Quantum interference and phase-dependent spectrum of resonance fluorescence of a three-level V-type atom

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The spectrum of quantum fluctuations in phase quadratures of resonance fluorescence of a three-level V-type atom driven by a coherent field is investigated. We find that quantum interference between two decay channels of the atom can greatly modify the spectrum. We show that for weak excitation, quantum interference can greatly enhance squeezing and change the phase quadrature in which the maximum squeezing occurs. We also show that for strong and far-off-resonance excitation two deeply squeezing peaks appear at the Rabi sideband frequencies and quantum interference broadens the squeezing peaks.

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I. INTRODUCTION

Resonance fluorescence of atoms illuminated by coherent fields can display a number of interesting quantum features of the electromagnetic field. Theoretical investigations and experimental observations have revealed nonclassical phenomena in resonance fluorescence of driven atoms such as photon antibunching [1] and sub-Poissonian photon statistics [2]. Theoretical studies also showed that the quantum noise level in one of the phase quadratures of resonance fluorescence of a driven two-level atom can be lower than the shot-noise limit for the vacuum of the electromagnetic field [3]. Squeezing of the quantum noise in phase quadratures of resonance fluorescence of a driven three-level atom with various level configurations was also extensively studied [4]. The phenomenon of squeezing in resonance fluorescence may be investigated by two ways: observing variances either in total phase quadratures or in frequency components of phase quadratures. In the latter one, the radiation field of a driven atom is mixed with a local oscillator field having a controllable fixed phase relative to the field that drives the atom, and then quantum fluctuations in a selected phase quadrature of resonance fluorescence can directly be measured [5]. Using a spectrum analyzer, this homodyne detection results in a phase-dependent spectrum of resonance fluorescence, i.e., a squeezing spectrum [6,7]. Collett, Walls, and Zoller [6] showed that the quantum noise in the out-of-phase quadrature of resonance fluorescence of a driven two-level atom around the atomic frequency for resonance weak excitation can be squeezed down lower than the shot-noise limit. Zhou and Swain [8] calculated the normally ordered noise spectra of the in-phase and the out-of-phase quadratures of resonance fluorescence of a single two-level atom strongly driven by a coherent field, including either the effects of a finite laser bandwidth due to phase diffusion or a frequency-tunable cavity with a low-Q value. They showed that for far-off-resonance and strong excitation the resonance fluorescence can exhibit two-mode squeezing at the Rabi sideband frequencies. In contrast to the theoretical work that paid attention to squeezing in the in- and out-of-phase quadratures, the recent experimental work [5] has measured the noise spectra in different phase quadratures of the resonance fluorescence of a coherently driven two-level atom with a long lifetime and found that the maximum squeezing occurs in the quadrature with a phase near $\pm\pi/4$ relative to the driving field. This experiment makes the study of the spectra of quantum fluctuations in phase quadratures of the resonance fluorescence of a coherently driven atom a more attractive subject.

The spectrum of quantum fluctuations in the positive frequency part of the resonance fluorescence of a driven atom is in general not phase dependent. Recent studies have shown that quantum intereference between different atomic decay channels can lead to quenching spontaneous emission and then greatly modify the phase-independent spectrum such as making ultranarrow central lines [9–11]. Swain, Zhou, and Ficek [12] also showed that under strong-field excitation, quantum interference can lead to anticorrelations of photons emitted from excited levels of a three-level V-type atom. Rice and Carmichael [13] found that the incoherent part of the intensity spectrum may be decomposed into the sum of squeezing spectra of the in-phase and the out-of-phase quadratrues of resonance fluorescence. So, one could expect that quantum interference may also greatly modify the squeezing spectrum. On the other hand, as pointed out in Ref. [5], phase-dependent resonance fluorescence spectra obtained by the homodyne detection of scattered radiation from atoms that are driven by a coherent field are much richer than ordinary phase-independent resonance fluorescence spectra and present many novel features. For example, Zhao and co-workers [14] showed that for off-resonance excitation pairs of phase-dependent spectra with phases of opposite sign are striking differences, which arise entirely from time ordering. To our knowledge, however, it has not been investigated until now how quantum interference affects squeezing phenomena in atomic resonance fluorescence and squeezing spectra. In this paper, employing a three-level V-type atom that is driven by a coherent light and coupled to vacuum modes of the electromagnetic field in free space, we will study this open problem.

