Decoherence slowing down in a symmetry-broken environment

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We show how to characterize the temporal evolution of a single qubit interacting with a thermal bath by correlation functions of the bath under quite general hypotheses. The aim is to show how the qubit decoherence time depends on the degree of correlation of the bath. We explicitly study some spin-bath models where every spin of the environment interacts with the qubit either directly or by the mediation of a harmonic oscillator. In both cases we find an increase of the decoherence time as soon as a symmetry breaking occurs in the environment. In the latter case a possible experimental setup is discussed.

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I. INTRODUCTION

Decoherence is one of the problems we have to face in the hypothetical process of quantum computation [1,2]. Here the minimal information unit is called the quantum bit (qubit). The state of a qubit is a generic coherent superposition of the only two possible logical values that the classical bit can assume. Computation can be realized by a sequence of unitary transformations that affects simultaneously each element of the superposition, the result being parallel data processing (quantum parallelism). Taking a register of N qubits the calculus speed increases exponentially, as we can perform 2^N operations simultaneously instead of one (dense coding) [3].

When a qubit interacts with an environment, the resulting entanglement destroys the coherence of the initial state, so the gain of information we wished to obtain, in comparison to that a classical bit can store, quickly disappears. This mechanism is called *decoherence* and it brings about the transition from a pure ensemble of quantum bits to a mixture of classical ones [4–8]; the states of this mixture are selected by the form of the interaction and are usually called *pointer states*. We underline that the time scale of decoherence is usually much smaller than that of the expected loss of energy of the qubit due to its relaxation [9].

Two different strategies has been adopted to fight decoherence. The first, in analogy with the classical computation, is to resort to redundancy in encoding information, by means of a so-called *error-correcting code* [10–13]. The second, introduced by Zanardi and Rasetti [14], considers qubits that are symmetrically coupled with the same environment to design states that are hardly corrupted by the decoherence. This approach, however, reduces drastically the number of quantum states that can be used for the quantum computation. We suggest a strategy that consists in making system-environment entanglement difficult because the environment ensemble is close to a macroscopic state in the direction of the pointer basis.

A generic qubit is represented by a state vector in an abstract two-dimensional Hilbert space, and, without loss of of generality, it can be formally associated with a $\frac{1}{2}$ spin state in the form

$$|S\rangle=\cos\frac{\theta}{2}|\uparrow\rangle+e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle. \tag{1}$$

An ensemble of such identical pure states can be statistically described by the density operator

$$\rho_S=|S\rangle\langle S|=\frac{1}{2}+\mathbf{n}\cdot\mathbf{S}, \tag{2}$$

where $\mathbf{n}=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$ is a vector that lies on the surface of the Bloch sphere. In the basis of the S^z eigenvalues the density matrix is

$$\rho_S=\frac{I}{2}+\cos\theta S^z+\frac{1}{2}\sin\theta(e^{-i\phi}S^+ +\text{H. c.}). \tag{3}$$

The diagonal terms of Eq. (3) represent the probabilities of the two possible values "up" and "down" and are usually referred to as the "occupation" or "population." The off-diagonal terms are associated, with their absolute value, with the amplitude of the interference patterns and quantify the coherence of the pure state. As a consequence of the Schrödinger equation, the temporal evolution of the isolated qubit is unitary and so an initial pure state evolves into another one, maintaining its coherence. Likewise, the evolution of the density matrix allows the vector $\mathbf{n}$ to move only on the surface of the sphere.

Introducing an external environment, we have to enlarge the Hilbert space and consider the evolution of the overall system including the bath. If the density matrix of the composed system is initially $\rho(0)=\rho_S\otimes\rho_B$, it will not generally evolve into a product because of the entanglement between qubit and environment.

In this paper we consider only a thermal bath even though this is not the most general situation [15].

