is defined by using the generalized Schmidt decomposition as [11]

$$|\phi_4\rangle_{AB}=\sum_{i=0}^{1}\lambda_i|\alpha_i\rangle_{A_1}\otimes|\beta_i\rangle_{A_2}\otimes|\gamma_i\rangle_{B_1}\otimes|\delta_i\rangle_{B_2}, \tag{7}$$

where $\{|\alpha_i\rangle\}$, $\{|\beta_i\rangle\}$, $\{|\gamma_i\rangle\}$, and $\{|\delta_i\rangle\}$ are orthonormal vector sets and the λ_i's are positive. A pure generalized GHZ state is not a good candidate for a quantum channel because it does not fulfill the requirement (3) of maximal entanglement of two parts A and B. More explicitly,

$$\mathrm{Tr}_B(|\phi_4\rangle_{AB}\langle\phi_4|)=\sum_{i=0}^{1}|\lambda_i|^2|\alpha_i,\beta_i\rangle_A\langle\alpha_i,\beta_i|, \tag{8}$$

which is not proportional to $\mathbb{1}_A$. In fact it is proportional to a projector that projects a state into a subspace spanned by $\{|\alpha_i,\beta_i\rangle_A\}$. We note, however, that a single-qubit teleportation can be performed via a quantum channel of three qubits which is in a maximal GHZ state [12], and thus there could be the possibility that a GHZ state of more than twice the teleporting qubits may lead to perfect teleportation.

Alice performs a joint measurement on the four qubits A_1, A_2, U_1, and U_2. The joint measurement is constructed using a set of 16 projectors $\{\hat{M}_{\alpha\beta}=|\mu_{\alpha\beta}\rangle_{AU}\langle\mu_{\alpha\beta}|\}$, where

$$|\mu_{\alpha\beta}\rangle_{AU}=(\mathbb{1}_A\otimes\hat{U}_{\alpha\beta})|\phi_c\rangle_{AU}. \tag{9}$$

Here $|\phi_c\rangle_{AU}$ is the same as the state given in Eq. (5) and $\hat{U}_{\alpha\beta}=\hat{\sigma}_\alpha\otimes\hat{\sigma}_\beta$ is a local unitary operator with Pauli spin operators $\hat{\sigma}_\alpha=\mathbb{1}$, $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$. The set $\{\hat{M}_{\alpha\beta}\}$ satisfies a completeness relation as

$$\sum_{\alpha,\beta=1}^{4}\hat{M}_{\alpha\beta}=\mathbb{1}_A\otimes\mathbb{1}_U. \tag{10}$$

Further, the 16 projectors are orthogonal such that

$$\hat{M}_{\alpha\beta}\hat{M}_{\gamma\delta}=\mathrm{Tr}(\hat{U}^{\dagger}_{\alpha\beta}\hat{U}_{\gamma\delta})|\mu_{\gamma\delta}\rangle_{AU}\langle\mu_{\alpha\beta}|=\delta_{\alpha\gamma}\delta_{\beta\delta}\hat{M}_{\alpha\beta}. \tag{11}$$

This implies that the joint measurement represented by $\{\hat{M}_{\alpha\beta}\}$ is an orthogonal measurement on the composite system of A and U.

A key step is to evaluate a partial inner product ${}_{AU}\langle\mu_{\alpha\beta}|\phi_c\rangle_{AB}$ by applying an identity operator $\mathbb{1}_U=\Sigma_{i,j}|i,j\rangle_U\langle i,j|$ on the right side:

$$ {}_{AU}\langle\mu_{\alpha\beta}|\phi_c\rangle_{AB}=\frac{1}{4}\hat{U}^{\dagger}_{\alpha\beta}\hat{\mathcal{T}}_{BU}, \tag{12}$$

where $\hat{\mathcal{T}}_{BU}=\Sigma_{i,j}|i,j\rangle_{B\,U}\langle i,j|$ is a transfer operator from a state of U to that of B such that $\hat{\mathcal{T}}_{BU}|\phi\rangle_U=|\phi\rangle_B$. The form of $\hat{U}^{\dagger}\hat{\mathcal{T}}$ plays a crucial role in revealing an invariance of entanglement teleportation which will be discussed in the next section.