II. MODEL AND PHASE-DEPENDENT SPECTRUM

The atomic level scheme under consideration consists of two closely upper and one lower levels [10]. The excited levels $|2\rangle$ with energy $\hbar\omega_2$ and $|1\rangle$ with energy $\hbar\omega_1$ are seperated by $\hbar\omega_{21}$ and are coupled to a common ground level $|0\rangle$ with zero energy by a single-mode laser field of frequency ω_L, amplitude E_L, and polarization vector $\mathbf{e}_L$. The excited levels are also coupled to the ground level by vacuum modes of the electromagnetic field. Here, considering that the level splitting between the excited states is very small, we assume that the vacuum mode coupling $|2\rangle$ to $|0\rangle$ is the same as one coupling $|1\rangle$ to $|0\rangle$. The direct transition between the excited levels is electric-dipole forbidden. Using a unit such that $\hbar=1$, one can write the Hamiltonian of the system in the frame rotating with the laser frequency ω_L as the form

$$H=H_C+H_R, \tag{1}$$

where

$$H_C=(\Delta-\omega_{21})|1\rangle\langle 1|+\Delta|2\rangle\langle 2|+[\Omega_1 A_{10}+\Omega_2 A_{20}+\text{H.c.}], \tag{2}$$

$$H_R=\sum_k [g_k^{(1)}e^{i\Delta_k t}a_k A_{10}+g_k^{(2)}e^{i\Delta_k t}a_k A_{20}+\text{H.c.}]. \tag{3}$$

Above, $\Delta=\omega_2-\omega_L$ is detuning between the transition from the uppermost level to the ground level and the laser field, $\Delta_k=\omega_L-\omega_k$ detuning between the laser field and the kth vacuum mode at frequency ω_k, $\Omega_i=E_L\mathbf{e}_L\cdot\vec{\mu}_{i0}(i=1,2)$ are the Rabi frequencies of the driving laser, and $\vec{\mu}_{i0}$ is the dipole moment of the atomic transition from $|i\rangle$ to $|0\rangle$, which is assumed to be real. In Eqs. (2) and (3), the atomic operator $A_{ij}=|i\rangle\langle j|(i,j=1,2,0)$, $a_k(a_k^\dagger)$ is the annihilation(creation) operator for the kth vacuum mode and $g_k^{(i)}(i=1,2)$ is the coupling constant between the kth vacuum mode and the atomic transition from $|i\rangle$ to $|0\rangle$.

According to the general reservoir theory with the Weisskopf-Wigner approximation [16], we can derive the following equations of motion for the reduced atomic-density-matrix elements $\rho_{ij}=\text{tr}(\rho A_{ji})$ where ρ is the density-matrix operator of the system [11],

$$\dot{\rho}_{10}=-[i(\Delta-\omega_{21})+\tfrac{1}{2}\gamma_1]\rho_{10}-\tfrac{1}{2}\gamma_{12}\rho_{20}+i\Omega_2\rho_{12}+i\Omega_1(1-\rho_{22}-2\rho_{00}), \tag{4}$$

$$\dot{\rho}_{20}=-[i\Delta+\tfrac{1}{2}\gamma_2]\rho_{20}-\tfrac{1}{2}\gamma_{12}\rho_{10}+i\Omega_2(\rho_{22}-\rho_{00})+i\Omega_1\rho_{21}, \tag{5}$$

$$\dot{\rho}_{00}=\gamma_1(1-\rho_{00})+(\gamma_2-\gamma_1)\rho_{22}+\gamma_{12}(\rho_{12}+\rho_{21})-i\Omega_2(\rho_{20}-\rho_{02})-i\Omega_1(\rho_{10}-\rho_{01}), \tag{6}$$

$$\dot{\rho}_{21}=-[i\omega_{21}+\tfrac{1}{2}(\gamma_1+\gamma_2)]\rho_{21}-\tfrac{1}{2}\gamma_{12}(1-\rho_{00})+i\Omega_1\rho_{20}-i\Omega_2\rho_{01}, \tag{7}$$

$$\dot{\rho}_{22}=-\gamma_2\rho_{22}-\tfrac{1}{2}\gamma_{12}(\rho_{12}+\rho_{21})-i\Omega_2(\rho_{02}-\rho_{20}). \tag{8}$$

When arriving at Eqs. (4)–(8), we have used the normalization condition $\rho_{11}+\rho_{22}+\rho_{00}=1$ to cancel ρ_{11}. In the above equations, γ_1 and γ_2 are spontaneous decay rates from the two excited levels to the ground level, which are given by $\gamma_i=|\vec{\mu}_{i0}|^2\omega_i^3/3\pi\varepsilon_0 c^3\hbar(i=1,2)$. In addition, using the generalized reservior theory in deriving Eqs. (4)–(8), we have the crossing term $\Sigma_k(g_k^{(1)})^* g_k^{(2)}$. With the Weisskopf-Wigner approximation, these cross terms can easily be written as $\vec{\mu}_{10}\cdot\vec{\mu}_{20}\omega_{1,2}^3/3\pi\varepsilon_0 c^3\hbar$. Considering that the upper level splitting ω_{21} is much smaller than the optical transition frequencies $\omega_{1,2}$, we have relpaced these terms by $\gamma_{12}=p\sqrt{\gamma_1\gamma_2}$, where $p=\vec{\mu}_{10}\cdot\vec{\mu}_{20}/|\vec{\mu}_{10}||\vec{\mu}_{20}|$. In Eqs. (4)–(8), the terms related to $\gamma_{12}=p\sqrt{\gamma_1\gamma_2}$ represent the effect of quantum interference between spontaneous-emission pathways from $|1\rangle$ to $|0\rangle$ and from $|2\rangle$ to $|0\rangle$. It reflects the fact that as the atom decays from the excited sublevel $|1\rangle$, it drives the other excited sublevel $|2\rangle$ to decay and vice versa, because of the existence of atomic coherence between $|1\rangle$ and $|2\rangle$. We notice that in the above equations the usual decay terms proportional to $\gamma_{1,2}$ are always paired with the decay interference terms proportional to $\gamma_{12}=p\sqrt{\gamma_1\gamma_2}$ as long as $p\neq 0$. These two terms may cancel each other and result in the depression of spontaneous emission. This quantum destructive interference can greatly modify the intensity spectrum of resonance fluorescence [9–11]. Here, we are interested in quantum interference how to modify the squeezing spectrum of resonance fluorescence, which is phase dependent.