The single-qubit dynamics, relative to the entangled total system, can be characterized by the reduced density operator obtained by a partial trace of $\rho(t)$ on the states of the bath:

$$\rho_S(t)=\mathrm{Tr}_B\{\rho(t)\}=\mathrm{Tr}_B\{e^{-i\mathcal{H}t}\rho(0)e^{i\mathrm{H}t}\}. \tag{4}$$

Here the Hamiltonian $\mathcal{H}$ is given by the unperturbed contribution of qubit and bath and by their mutual interaction:

$$\mathcal{H}=\mathcal{H}_B+\mathcal{H}_S+\mathcal{H}_I. \tag{5}$$

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The reduced matrix evolution is no longer unitary and it can determine a suppression of the off-diagonal elements (typically much faster than the evolution of the diagonal ones). Likewise, the vector $\mathbf{n}$ leaves the surface of the Bloch sphere and evolves into its internal vectors, so the initial pure state of the qubit collapses into a statistical mixture of classical bits, losing its coherence.

The decoherence time depends both on the thermal properties of the bath and on the quantum fluctuations [16]. The latter are a characteristic of the delocalization of the environment particles and they appear, for example, in the commonly studied boson baths of harmonic oscillators [17–21].

Usually a master equation formalism, together with a Markovian approximation [22–24], is used to obtain a first-order differential equation for the reduced density operator. The approximation amounts to considering the evolution "local" in time, that is, $\rho_S(t+dt)$ is assumed completely determined by $\rho_S(t)$, so the evolution become intrinsically irreversible. In this work we do not use this approximation and, even though a perturbative expansion is adopted, the evolution obtained remains formally reversible. In our approach decoherence is the relaxation occurring in the temporal range between the characteristic time of interaction and the Poincaré recurrence time. This temporal range increases as the environment gains more degrees of freedom.

In the next section we obtain a general equation for the evolution (4) involving only the temperature-dependent correlation functions of the bath. This result can be applied to a wide variety of situations and it provides a useful tool to show how the qubit dynamics, and in particular its decoherence time, is influenced by a progressive ordering of the environment.

In the second and third sections we use this result to analyze two particular models where phase transitions can occur in a spin-bath environment. Treating each spin as a localized particle, we show that only thermal effects produce the decoherence, which can be reduced increasing the order parameter at low temperatures.

Spontaneous symmetry breaking in a system modifies its correlation properties. This effect has been widely studied, in the context of quantum optics [24] or of atomic Bose-Einstein condensation [25], by means of n-order coherence functions. Here we do not investigate a bosonic environment but a spinlike one. Furthermore, by making use of the environment properties, we want to study the temporal evolution of a two-level system inside the environment.

II. EVOLUTION OF THE REDUCED DENSITY MATRIX

Equation (4) can be directly associated with the evolution of the qubit operators which define the reduced density matrix. As we can see in Eq. (2), the number of operators we have to take is three and they can be chosen from every independent linear combination of $\{S^z(t), S^+(t), S^-(t)\}$. These operators can be considered as elements of a vector $\mathbf{v}(t)$ evolving with a system of differential Heisenberg equations:

$$\dot{\mathbf{v}}(t) = i e^{-i\mathcal{H}t}[\mathbf{v}(t),\mathcal{H}]e^{i\mathcal{H}t}, \tag{6}$$

where the Hamiltonian is given by the general form (5).

Here we compute Eq. (6) under the following hypotheses on the model.

(1) The Hamiltonian of the bath is separable into the Hamiltonans of each single element,

$$\mathcal{H}_B = \sum_k \mathcal{H}_B^k \tag{7}$$

and $[\mathcal{H}_B^k, \mathcal{H}_B^{k'}]=0$. This condition can be exactly satisfied (for example, in a set of noninteracting spins in an external magnetic field), or a mean field approximation can be introduced, as we will see later.