The state $|\Psi\rangle_{UAB}$ of the whole composite system of U, A, and B can be represented with respect to the basis set $\{|\mu_{\alpha\beta}\rangle_{AU}\}$ of the joint measurement as follows:

$$\begin{aligned}|\Psi\rangle_{UAB}&=|\phi_u\rangle_U\otimes|\phi_c\rangle_{AB}\\&=\left(\sum_{\alpha,\beta=1}^{4}\hat{M}_{\alpha\beta}\right)|\phi_c\rangle_{AB}\otimes|\phi_u\rangle_U\\&=\frac{1}{4}\sum_{\alpha,\beta=1}^{4}|\mu_{\alpha\beta}\rangle_{AU}\otimes\hat{U}^{\dagger}_{\alpha\beta}\hat{\mathcal{T}}_{BU}|\phi_u\rangle_U.\end{aligned} \tag{13}$$

Suppose Alice obtains an outcome (α,β) when she performs the joint measurement on the composite system of A and U. Bob's two qubits come to be in the state of $\hat{U}^{\dagger}_{\alpha\beta}|\phi_u\rangle_B$. When he receives through a classical communication the four-bit message concerning the outcome (α,β), Bob applies the corresponding unitary operation $\hat{U}_{\alpha\beta}$ on his qubits, which completes the two-qubit teleportation process.

III. RELATIONS OF SEPARABILITIES FOR JOINT MEASUREMENTS AND FOR UNITARY OPERATIONS

In the proposed protocol of two-qubit teleportation, we employed an orthogonal measurement for the joint measurement. We may consider a positive operator valued measurement for a joint measurement, such that for a set of unitary operators $\{\hat{U}_g\}$ with order G,

$$\frac{1}{G}\sum_g \mathbb{1}\otimes\hat{U}_g|\phi\rangle_{AU}\langle\phi|\mathbb{1}\otimes\hat{U}^{\dagger}_g=\frac{1}{4^2}\mathbb{1}_A\otimes\mathbb{1}_U, \tag{14}$$

where $|\phi\rangle_{AU}$ is a maximally entangled state of A and U. This type of positive operator valued measurement was studied for universal teleportation [13]. If $\hat{U}_g=\hat{\sigma}_\alpha\otimes\hat{\sigma}_\beta$, this measurement is simply equal to the orthogonal joint measurement represented by the bases in Eq. (9).

We shall show the invariance of entanglement teleportation under an arbitrary two-qubit unitary transformation. For a maximally entangled state $|\phi\rangle$ of two parts, let $\{|\mu_g\rangle_{AU}=\mathbb{1}_A\otimes\hat{U}_g|\phi\rangle_{AU}\}$ be a set of joint measurement bases and $\{|\phi_{g'}\rangle_{AB}=\mathbb{1}_A\otimes\hat{U}_{g'}|\phi\rangle_{AB}\}$ be a set of unitarily transformed channel states. The partial inner product of $|\mu_g\rangle_{AU}$ and $|\phi_{g'}\rangle_{AB}$ is obtained as

$$ {}_{AU}\langle\mu_g|\phi_{g'}\rangle_{AB}=\frac{1}{4}\hat{U}_{g'}\hat{U}^{\dagger}_g\mathcal{T}_{BU}. \tag{15}$$

When $g=g'$, this is just a transfer operator. The teleportation is completely specified by G pairs of joint measurement bases and their corresponding channel states $\{|\mu_g\rangle,|\phi_g\rangle\}$. The partial inner product in Eq. (15) is invariant under the transformation of

$$|\mu_g\rangle_{AU}\rightarrow\hat{W}^T_r\otimes\hat{W}_l|\mu_g\rangle_{AU} \tag{16}$$

and

$$|\phi_g\rangle_{AB}\rightarrow\hat{W}^T_r\otimes\hat{W}_l|\phi_g\rangle_{AB} \tag{17}$$

for each g with some two-qubit unitary operators $\hat{W}_l$ and $\hat{W}_r$. Thus one may have variant protocols of entanglement teleportation under the transformation in Eqs. (16) and (17), due to the arbitrariness of $\hat{W}_l$ and $\hat{W}_r$. We note here that the invariance of entanglement teleportation may be extended further with respect to a rather general completely positive operation [14].

The invariance of entanglement teleportation raises the relations of separabilities for joint measurements and for unitary operations. In particular, an inseparable joint measurement may be transformed into two independent Bell-state measurements and/or a nonlocal unitary operation into a local operation. A joint measurement is said to be separable when each measurement basis can be decomposed into a product state of either (A_1,U_1) and (A_2,U_2) or (A_1,U_2) and (A_2,U_1). Further, a protocol of entanglement teleportation is called a series teleportation of entanglement when its joint measurement is separable *and* the corresponding unitary operation is local. The series teleportation of entanglement consists of independent Bell-state measurements and local unitary operations [4,6]. Otherwise, it is called a total teleportation of entanglement in the sense that it is not decomposable into a series of single-qubit teleportation [4].