Let $\mathbf{E}^{(+)}(\mathbf{r},t)$ and $\mathbf{E}^{(-)}(\mathbf{r},t)$ be operators for the positive and negative frequency parts of the fluorescent field, respectively. We introduce a slowly varying electric-field operator with phase θ,

$$\mathbf{E}_\theta(\mathbf{r},t)=\tfrac{1}{2}[\mathbf{E}^{(+)}(\mathbf{r},t)e^{i(\omega_L t+\theta)}+\mathbf{E}^{(-)}(\mathbf{r},t)e^{-i(\omega_L t+\theta)}] \tag{9}$$

$$=\mathbf{E}_1(\mathbf{r},t)\cos\theta+\mathbf{E}_2(\mathbf{r},t)\sin\theta, \tag{10}$$

where $\mathbf{E}_1(\mathbf{r},t)=[\mathbf{E}^{(+)}(\mathbf{r},t)\exp(i\omega_L t)+\mathbf{E}^{(-)}(\mathbf{r},t)\exp(-i\omega_L t)]/2$ and $\mathbf{E}_2(\mathbf{r},t)=i[\mathbf{E}^{(+)}(\mathbf{r},t)\exp(i\omega_L t)-\mathbf{E}^{(-)}(\mathbf{r},t)\exp(-i\omega_L t)]/2$ are the in-phase and out-of-phase quadratures of the fluorescent field relative to the coherent driving field, respectively. The normally ordered variance in $\mathbf{E}_\theta(\mathbf{r},t)$ can be written as

$$\langle:[\Delta\mathbf{E}_\theta(\mathbf{r},t)]^2:\rangle=\langle:\mathbf{E}_\theta(\mathbf{r},t),\mathbf{E}_\theta(\mathbf{r},t):\rangle=\left(\frac{1}{2\pi}\right)^2\int_{-\infty}^{\infty}d\omega_1 d\omega_2\int_{-\infty}^{\infty}dt_1 dt_2\times e^{i[\omega_1(t-t_1)+\omega_2(t-t_2)]}\times\hat{T}\langle:\mathbf{E}_\theta(\mathbf{r},t_1),\mathbf{E}_\theta(\mathbf{r},t_2):\rangle, \tag{11}$$

where $\langle\mathbf{A},\mathbf{B}\rangle=\langle\mathbf{A}\cdot\mathbf{B}\rangle-\langle\mathbf{A}\rangle\cdot\langle\mathbf{B}\rangle$. As Knöll, Vogel, and Welsch did [15], to ensure each of correlation functions of the positive and negative frequency parts of the electric field in $\langle:\mathbf{E}_\theta(\mathbf{r},t_1),\mathbf{E}_\theta(\mathbf{r},t_2):\rangle$ has a correct time order and is measurable, we explicitly introduce the time ordering operator $\hat{T}$ in Eq. (11), which is defined as

$$\hat{T}\langle:\mathbf{E}_\theta(\mathbf{r},t_1),\mathbf{E}_\theta(\mathbf{r},t_2):\rangle$$
$$=\tfrac{1}{4}[\langle\mathbf{E}_\theta^{(+)}(\mathbf{r},t_1),\mathbf{E}_\theta^{(+)}(\mathbf{r},t_2)\rangle e^{i[\omega_L(t_1+t_2)+2\theta]}\Theta(t_1-t_2)$$
$$+\langle\mathbf{E}_\theta^{(+)}(\mathbf{r},t_2),\mathbf{E}_\theta^{(+)}(\mathbf{r},t_1)\rangle e^{i[\omega_L(t_1+t_2)+2\theta]}\Theta(t_2-t_1)$$
$$+\langle\mathbf{E}_\theta^{(-)}(\mathbf{r},t_1),\mathbf{E}_\theta^{(-)}(\mathbf{r},t_2)\rangle e^{-i[\omega_L(t_1+t_2)+2\theta]}\Theta(t_2-t_1)$$
$$+\langle\mathbf{E}_\theta^{(-)}(\mathbf{r},t_2),\mathbf{E}_\theta^{(-)}(\mathbf{r},t_1)\rangle e^{-i[\omega_L(t_1+t_2)+2\theta]}\Theta(t_1-t_2)$$
$$+\langle\mathbf{E}_\theta^{(-)}(\mathbf{r},t_1),\mathbf{E}_\theta^{(+)}(\mathbf{r},t_2)\rangle e^{i\omega_L(t_2-t_1)}$$
$$+\langle\mathbf{E}_\theta^{(-)}(\mathbf{r},t_2),\mathbf{E}_\theta^{(+)}(\mathbf{r},t_1)\rangle e^{i\omega_L(t_1-t_2)}], \quad (12)$$

where Θ is a unit step function. In steady state, Eq. (11) can be written as

$$\langle:[\Delta\mathbf{E}_\theta(\mathbf{r})]^2:\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\tau e^{i\omega\tau}\hat{T}\langle:\mathbf{E}_\theta(\mathbf{r},t),\mathbf{E}_\theta\times(\mathbf{r},t+\tau):\rangle. \quad (13)$$