(2) $\mathcal{H}_I$ and $\mathcal{H}_S$ are linear with respect to the qubit operators so the temporal evolution has the form

$$\dot{\mathbf{v}}(t) = i\sum_k g_k \hat{\Lambda}_k(t)\mathbf{v}(t), \tag{8}$$

where $\hat{\Lambda}(t)_k$ is a 3×3 matrix whose elements can include time-dependent Hermitian operators that act only on the kth element of the environment.

(3) The temporal range considered is small in comparison to the characteristic time $\tau \sim 1/g_k$,

$$g_k t \ll 1. \tag{9}$$

If the coupling between qubit and environment is weak g_k is small too, and this range becomes larger. In particular, we will study the case in which $g_k \propto 1/\sqrt{N}$, where N is the number of environment elements, typically very large, so that in this case the hypothesis is not restrictive. Note that the previous assumption prevents us from observing any periodic recoherence phenomenon, intrinsic in the Schrödinger evolution, but it occurs after a time longer than the effectiveness of the model.

(4) The interaction with the qubit does not affect the evolution of the bath, or its contribution is negligible. If a bath operator O_B^k is given we have

$$[\mathcal{H}, O_B^k] \simeq [\mathcal{H}_B, O_B^k]. \tag{10}$$

As a consequence, for a statistical bath in thermal equilibrium described by the Boltzmann distribution $\rho_B(0) = Z^{-1}e^{-\beta\mathcal{H}_B}$, the mean value of O_B can be considered time independent. From a physical point of view, small excitations of the bath degrees of freedom, due to the interaction with the qubit, are expected to relax very quickly and therefore to be irrelevant for the reduced matrix evolution. In our approximation these excitations are completely neglected. Within this assumption the evolution is still reversible because it is associated with unitary operators.

These assumptions are not very restrictive and require only a macroscopic spinlike bath or a bosonic one linearly and weakly interacting with an external two-level system. Equation (6) can be formally integrated using the chronological T product:

$$\mathbf{v}(t)=T\left[e^{i\sum_k g_k\int_0^t dt_1\hat{\Lambda}_k(t_1)}\right]\mathbf{v}(0)$$

$$=\prod_k T\left[e^{ig_k\int_0^t dt_1\hat{\Lambda}_k(t_1)}\right]\mathbf{v}(0). \tag{11}$$

Here, for factorizing the T product, we used the approximation $\Sigma_k g_k\hat{\Lambda}_k(t)\simeq e^{-i\mathcal{H}_B t}\Sigma_k g_k\hat{\Lambda}_k e^{i\mathcal{H}_B t}$ arising from the condition (8), and the well known properties of the exponential noncommuting operators.

Defining the quantity

$$\Omega=\ln \mathrm{Tr}_B\left\{\prod_k T\left[e^{ig_k\int_0^t dt_1\hat{\Lambda}_k(t_1)}\right]\right\} \tag{12}$$

we have

$$\mathbf{v}(t)=e^{\Omega}\mathbf{v}(0), \tag{13}$$

so we can use the third condition (9) to expand first the exponential given by Eq. (11) and then the logarithm, up to second order in the powers of g_k. We obtain

$$\Omega\simeq\sum_k\left(ig_k m_k t-\frac{g_k^2}{2}t^2[C_k(t)-m_k^2]\right), \tag{14}$$

where

$$m_k=\frac{1}{Z}\mathrm{Tr}_B\{e^{-\beta\mathcal{H}_B^k}\hat{\Lambda}_k(t)\}, \tag{15}$$

$$C_k(t)=\frac{2}{t^2}\int_0^t\int_0^{t_1}dt_1dt_2\frac{1}{Z}\mathrm{Tr}_B\{e^{-\beta\mathcal{H}_B^k}\hat{\Lambda}_k(t_1)\hat{\Lambda}_k(t_2)\}$$

are the mean value and the first-order temporal correlation function of the matrix $\hat{\Lambda}_k(t)$.

