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Strong-field and plasma aspects of multiphoton radiative recombination

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A strong enhancement of the power emitted in laser assisted radiative recombination is reported. Also, strong modification and enhancement of the spectrum of the radiation emitted during radiative recombination in a plasma are reported. The reported results take place when the electron average translation velocity and the thermal velocity are comparable to or smaller than the amplitude of the quiver velocity.

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The recombination of a free electron with a positive ion followed by the spontaneous emission of a photon has been studied extensively since many years, as the results of such studies are relevant to the analysis of astrophysical and laboratory plasmas. As the photon emission time is very long as compared to the collision time, the probability of the process is very small and its direct experimental observation difficult. In the experimental investigation, great progress has been achieved recently by using merged electron and ion beams [1]. In particular, using laser induced recombination, performed with merged beams of protons, electrons and photons in an ion storage ring, it has been possible to study the photorecombination spectrum with high resolution [2]. The experimental results have shown also that the recombination rates are enhanced by the presence of an external field. Rogelstad *et al.* [3] have observed stimulated recombination in $e^- + \mathrm{H}^+$ and $e^- + \mathrm{He}^+$ by using a merged electron-ion apparatus and have found an enhancement of the stimulated recombination over the spontaneous one by a gain factor of about three orders of magnitude in the case of final states with the principal quantum numbers $n = 11$, 12, and 13. The gain factor is given by the ratio between the induced recombination rate and the spontaneous one. In the aforementioned experiments the laser field is generally weak. Recently, the study of multiphoton radiative recombination in moderately strong field too has been addressed [4,5a], as it is viewed as an interesting process for production of high-energy photons. Rescattering effects in this process have been considered in Ref. [5b].

In the strong-field context, the recombination of an electron initially in a laser-dressed continuum state into an atomic bound state generally results in the emission of photons with energy given in atomic units by

$$\omega_x = |\epsilon_0| + \Delta + n\omega_L + \epsilon_q, \tag{1}$$

where ϵ_0 is the quasienergy of the final state, $\Delta = E_{0L}/4\omega_L$ is the ponderomotive shift, $\epsilon_q = q^2/2$ is the translation energy of the incoming electron, $\mathbf{q}$ is the translation momentum, n is the number of exchanged laser photons, and E_{0L} and ω_L are, respectively, the electric-field amplitude and the photon energy of the laser field.

As stated, the problem has been recently considered by Kuchiev and Ostrovsky [4], by employing a Keldysh-type approximation, i.e., neglecting the interaction of the incoming electron with the atomic core in the initial continuum state, and by Jaron, Kaminski, and Ehlotzky [5a] as well, who took approximately into account the interaction between the incoming electron and the atomic ion. Definite results concerning the emitted photon spectrum as a function of the laser intensity and of the incoming electron translation energy, reported in Refs. [4,5a], are restricted to the regime in which the amplitude of the quiver velocity $V_E = E_{0L}/\omega_L$ of the incoming electron is much smaller than the average velocity $\mathbf{V}_q$ related to the average momentum q. In particular, for this (weak-field) regime, Kuchiev and Ostrovsky [4] have found that the total cross section of electron recombination summed over all the channels of exchanged laser photons exhibits negligible dependence on the laser field intensity, and its value is close to that of the spontaneous process. Accordingly, the presence of the laser field does not modify the total yield of emitted photons, but only redistributes the photons in different channels, with energies given by Eq. (1).

As it will be shown below, this "sum rule" is restricted to the case when the energy exchanged between the electrons and the laser field is a very small fraction of the initial translation energy of the incoming electron, and clearly does not apply when this condition is not met, such as, for instance, in the strong-field regime.

In this Rapid Communication we give a theoretical and numerical analysis of the elementary process of the laser assisted radiative recombination for the regime in which the amplitude V_E of the quiver velocity of the incoming electron is equal to or larger than the average translation velocity $\mathbf{V}_q$ (strong-field regime). We show that in this regime the power emitted during the laser assisted radiative recombination is significantly enhanced over the field-free process, at variance with the weak-field results of Ref. [4]. For the strong-field case, we take also into account the influence of the medium (a plasma), in which the elementary process is assumed to take place. Here, to the best of our knowledge, in the strong-field multiphoton context, this is done for the first time. In particular, we show that the shape of the spectrum of the radiation emitted during the laser assisted radiative recombination in a plasma is strongly modified and enhanced, when the amplitude of quiver velocity V_E is of the same order as or larger than the plasma electron thermal velocity V_T.