According to this expression, one can introduce the squeezing spectral density [15]

$$:S(\mathbf{r},\omega,\theta):=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\tau e^{i\omega\tau}\hat{T}\langle:\mathbf{E}_\theta(\mathbf{r},t),\mathbf{E}_\theta(\mathbf{r},t+\tau):\rangle. \quad (14)$$

In the radiation zone, the positive frequency part of the fluorescent light emitted by the atom takes the form [17]

$$\mathbf{E}_\theta^{(+)}(\mathbf{r},t)=-\frac{1}{c^2r}\left[\omega_1^2\mathbf{n}\times(\mathbf{n}\times\vec{\mu}_{01})A_{01}\left(t-\frac{r}{c}\right)+\omega_2^2\mathbf{n}\times(\mathbf{n}\times\vec{\mu}_{02})A_{02}\left(t-\frac{r}{c}\right)\right]e^{-i\omega_L(t-r/c)}, \quad (15)$$

where $\mathbf{n}$ is a unit vector in the direction of observation. Suppose that $\omega_1\approx\omega_2$ and $\mathbf{n}$ is perpendicular to the atomic dipole moments $\vec{\mu}_{01}$ and $\vec{\mu}_{02}$. Equation (15) can be rewritten as

$$\mathbf{E}_\theta^{(+)}(\mathbf{r},t)=f(r)[\vec{\mu}_{01}A_{01}(\hat{t})+\vec{\mu}_{02}A_{02}(\hat{t})]e^{-i\omega_L\hat{t}}, \quad (16)$$

where $f(r)=\omega_1^2/c^2r$ and $\hat{t}=t-r/c$. Substituting Eq. (16) into Eq. (12), we obtain

$$\hat{T}\langle:\mathbf{E}_\theta(\mathbf{r},t),\mathbf{E}_\theta(\mathbf{r},t+\tau):\rangle$$
$$=\tfrac{1}{2}\mu^2f^2(r)\mathrm{Re}\{[\langle A_{01}(\hat{t}+\tau),A_{01}(t)\rangle+\langle A_{02}(\hat{t}+\tau),\times A_{02}(\hat{t})\rangle+p(\langle A_{01}(\hat{t}+\tau),A_{02}(\hat{t})\rangle+\langle A_{02}(\hat{t}+\tau),A_{01}(\hat{t})\rangle)]e^{2i(\theta+\omega_Lr/c)}$$
$$+\langle A_{10}(\hat{t}+\tau),A_{01}(\hat{t})\rangle+\langle A_{20}(\hat{t}+\tau),A_{02}(\hat{t})\rangle+p(\langle A_{10}(\hat{t}+\tau),A_{02}(\hat{t})\rangle+\langle A_{20}(\hat{t}+\tau),A_{01}(\hat{t})\rangle)\}, \quad (17)$$

where $\tau>0$, $\langle A_{ij}(t)\rangle=tr[\rho(t)A_{ji}]=\rho_{ji}(t)$ and $\mu=\mu_{01}\approx\mu_{02}$. So, to calculate the squeezing spectrum (14), one needs to know the atomic two-time correlation functions in Eq. (17). In the following, for simplicity, we will drop the hat symbol on time variable *t*.

We now introduce a column vector

$$\hat{\psi}(t)=[\langle A_{01}(t)\rangle,\langle A_{02}(t)\rangle,\langle A_{10}(t)\rangle,\langle A_{00}(t)\rangle,\times\langle A_{12}(t)\rangle,\langle A_{20}(t)\rangle,\langle A_{21}(t)\rangle,\langle A_{22}(t)\rangle]^T. \quad (18)$$

In this way, Eqs. (4)–(8) can be rewritten as the compact matrix form

$$\frac{d\hat{\psi}(t)}{dt}=\hat{M}\hat{\psi}(t)+\hat{I}, \quad (19)$$

where $\hat{M}$ is an 8×8 matrix of coefficients of equations (4)–(8) and $\hat{I}$ is a column vector whose elements $I_1=-I_3=-i\Omega_1, I_4=\gamma_1, I_5=I_7=-\gamma_{12}/2$, and the others are zero. Let $\Delta\hat{\psi}(t)=\hat{\psi}(t)-\hat{\psi}(\infty)$, where $\hat{\psi}(\infty)=-\hat{M}^{-1}\hat{I}$ is the steady-state solution of Eq. (19). Then, Eq. (19) is rewritten as