This approximation does not affect the trace invariance of the density matrix because it transforms Eq. (2) into $\rho_S(t)=\frac{1}{2}+\mathbf{n}'(\Omega)\cdot\mathbf{S}$, whose trace is always 1.

Now the evolution of the reduced density operator depends only on the statistical properties of the bath and it is described by Eq. (4). In the last part of this work we use this result to analyze two particular models where phase transitions can occur in a spin-bath environment. By treating each spin as a localized particle [26] we show that only thermal effects produce the decoherence, which can be reduced by increasing the order parameter at low temperatures.

III. RANDOM INTERACTION WITH A SPIN BATH

As a first example we consider a simple generalization of one of the first models introduced by Zurek [4]. Here the bath is described by a simple long-range Ising model $\mathcal{H}_B=-(J/N)\Sigma_{i,j}S_i^zS_j^z$, and the coupling is scaled with N to allow the free energy to be extensive. We assume that the qubit interacts randomly with each spin of the environment and the coupling constant is taken proportional to the standard deviation $1/\sqrt{N}$ of a Poissonian distribution. Neglecting the self-Hamiltonian of the qubit, we have the following Hamiltonian:

$$\mathcal{H}=-\frac{J}{N}\sum_{i,j}S_i^zS_j^z-\frac{J_0}{\sqrt{N}}S^z\sum_k\chi_kS_k^z, \tag{16}$$

where χ_k can assume randomly the values 1 or -1.

To obtain the condition (7) we introduce a mean field approximation for the environment,

$$\left(\frac{1}{N}\sum_k S_k^z-m\right)^2\simeq 0, \tag{17}$$

where the order parameter m is the S_k^z mean value, to be determinated self-consistently. Expanding Eq. (16) and replacing it in $\mathcal{H}_B$ we obtain a new Hamiltonian $\mathcal{H}_B^{cm}=-2Jm\Sigma_kS_k^z+m^2\tilde{J}N$.

The order parameter can be evaluated by minimizing the free energy $f=-(1/N\beta)\ln\mathrm{Tr}\{e^{-\beta\mathcal{H}_B^{cm}}\}$ and its value is given by the Curie-Weiss equation

$$m=\frac{1}{2}\tanh(\beta Jm). \tag{18}$$

When the phase transition occurs, under the critical temperature $T_c=J/2K$, the magnetization is different from zero and every spin is under the influence of the self-consistent magnetic field.

Consider the vector

$$\mathbf{v}(t)=\begin{pmatrix}S^+(t)\\S^-(t)\\S^z(t)\end{pmatrix}. \tag{19}$$

Its evolution has the form (8)

$$\dot{\mathbf{v}}(t)=i\sum_k\frac{J_0}{\sqrt{N}}\chi_kS_k^z\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\mathbf{v}(t). \tag{20}$$

We see that S^z component, relative to the population, remains unaltered; instead, the spin-flip operators evolve as $S^{\pm}(t)=r(t)S^{\pm}$. Here $|r(t)|$ is the so-called correlation amplitude and it quantifies the coherence of the qubit. Using Eq. (14) and observing that $C_k(t)=\frac{1}{4}$, we obtain

$$|r(t)|^2\simeq e^{-J_0^2t^2(1/4-m^2)}. \tag{21}$$

In the temporal range given by condition (9), we see a Gaussian suppression of the interference terms. The decoherence time is given by $\tau_{dec}=1/J_0\sqrt{\frac{1}{4}-m^2}$. As the order parameter tends to saturate to its maximum value $m^2=\frac{1}{4}$, τ_{dec} becomes larger and larger. This is a very simple example of how a symmetry-breaking phenomenon in the environment reduces the effect of the decoherence even if no external field is applied. Now we improve the model by removing the very unrealistic assumption of long-range interaction between spins.

We underline that in this model the approximation (10) has not been used because the only bath operator involved in the reduced density evolution is S_k^z that exactly commutes with $\mathcal{H}_I$.