In Sec. II we presented a protocol of total teleportation of entanglement with an inseparable joint measurement and a local unitary operation when the quantum channel state in Eq. (5) is inseparable. One may try to construct a series teleportation of entanglement by using the invariance of entanglement teleportation under the transformation of Eqs. (16) and (17). Suppose that a joint measurement becomes separable in (A_1,U_1) and (A_2,U_2) for some $\hat{W}_l$ and $\hat{W}_r$ such that

$$|\mu_{\alpha\beta}\rangle_{AU}\rightarrow|\tilde{\mu}_{\alpha\beta}\rangle_{AU}=\mathbb{1}_A\otimes\overline{U}_{\alpha\beta}|\overline{\phi_c}\rangle_{AU}, \tag{18}$$

where $\overline{U}_{\alpha\beta}=\hat{W}_l\hat{U}_{\alpha\beta}\hat{V}\hat{U}^T\hat{W}_r=\sigma_\alpha\otimes\sigma_\beta$. Then, the corresponding unitary operators are transformed as

$$\hat{U}_{\alpha\beta} \rightarrow \overline{U}_{\alpha\beta}(\hat{V}\hat{U}^T)^\dagger. \quad (19)$$

The transformed unitary operators are clearly nonlocal since $\hat{V}\hat{U}^T$ is nonlocal due to the inseparability of $|\phi_c\rangle$. This protocol consists of the separable joint measurement and a nonlocal unitary operation, which is the opposite case to the untransformed protocol of the inseparable joint measurement and local unitary operation. However, the altered protocol is a total teleportation as well. An inseparable quantum channel always leads to a total teleportation of entanglement.

It is possible to obtain two EPR pairs by applying some two-qubit unitary operation to an inseparable quantum channel, which enable a series teleportation of entanglement with a rather simple Bell-state measurement [15] and local unitary operation. Unless a quantum channel is likely to suffer from a reservoir, it may be the simplest protocol that employs EPR pairs as a quantum channel. However, when a reservoir is present, it is important to study inseparable quantum channels because some inseparable channel can be more robust against decoherence than EPR pairs. It is known that some particular state is robust against decoherence once the interaction with a reservoir is known. For example, a decoherence-free state, an eigenstate with zero eigenvalue of the interaction Hamiltonian, never becomes decoherent in the given reservoir. We will not further discuss the effects of decoherence, which are beyond the scope of this paper.

IV. MANY-QUBIT ENTANGLEMENT OF AN INSEPARABLE QUANTUM CHANNEL

Any quantum channel in a maximally entangled state of two parts A and B can be employed for a perfect teleportation of entanglement. Entanglement of four qubits may be classified into two-qubit entanglement, three-qubit entanglement, and four-qubit entanglement. A state of four qubits is said to have two-qubit entanglement when some two qubits among the four qubits are in an entangled state, three-qubit entanglement when some three qubits are in a three-qubit GHZ state, and four-qubit entanglement when the four qubits are in a four-qubit GHZ state. Note that W-class states and biseparable states [16] belong to two-qubit entanglement by our definition. As shown in Sec. II a four-qubit GHZ state is not a good candidate for a perfect teleportation of entanglement.

The entanglement structure of a possible quantum channel state (4) depends on two-qubit unitary operator $\hat{V}\hat{U}^T$. A quantum channel of two EPR pairs is in the state $|\overline{\phi_c}\rangle_{AB}$, which is separable into (A_1,B_1) and (A_2,B_2), and it has only two-qubit entanglement. We shall present an example of an inseparable quantum channel that has many-qubit entanglement; the channel state is written as

$$|\phi_c\rangle_{AB} = \frac{1}{2\sqrt{2}}(|0000\rangle - |0011\rangle + |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle)_{A_1A_2B_1B_2}. \quad (20)$$

This state is obtained from Eq. (6) with the two-qubit unitary operator

$$\hat{V}\hat{U}^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad (21)$$

which is represented in a product basis set of $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$. The operator $\hat{V}\hat{U}^T$ transforms the product bases to Bell bases.