$$\frac{d\Delta\hat{\psi}(t)}{dt}=\hat{M}\Delta\hat{\psi}(t). \quad (20)$$

The formal solution of Eq. (20) is

$$\Delta\hat{\psi}(t+\tau)=e^{\tau\hat{M}}\Delta\hat{\psi}(t). \quad (21)$$

In order to calculate two-time correlation functions

$$\langle\Delta A_{lk}(t+\tau)\Delta A_{ij}(t)\rangle=\langle[A_{lk}(t+\tau)-A_{lk}(\infty)][A_{ij}(t)-A_{ij}(\infty)]\rangle$$

with $l,k,i,j=0,1,2$, we define two-time column vectors

$$\hat{U}^{(ij)}(t+\tau)=[\langle\Delta A_{01}(t+\tau)\Delta A_{ij}(t)\rangle,\langle\Delta A_{02}(t+\tau)\times\Delta A_{ij}(t)\rangle,\langle\Delta A_{10}(t+\tau)\times\Delta A_{ij}(t)\rangle,\langle\Delta A_{00}(t+\tau)\Delta A_{ij}(t)\rangle,\langle\Delta A_{12}(t+\tau)\times\Delta A_{ij}(t)\rangle,\langle\Delta A_{20}(t+\tau)\Delta A_{ij}(t)\rangle,\langle\Delta A_{21}(t+\tau)\times\Delta A_{ij}(t)\rangle,\langle\Delta A_{22}(t+\tau)\Delta A_{ij}(t)\rangle]^T. \quad (22)$$

According to the quantum regression theorem [18], the two-time column vectors must satisfy the equation