That model can also be solved exactly, the result for the correlation amplitude being

$$|r(t)|^2=\left(1-(1-m^2)\sin^2\frac{Jt}{2\sqrt{N}}\right)^N, \tag{22}$$

which is a periodic function of period $T=2\pi\sqrt{N}/J$ and shows explicitly the reversible nature of the evolution. Nevertheless, such a *recoherence* phenomenon requires a time proportional to $\sqrt{N}$ that is of no physical interest as soon as the environment has a macroscopic number of degrees of freedom. We see that in the limit (9) the two equations (22) and (21) coincide.

IV. INTERACTION BETWEEN SPINS AND A HARMONIC OSCILLATOR

Consider, as a model for the environment, a set of N two-level systems (TLSs) coupled, in the dipole approximation, with a unidimensional harmonic oscillator, with frequency ω. If the wavelength is larger than the spatial range where the TLSs are confined, the interaction does not depend on the position of the TLS inside the cavity. The level separation is μ and each TLS can be described by a spin formalism. One of these spins is selected to play the qubit rule, its level separation being μ_0. The Hamiltonian is

$$\mathcal{H}=\omega a^\dagger a-\mu\sum_k S_k^x-g(a+a^\dagger)\sum_k S_k^z$$
$$-\mu_0 S^x-g_0(a+a^\dagger)S^z. \tag{23}$$

The model describes, roughly speaking, an array of Rydberg atoms interacting with a mode of the radiation field. Here the coupling constant has the form $g\propto\mu/\sqrt{\omega V}$, where V is the coherence volume of the mode and, assuming a constant density for the atoms, is proportional to N.

The effect of the delocalized oscillator is to induce an effective long-range interaction between spins and, to make it clear, we operate a displacement transformation by the unitary operator $D=e^{(g/\omega)(a^\dagger-a)\Sigma_k S_k^z}$. If a phase transition occurs the oscillator becomes strongly polarized and we can assume that

$$D^\dagger(a+a^\dagger)D=a+a^\dagger+2\frac{g}{\omega}\sum_k S_k^z\simeq 2\frac{g}{\omega}\sum_k S_k^z. \tag{24}$$

The spin-flip operators change in

$$D^\dagger S_k^\pm D\simeq e^{-(g^2/2\omega^2)}S_k^\pm, \tag{25}$$

where we substituted $e^{\mp(g/\omega)(a^\dagger-a)}$ with its mean value in the oscillator vacuum state. The transformation allows us to separate the contribution of the oscillator from that of the spins; as a result we obtain a new Ising-like model for the spins:

$$\tilde{\mathcal{H}}=-\frac{J}{N}\left(\sum_k S_k^z\right)^2-w\sum_k S_k^x-\mu_0 S^x$$
$$-\frac{J_0}{\sqrt{N}}S^z\sum_k S_k^z, \tag{26}$$

where $J=Ng^2/\omega$, $J_0/\sqrt{N}=2g_0 g/\omega$, and $w=\mu e^{-g^2/2\omega^2}$.

The mean field approximation (17) can be introduced again, obtaining the Curie-Weiss equation $\Theta/J=\tanh(\beta\Theta/2)$ for the quantity $\Theta=\sqrt{w^2+4m^2J^2}$. Although the critical temperature $T_c=J/2K$ remains the same, the added transverse component S_k^x hampers the transition, so the additional condition $w/J<\tanh(\beta w/2)$ is required.

In the symmetry-broken phase, the resulting Hamiltonian

$$\mathcal{H}_{cm}=-2Jm\sum_k S_k^z-w\sum_k S_k^x-\mu_0 S^x$$
$$-\frac{J_0}{\sqrt{N}}S^z\sum_k S_k^z \tag{27}$$

satisfies all the conditions previously required for Eq. (13) to be valid.