Let us consider the recombination of a free electron with a hydrogenic positive ion with Z charges in the presence of a spatially and temporally homogeneous laser field linearly polarized with the polarization along the z axis in the dipole approximation $[\mathbf{E}_L = E_{0L}\sin(\omega_L t)\mathbf{z}]$. The probability rate of emission of a high-frequency photon ω_x by an electron recombining from a scattering state $\Psi^+_{\mathbf{q}}(\mathbf{r},t)$ of translation momentum $\mathbf{q}$ into a bound state $\Psi_0(\mathbf{r},t)$ is given by

$$W(\hat{\mathbf{n}}_x,\hat{\epsilon}_X,\omega_X,\mathbf{q}) = \sum_n W_n(\hat{\mathbf{n}}_x,\hat{\epsilon}_X,\omega_X,\mathbf{q}), \tag{2}$$

$$W_n(\mathbf{n}_X,\hat{\epsilon}_X,\omega_X,\mathbf{q}) = (2\pi)^2\omega_X\delta(\omega_X - |I_0| - \epsilon_q - \Delta - n\hbar\omega_L), \tag{2a}$$
$$\times\left|\frac{1}{T}\int_0^T dt\left\langle \Psi_0\right|\exp\left\{-i\omega_X + i\frac{\omega_x}{c}\hat{\mathbf{n}}_X\cdot\mathbf{r}\right\}\hat{\epsilon}_X\cdot\mathbf{r}\left|\Psi_q^+(r,t)\right\rangle\right|^2,$$

where $\mathbf{n}_x$ and $\boldsymbol{\epsilon}_x$ are, respectively, the propagation and the polarization directions of the emitted photon ω_x. Equation (2a) is the probability rate that n laser photons are exchanged in the recombination process. Formula (2) is derived following the standard procedure for this kind of process [6]. The only essentially far-reaching point is the use of the so-called Volkov-Coulomb wave function $\Psi^+_{\mathbf{q}}(\mathbf{r},t)$ to describe the electrons in the simultaneous presence of the ion Coulomb field and of the strong laser radiation,

$$\Psi_q^+(\mathbf{r},t) = \exp\left\{-\frac{i}{2}\int^t d\tau[\mathbf{q}+\mathbf{k}_L(\tau)]^2\right\}\exp\{i\mathbf{k}_L\cdot\mathbf{r}\}u_q^+(r), \tag{3}$$

where $\mathbf{K}_L = V_E\sin(\omega_L t)\mathbf{z}$ is the oscillating momentum imparted to the free electron by the laser field, and $u_{\mathbf{q}}(\mathbf{r})$ is the field-free outgoing Coulomb wave.

The Volkov-Coulomb wave function is an approximation [7] expected to reproduce satisfactorily the basic physics of the process: We note that a crucial test of this wave function as compared to other approximations has been the correct explanation of the first measurements of angular distributions of a photoelectron ionized by a strong elliptically polarized laser field [8,9].

Formula (2) is valid in the nonrelativistic regime; accordingly the laser intensity must be such that the amplitude of the quiver velocity V_E is much smaller than the light velocity c. Finally, in Eq. (2) an average over the field period is required.

In the calculations reported below the amplitude of the electric laser field satisfies the condition $E_{0L} < ZE_{\mathrm{at}}$ (E_{at} is the infratomic electric field of the hydrogen ground state), so that the dressing of the final bound state by the laser field may be safely neglected and Ψ_0 approximated by the field-free bare state of the hydrogenic ion with $Z-1$ charge and $\epsilon_0 = Z^2 I_0$ (I_0 is the energy of the hydrogen ground bound state).