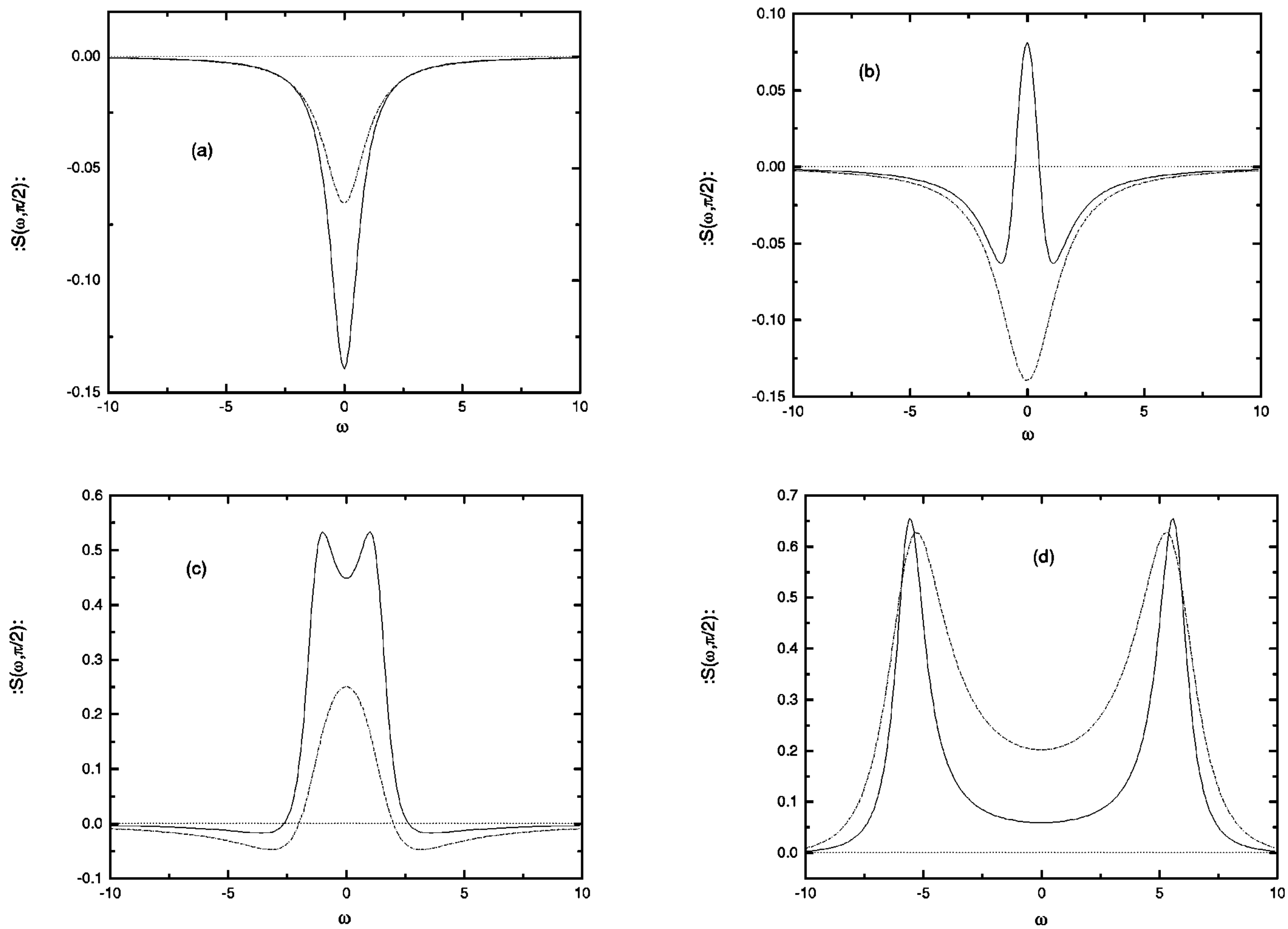

FIG. 1. Squeezing spectra of the out-of-phase quadrature for $\Delta=0$, $\omega_{21}=0$, $p=0.999$ (dot-dashed lines) and 0.0 (solid lines). (a) $\Omega=0.1$; (b) $\Omega=0.2$; (c) $\Omega=0.5$; (d) $\Omega=2.0$.

$$\frac{d\hat{U}^{(ij)}(t+\tau)}{d\tau}=\hat{M}\hat{U}^{(ij)}(t+\tau). \tag{23}$$

The formal solution of Eq. (23) is

$$\hat{U}^{(ij)}(t+\tau)=e^{\tau\hat{M}}\hat{U}^{(ij)}(t). \tag{24}$$

From Eqs. (21) and (22), we obtain the following two-time atomic correlation functions:

$$\begin{aligned}\langle A_{01}(t+\tau),A_{01}(t)\rangle=&(e^{\tau\hat{M}})_{1,3}[1-\psi_4(t)-\psi_8(t)]\\&+(e^{\tau\hat{M}})_{1,4}\psi_1(t)+(e^{\tau\hat{M}})_{1,6}\psi_7(t)\\&-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{1,j}\psi_j(t)\psi_1(t),\end{aligned} \tag{25}$$

$$\begin{aligned}\langle A_{02}(t+\tau),A_{02}(t)\rangle=&(e^{\tau\hat{M}})_{2,3}\psi_5(t)+(e^{\tau\hat{M}})_{2,4}\psi_2(t)\\&+(e^{\tau\hat{M}})_{2,6}\psi_8(t)\\&-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{2,j}\psi_j(t)\psi_2(t),\end{aligned} \tag{26}$$

$$\begin{aligned}\langle A_{01}(t+\tau),A_{02}(t)\rangle=&(e^{\tau\hat{M}})_{1,3}\psi_5(t)+(e^{\tau M})_{1,4}\psi_2(t)\\&+(e^{\tau M})_{1,6}\psi_8(t)\\&-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{1,j}\psi_j(t)\psi_2(t),\end{aligned} \tag{27}$$

$$\begin{aligned}\langle A_{02}(t+\tau),A_{01}(t)\rangle=&(e^{\tau\hat{M}})_{2,3}[1-\psi_4(t)-\psi_8(t)]\\&+(e^{\tau\hat{M}})_{2,4}\psi_1(t)+(e^{\tau\hat{M}})_{2,6}\psi_7(t)\\&-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{2,j}\psi_j(t)\psi_1(t),\end{aligned} \tag{28}$$

$$\begin{aligned}\langle A_{10}(t+\tau),A_{01}(t)\rangle=&(e^{\tau\hat{M}})_{3,3}[1-\psi_4(t)-\psi_8(t)]\\&+(e^{\tau\hat{M}})_{3,4}\psi_1(t)+(e^{\tau\hat{M}})_{3,6}\psi_7(t)\\&-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{3,j}\psi_j(t)\psi_1(t),\end{aligned} \tag{29}$$

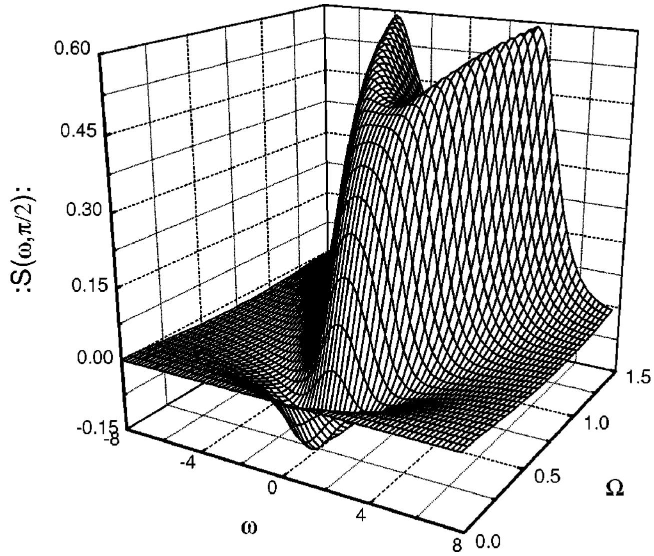

FIG. 2. Three-dimensional spectra of the out-of-phase quadrature as a function of the Rabi frequency Ω for $\omega_{21}=0$, $\Delta=0.0$, and $p=0.999$.