The reduced density operator of each qubit i is proportional to an identity operator $\rho_i = \mathbb{1}_i/2$. Noting that $|\phi_c\rangle$ is pure, this implies that $|\phi_c\rangle$ has no individual information but contains entanglement of a given qubit and the rest.

To investigate two-qubit entanglement, we employ the Peres-Horodecki criterion [4,17] for two qubits that their density operator ρ is entangled if and only if its partial transposition has any negative eigenvalue. The partial transposition of ρ is defined as

$$\rho^{T_1} = \sum_{ijkl} \rho_{jikl}|i\rangle\langle j| \otimes |k\rangle\langle l| \quad (22)$$

when $\rho = \Sigma_{ijkl}\rho_{ijkl}|i\rangle\langle j| \otimes |k\rangle\langle l|$. As an example, consider a reduced density operator of a pair among four qubits which are in a symmetric W state,

$$|W\rangle = \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle). \quad (23)$$

The partial transposition of the reduced density operator has a negative eigenvalue $(1-\sqrt{2})/4$ for all pairs.

We shall show below that all pairs that can be selected out of four qubits in the state $|\phi_c\rangle$ are in separable states. Every pair (i,j) except (A_1,B_1) and (A_2,B_2) has the reduced density operator $\rho_{ij} = \frac{1}{4}\mathbb{1}_{ij}$ and it is disentangled. In addition, the reduced density operator of (A_1,B_1) or (A_2,B_2) is given as

$$\rho_{A_1B_1(A_2B_2)} = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

The partial transposition of $\rho_{A_1B_1(A_2B_2)}$ has only positive eigenvalues of $(0,0,1/2,1/2)$. These results imply that the state $|\phi_c\rangle$ in Eq. (20) has no two-qubit entanglement. However, the state $|\phi_c\rangle$ is entangled as shown in the consideration of the reduced density operators for single qubits, and it has three-qubit entanglement.

A reduced density operator ρ of each triad is obtained by tracing over the other qubit and is in the form of

$$\rho = \frac{1}{2}|\phi_0\rangle\langle\phi_0| + \frac{1}{2}|\phi_1\rangle\langle\phi_1|, \quad (25)$$

TABLE I. Amplitudes λ_i of two orthogonal GHZ states $|\phi_0\rangle$ and $|\phi_1\rangle$ in Eq. (25).

Triad	λ_1	λ_2	λ_3	λ_4
(A_1,A_2,B_1)	-1	1	-1	1
(A_1,A_2,B_2)	1	1	-1	-1
(A_1,B_1,B_2)	-1	1	1	-1
(A_2,B_1,B_2)	-1	-1	1	1

where $|\phi_0\rangle=|000\rangle+\lambda_1|011\rangle+|101\rangle+\lambda_2|110\rangle$ and $|\phi_1\rangle=\lambda_3|001\rangle+\lambda_4|010\rangle+|100\rangle+|111\rangle$ with λ_i given in Table I. By generalized Schmidt decomposition [11] it is found that both $|\phi_0\rangle$ and $|\phi_1\rangle$ are maximal three-qubit GHZ states.

To investigate three-qubit entanglement explicitly, one may employ an entanglement witness scheme that a density operator of three qubits ρ has three-qubit entanglement if $\mathrm{Tr}(\mathcal{W}\rho)<0$ for some three-qubit GHZ entanglement witness $\mathcal{W}$ [18]. However, it is nontrivial to find such an entanglement witness for a given density operator while a typical entanglement witness is known as [18]

$$\mathcal{W}=\frac{3}{4}\mathbb{1}-|\phi\rangle\langle\phi|, \tag{26}$$

where $|\phi\rangle=(1/\sqrt{2})\Sigma_{i=0}^{1}|\alpha_i,\beta_i,\gamma_i\rangle$ is a maximal three-qubit GHZ state. We perform numerical calculations with the steepest decent method to search for some local trilateral rotation for the typical witness (26) to minimize $\mathrm{Tr}(\mathcal{W}\rho)$, and we find $\mathrm{Tr}(\mathcal{W}\rho)\geqslant 1/4$. This implies that the typical entanglement witness cannot detect three-qubit entanglement of triads.

V. REMARKS

We proposed a scheme for entanglement teleportation of a completely unknown state so as to incorporate a multipartite entangled state as the quantum channel. By deriving the relations of separabilities for joint measurements and for corresponding unitary operations, it was found that an inseparable quantum channel always leads to a total teleportation of entanglement. We gave an example of an inseparable quantum channel with each triad in three-qubit entanglement.

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