For a given value of the incoming electron energy, the total probability rate integrated over all directions of electron incidence and all directions of high-frequency photon emission,

$$W(\epsilon_q) = \sum_n W_n(\epsilon_q), \tag{4}$$

$$W_n(\epsilon_q) = \frac{1}{(2\pi)^6}\int d^3k_X d\Omega_q q W_n(\hat{\mathbf{n}}_x,\hat{\epsilon}_X,\omega_X,\mathbf{q}). \tag{4a}$$

In Eq. (4a) $\mathbf{k}_x$ is the momentum of high-frequency photon ω_x. The total power of the emitted high-frequency photons ω_x, summed over all the channels of exchanged laser photons, is given by

$$P(\epsilon_q) = \sum_n P_n(\epsilon_q) = \int d\Omega_X\left(\frac{dP(\epsilon_q)}{d\Omega_X}\right), \tag{5}$$

$$P_n(\epsilon_q) = \int d\Omega_q d\Omega_X d\omega_X \frac{\omega_X^3 q}{(2\pi)^6 c^3} W_n(\hat{\mathbf{n}}_x,\hat{\epsilon}_X,\omega_X,\mathbf{q}). \tag{5a}$$

In Eqs. (4a) and (5a), $W_n(\epsilon_q)$ and $P_n(\epsilon_q)$ are, respectively, the total probability and the total power emitted for the channel in which n laser photons are exchanged. Below, among others, we calculate

$$Q(\epsilon_q) = \frac{dP(\epsilon_q)}{d\Omega_X} = \sum_n Q_n(\epsilon_q) \tag{6}$$

and

$$Q_n(\epsilon_q) = \frac{dP_n(\epsilon_q)}{d\Omega_X} = \int d\Omega_q d\omega_X\left[\frac{\omega_X^3 q}{(2\pi)^6 c^3} W_n(\hat{\mathbf{n}}_x,\hat{\epsilon}_X,\omega_X,\mathbf{q})\right]. \tag{7}$$

$Q(\epsilon_q)$ is the total power of the ω_x photon emission per steradian summed over all the channels of exchanged laser photons, and $Q_n(\epsilon_q)$ is the total power of the ω_x photon emission per steradian for a given channel in which n laser photons are exchanged. In general, from the present and other calculations not reported here for different values of the laser intensity and frequencies in the range $0.117 < \omega_L < 1.55$ eV, we find that $Q(\epsilon_q)$ depends on the laser parameters only through the amplitude of the quiver velocity V_B.

In Fig. 1 we show the gain factor $G = Q(\epsilon_q)/Q_{\mathrm{ff}}(\epsilon_q)$ defined as the total emitted power per steradian summed over all the channels of exchanged laser photons divided by the emitted power per steradian in the field-free process $Q(\epsilon_q)_{\mathrm{ff}}$ as a function of V_E for different values of ion charge Z and incident electron translation energy $\epsilon_q = 6$ a.u. Insofar as the amplitude of the quiver velocity V_E is smaller than the translation velocity $\mathbf{V}_q$, the gain factor G is near 1, i.e., the process exhibits a negligible dependence on the laser field intensity (weak-field domain). As soon as the amplitude of the quiver velocity becomes of the same order of magnitude as the average velocity $\mathbf{V}_q$ (the latter is indicated by the arrow on the axis of abscissas), the gain factor increases rather rapidly, especially for $Z = 1$.