$$\langle A_{20}(t+\tau),A_{02}(t)\rangle=(e^{\tau\hat{M}})_{6,3}\psi_5(t)+(e^{\tau\hat{M}})_{6,4}\psi_2(t)+(e^{\tau\hat{M}})_{6,6}\psi_8(t)-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{6,j}\psi_j(t)\psi_2(t), \tag{30}$$

$$\langle A_{10}(t+\tau),A_{02}(t)\rangle=(e^{\tau\hat{M}})_{3,3}\psi_5(t)+(e^{\tau\hat{M}})_{3,4}\psi_2(t)+(e^{\tau\hat{M}})_{3,6}\psi_8(t)-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{3,j}\psi_j(t)\psi_2(t), \tag{31}$$

$$\langle A_{20}(t+\tau),A_{01}(t)\rangle=(e^{\tau\hat{M}})_{6,3}[1-\psi_4(t)-\psi_8(t)]+(e^{\tau\hat{M}})_{6,4}\psi_1(t)+(e^{\tau\hat{M}})_{6,6}\psi_7(t)-\sum_{j=1}^{8}(e^{\tau\hat{M}})_{6,j}\psi_j(t)\psi_1(t). \tag{32}$$

Inserting these two-time correlation functions into Eq. (14) and taking $t\rightarrow\infty$ [17], we obtain

$$:S(\omega,\theta):=-\frac{1}{4\pi}\mu^2 f^2(r)\mathrm{Re}\sum_{i=1}^{8}\sum_{\alpha=+,-}\hat{\Gamma}_i^{(\alpha)}(Z)|_{Z=i\omega}, \tag{33}$$

where

$$\hat{\Gamma}_1^{(\alpha)}(Z)=\Big(\hat{N}_{1,3}^{(\alpha)}(Z)[1-\psi_4(\infty)-\psi_8(\infty)]+\hat{N}_{1,4}^{(\alpha)}(Z)\psi_1(\infty)+\hat{N}_{1,6}^{(\alpha)}(Z)\psi_7(\infty)-\sum_{j=1}^{8}\hat{N}_{1,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_1(\infty)\Big)e^{2i(\theta+\omega_L r/c)}, \tag{34}$$

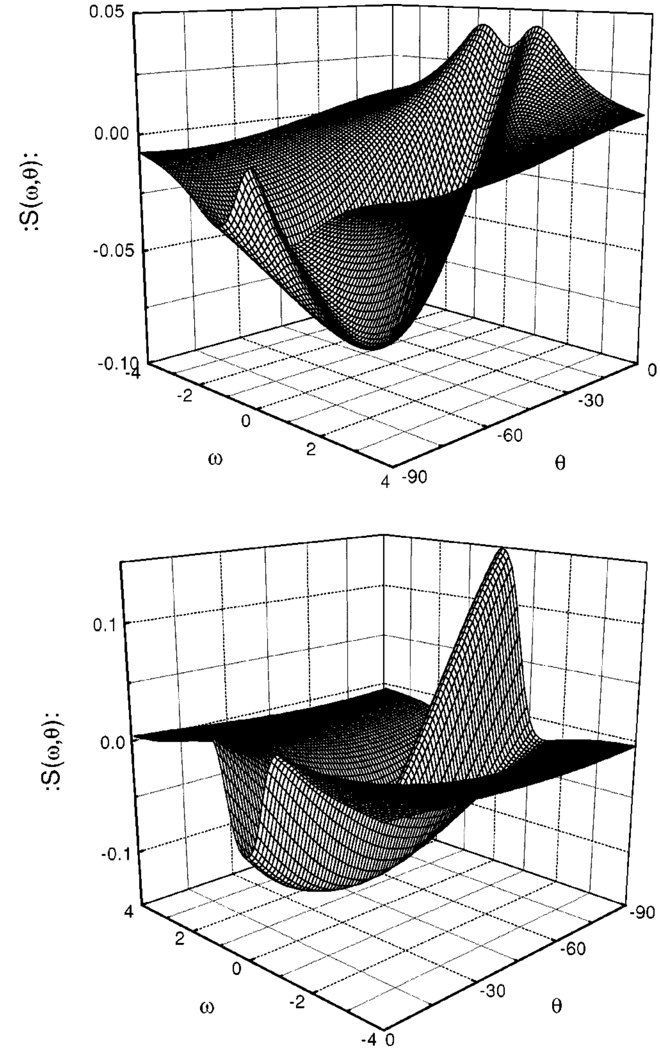

FIG. 3. Three-dimensional spectra of various phase quadratures for $\omega_{21}=0.0$, $\Omega=0.15$, $\Delta=0.5$. (a) $p=0.999$; (b) $p=0.0$.