It is convenient to choose, for the qubit, the vector

$$\mathbf{v}(t)=\begin{pmatrix}\tilde{S}^+(t)\\ \tilde{S}^-(t)\\ S^z(t)\end{pmatrix}, \tag{28}$$

where $\tilde{S}_\pm(t)=(1/\sqrt{2})S_\pm(t)$. Its evolution is given by

$$\dot{\mathbf{v}}(t)$$
$$=i\sum_k\begin{pmatrix}\frac{J_0}{\sqrt{N}}S_k^z(t) & 0 & -\frac{\mu_0}{N\sqrt{2}}\\ 0 & -\frac{J_0}{\sqrt{N}}S_k^z(t) & \frac{\mu_0}{N\sqrt{2}}\\ -\frac{\mu_0}{N\sqrt{2}} & \frac{\mu_0}{N\sqrt{2}} & 0\end{pmatrix}\mathbf{v}(t). \tag{29}$$

Here $S_k^z(t)$ can be computed by introducing a vector $\mathbf{v}_k(t)$, analogous to Eq. (28), that satisfies

$$\dot{\mathbf{u}}_k(t)=iM\mathbf{u}_k(t)$$
$$=i\begin{pmatrix} 2Jm & 0 & -\frac{w}{\sqrt{2}} \\ 0 & -2Jm & \frac{w}{\sqrt{2}} \\ -\frac{w}{\sqrt{2}} & \frac{w}{\sqrt{2}} & 0 \end{pmatrix}\mathbf{u}_k(t). \tag{30}$$

Integrating this equation, we obtain $\mathbf{u}_k(t)=e^{iMt}\mathbf{u}_k(0)$, whose exponential can be exactly calculated by the recursive property for the powers of M.

Consider the case where $\mu_0=0$, we have the following expression for the correlation amplitude:

$$|r(t)|^2\simeq\exp-\left\{J_0^2t^2\left[\frac{1}{4\Theta^2}\left(4J^2m^2+w^2\mathrm{sinc}^2\frac{\Theta t}{2}\right)-m^2\right]\right\} \tag{31}$$

which tends to the asymptotic function (see Fig. 1)

$$|r_{as}(t)|^2=e^{-J_0^2m^2t^2[J^2/\Theta^2-1]}. \tag{32}$$

We can see an oscillatory behavior with period $T_{osc}=2\pi/\Theta$, caused by the precession induced by w, damped by a Gaussian function with decoherence time $\tau_{dec}=\Theta/J_0m\sqrt{(J^2-\Theta^2)}$. Again, at low temperature, Θ saturates to J and the decoherence time increases.

If $\mu_0\neq 0$, integrating Eq. (29) becomes more laborious because the matrix Ω cannot simply be exponentiated by analytic methods. Nevertheless, an asymptotic study of the eigenvalues, for large t, can be done and the result is identical to Eq. (32).