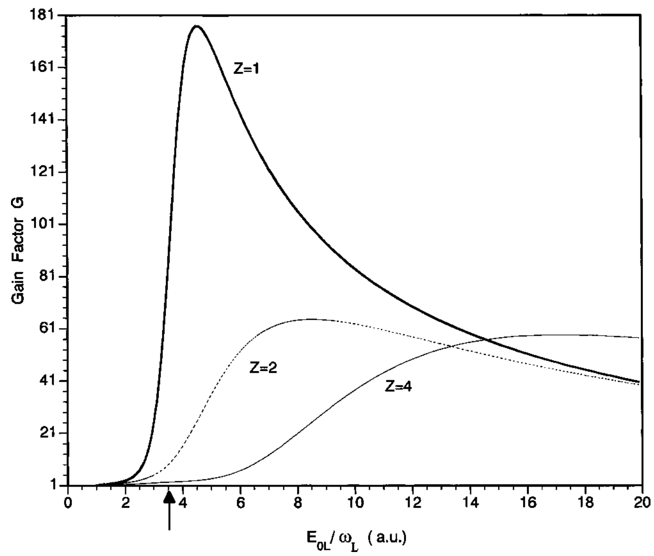

FIG. 1. Gain factor $G = Q(\epsilon_q)/Q_{ff}(\epsilon_q)$, defined as the total emitted power per steradian summed over all the channels of exchanged laser photons, divided by the emitted power per steradian in the spontaneous process, as a function of the amplitude of quiver velocity $V_E = E_{0L}/\omega$. Translation energy of the incident electron $\epsilon_q = 6$ a.u., propagation direction of the emitted radiation in the x direction with polarization vector along the z direction as the oscillating laser field. The arrow on the axis of the abscisses indicates the value of the average velocity $V_E = \mathbf{V}_q = 3.46$ a.u., related to the average momentum q.

As a rule, the radiative recombination as well as the bremsstrahlung are important elementary processes characterizing the continuum background emission spectrum of the plasmas. So it is appropriate to extend our consideration of strong-field laser assisted radiative recombination to take into account the medium influence. Here it is done by following the familiar and relatively simple two-step procedure [10], consisting in calculating, first, the quantities characterizing the elementary process (as we have done above) and then averaging them over the appropriate electron velocity distribution function (EDF) characteristic of the medium in the given physical conditions.

As in the strong-field context considered here the laser field is expected to modify significantly the medium EDF, a rigorous self-consistent procedure would require the simultaneous solution of the kinetic equation for the EDF, where all the pertinent interactions and processes, including recombination, are taken into account. Besides, in this context, an important issue, having autonomous interest, is that of the specific plasma electron velocity distribution created by the strong-field laser presence. However, an acceptable degree of self-consistency may be reached also within the two-step procedure. In fact, there is abundant literature (see, for instance, Ref. [11]), in which the issue of the EDF in laser fields is addressed in detail, and different EDFs are available corresponding to different assumptions and physical conditions. So, the two-step procedure may be profitably adopted using EDFs corresponding to given particular physical conditions, and *for the same conditions* calculating the elementary process.

Below we investigate how a given EDF jointly with the presence of a strong laser field is likely to influence the radiative recombination, which to be effective requires very slow electrons [12]. In fact a characteristic point on this issue is the presence of conflicting circumstances: (a) the radiative recombination performs well with slow electrons, and (b) plasma electrons especially the slow ones, in the presence of a laser field get heated.

Below, the influence of the plasma medium is taken into account by multiplying Eq. (2a) by an appropriate EDF $f(\mathbf{V})$, integrating the result over all the plasma electron velocities (or energies), and averaging over the laser field phase $\gamma = \omega_L t$, as the electron velocity $\mathbf{V}_q = \mathbf{q} + \mathbf{z} E_{0L}/\omega_L \sin\gamma$ is a function of γ. Therefore, in a plasma medium, the emitted total radiative power P has the following expression:

$$P = \frac{N_i N_e}{2\pi} \int_0^{2\pi} d\gamma \int d\Omega_q \, d\Omega_X \, d\epsilon_q \, d\omega_X \times f(\epsilon_q, \gamma) \frac{\omega_X^3 q}{(2\pi)^6 c^3} W_n(\hat{\mathbf{n}}_x, \hat{\epsilon}_X, \omega_X, \mathbf{q}), \quad (8)$$

where N_e and N_i are the electron and ion concentrations, and the EDF has been conveniently expressed as a function of the translation energy ϵ_q and the laser field phase γ. The interplay between the EDF and the rate of the laser assisted elementary process determines the features of the emitted power spectra in a plasma.

In calculating the power spectra per steradian and frequency unit of the emitted photon, $N_i N_e J(\omega_x) = dP/d\omega_x d\Omega_x$, Eq. (6), we confine below to two distinct physical situations of very weak and very strong fields, for which luckily enough, the same EDF may be used, namely, a Maxwellian. For very weak fields, in the early stages of laser-plasma interaction, the assumed Maxwellian EDF remains essentially unmodified. As it is known, subsequently the EDF will change, showing low electron depletion, but here these later stages will be not investigated. In the opposite case of very strong fields, if the plasma temperature and the laser parameters are such that the condition $ZV_T < V_E$ is fulfilled, one finds that the laser modified EDF is dominated by the electron-electron collisions. In such conditions, in a coordinate system oscillating with the laser field frequency, the EDF is known to be approximately Maxwellian [11].

In general, from the present and other calculations not reported here, we find that in any case the behavior of $J(\omega_x)$ vs ω_x is critically dependent on the ratio between the amplitude of the quiver velocity V_E and electron thermal velocity V_T. The ratio $R = V_E/V_T$ is a key parameter to characterize the laser-plasma interactions, with $R < 1$ identifying weak-field situations and $R > 1$ strong-field ones. As expected, for $R > 1$ the presence of the laser field influences strongly the shape of $J(\omega_x)$. Representative calculations are reported in Fig. 2. By assuming the electric fields of the laser radiation and of the emitted high-frequency radiation to be along z, $J(\omega_x)$ is calculated as a function of ω_x for ion charge $Z = 4$, plasma temperature $T = 1$ a.u., and different values of the laser intensity to give $R > 1$ and $ZV_T < V_E$.

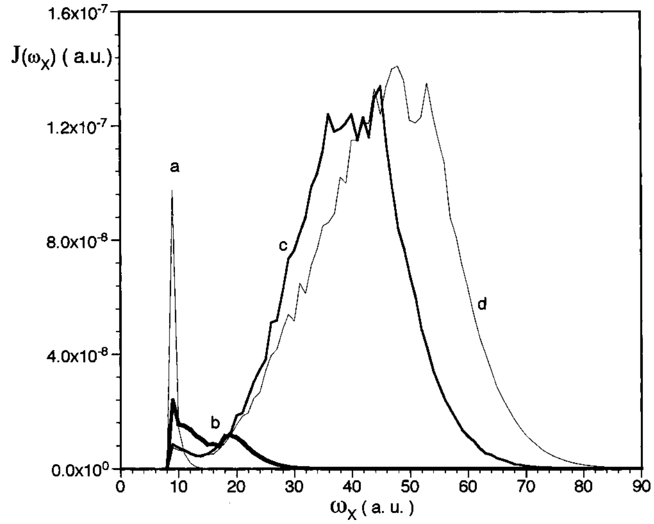

FIG. 2. Emitted power spectra per steradian and frequency unit of the emitted photon, $J(\omega_x)=(dP/d\omega_x d\Omega_x)/N_i N_e$, in a.u. as a function ω_x for ion charge $Z=4$, different values of the laser intensity (a) field free, (b) $I_L=2\times10^{15}$ W/cm^2, $R=V_E/V_T=4.2$, (c) $I_L=8\times10^{15}$ W/cm^2, $R=8.4$, (d) $I_L=10^{16}$ W/cm^2, $R=9.4$; plasma temperature $T=1$ a.u.; and a Maxwellian EDF.

The curves of Fig. 2 show plasma features, which are considered to be one of the results of the present work. First of all, a significant enhancement over the field-free case and a larger broadening of the emitted spectrum are found.

The reported curves exhibit a double-peak structure with the first peak roughly corresponding to the field-free value and the second one increasing and shifting to higher values of ω_x, with the increase of laser intensity. The field-free case is considered representative as well of the very weak field case ($R \ll 1$) in the early stages of the laser-plasma interaction. The first peak, related to the slow electrons, decreases with the increase of the laser intensity, the second broad peak corresponds to that large fraction of electrons having velocities close to the thermal velocity V_T and directed mostly along the laser field polarization. Thus, considering that the plasma electrons also oscillate with the quiver velocity amplitude V_E, for half-field period, in the plasma one has a large number of effective slow electrons (with velocities of the order of $|V_T - V_E|$ capable of yielding relatively large values of recombination. Besides, with the intensity increase, the number of photon channels significantly contributing is increased as well.

In conclusion, we have reported pilot calculations of laser induced radiative recombination, addressing both strong-field and plasma aspects. As to the strong-field aspects, we have found that a very significant enhancement of the field induced process over the field-free one takes place, provided the quiver velocity amplitude V_E is comparable with the average electron translation velocity $\mathbf{V}_q$.

As to the plasma aspects, calculations carried out in the regime where V_E is larger than the thermal velocity V_T, and many photons may be exchanged, confirm that on the one hand the field makes unfavorable the recombination of the slow electrons; on the other hand the process may be strongly enhanced for the group of electrons having a velocity near the thermal one.

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