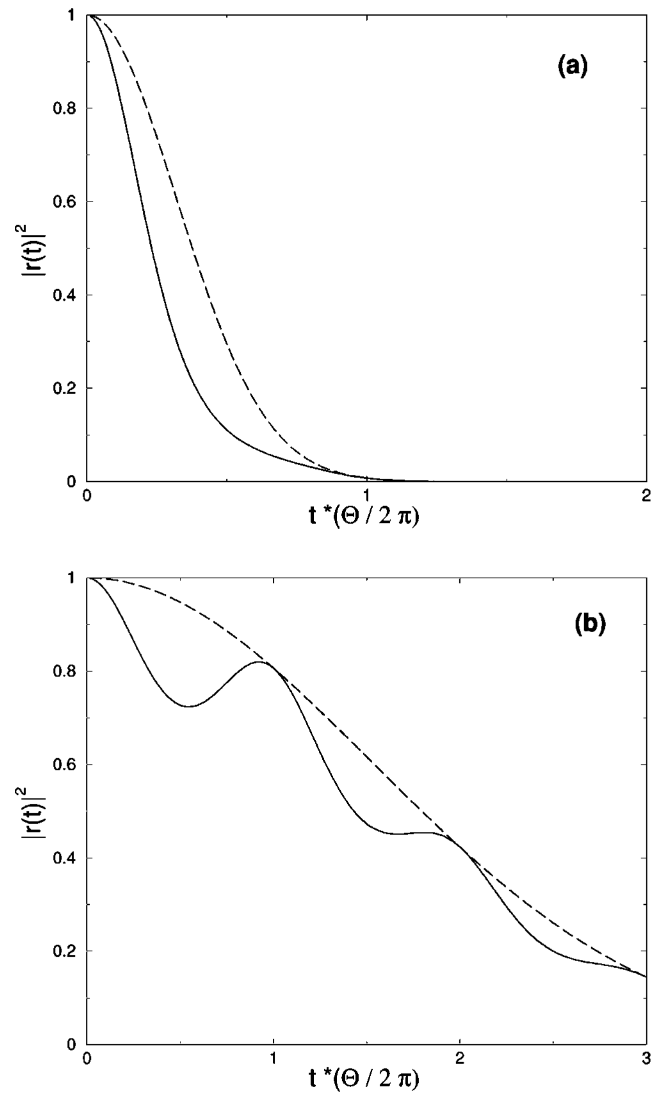

FIG. 1. Absolute value of the correlation amplitude (continuous lines) versus the asymptotic function (dashed lines) for different values of the ratio $R=\beta/\beta_c$ [(a) $R=1.25$, (b) $R=2.5$] at fixed values of $w/J=0.5$ and $J_0/J=1$.

V. A POSSIBLE EXPERIMENT

The last example suggests a possible experiment. A linear microcavity [27], i.e., a plane parallel faced Fabry-Pérot interferometer, can be used to select one mode of the electromagnetic field interacting with an array of Rydberg atoms inside. The interaction is analogous to the superradiance model [28,29] but the problem is different because no pumping is needed and no spontaneous, collective emission is studied. If the length of the cavity is one-half of the cutoff wavelength in the spectral density of the radiation, all the lower frequencies are cut off. All multiples of the fundamental frequency are permitted, but their contribution is negligible because of their small contribution to the spectrum and also because the coupling constant g becomes smaller on increasing the frequency. So just one frequency (or a thin band around it) is selected.

We consider a given number of atoms put in the center of the cavity, i.e., on the wave antinode, in a spatial range short in comparison with the selected wavelength, so that the coupling constants can be assumed independent of the individual position. At the same time, the atoms must be far enough apart to neglect the Coulomb interaction between them. The array can be oriented both longitudinally and transversely with respect to the cavity. In the last configuration the available volume to be filled by atoms does not depend on the cavity length but only on the transverse spatial coherence of the mode.

For a visible chaotic light spectrum, for example, a cavity of about 150 nm can be used to eliminate all the frequencies under the uv, that is, the visible cutoff. For this configuration a lateral open cavity is needed and the mode is not perfectly confined. In addition, the short distance between the mirrors prevents us from longitudinally putting a number of atoms larger than 50–100. Working in the microwave range, and closing the cavity in all directions, can also be convenient to completely confine the mode and to increase the number of atoms inside it.

A selected atom, with a different energy separation, represents the qubit and its state can be modified by a resonant laser pulse.

VI. CONCLUSION

The problem of decoherence of a given initial qubit, interacting with an environment, has been studied, relating the reduced matrix evolution to the environment correlation functions. This situation can be found in models of quantum computers to be realized by NMR, quantum dots, or atoms in an optical cavity. This approach, even if only with a mean field approximation for the environment, has the advantage of avoiding irreversible evolution as in the usual treatment with a Markovian master equation. As a consequence, decoherence is associated with a well defined temporal range. Spontaneous symmetry breaking of the environment implies an enhancement of coherence. This prediction can be verified by a suitable experiment.

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