$$\hat{\Gamma}_2^{(\alpha)}(Z)=\Big[\hat{N}_{2,3}^{(\alpha)}(Z)\psi_5(\infty)+\hat{N}_{2,4}^{(\alpha)}(Z)\psi_2(\infty)+\hat{N}_{2,6}^{(\alpha)}(Z)\psi_8(\infty)-\sum_{j=1}^{8}\hat{N}_{2,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_2(\infty)\Big]e^{2i(\theta+\omega_L r/c)}, \tag{35}$$

$$\hat{\Gamma}_3^{(\alpha)}(Z)=p\Big[\hat{N}_{1,3}^{(\alpha)}(Z)\psi_5(\infty)+\hat{N}_{1,4}^{(\alpha)}(Z)\psi_2(\infty)+\hat{N}_{1,6}^{(\alpha)}\psi_8(\infty)-\sum_{j=1}^{8}\hat{N}_{1,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_2(\infty)\Big]e^{2i(\theta+\omega_L r/c)}, \tag{36}$$

$$\hat{\Gamma}_4^{(\alpha)}(Z)=p\Big(\hat{N}_{2,3}^{(\alpha)}[1-\psi_4(\infty)-\psi_8(\infty)]+\hat{N}_{2,4}^{(\alpha)}(Z)\psi_1(\infty)+\hat{N}_{2,6}^{(\alpha)}(Z)\psi_7(\infty)-\sum_{j=1}^{8}\hat{N}_{2,j}(Z)\psi_j(\infty)\psi_1(\infty)\Big)e^{2i(\theta+\omega_L r/c)}, \tag{37}$$

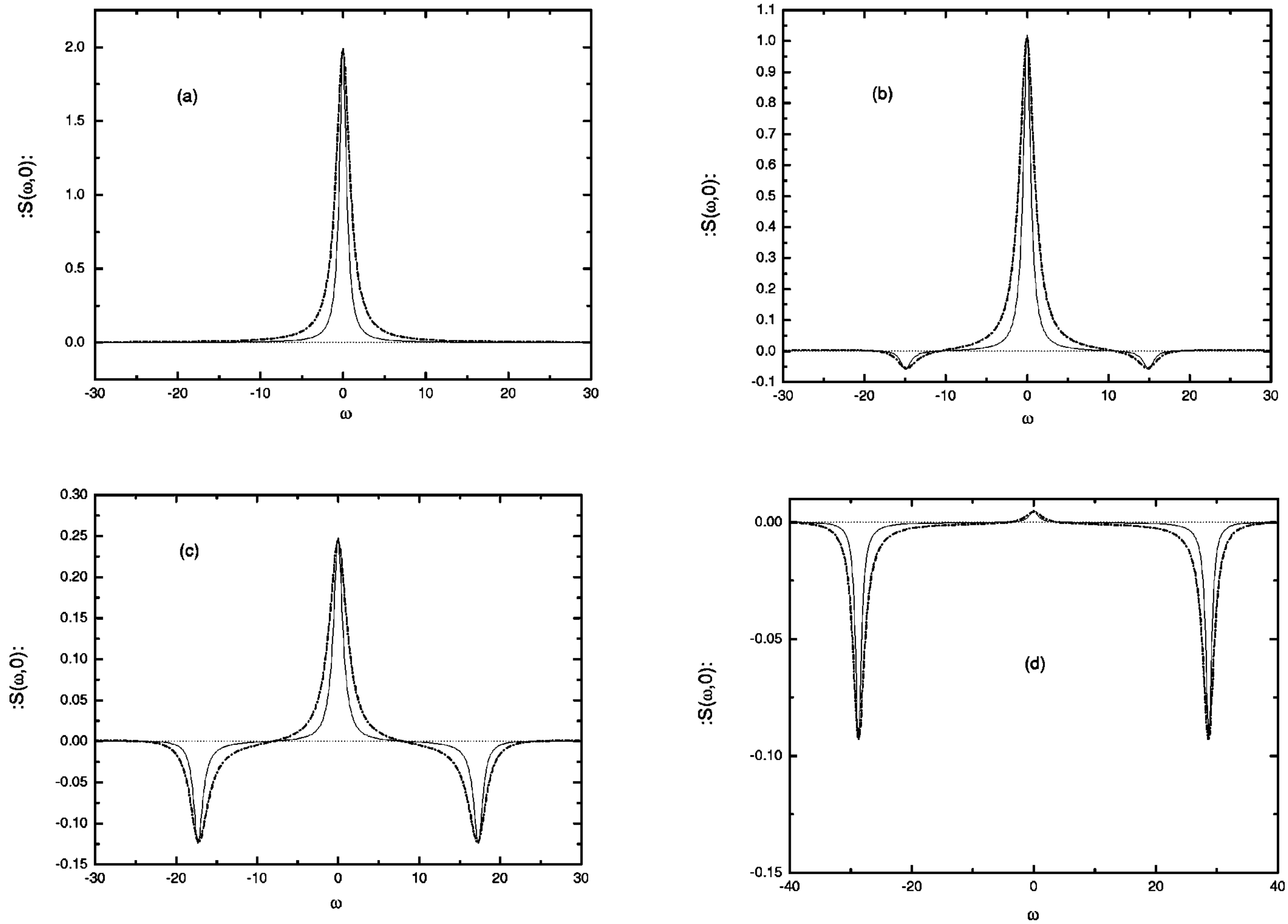

FIG. 4. Squeezing spectra of the in-phase quadrature for $\omega_{21}=0.0,\Omega=5$, $p=0.999$ (dot-dashed lines) and 0.0 (solid lines). (a) $\Delta=0.0$; (b) $\Delta=5.0$; (c) $\Delta=10$; (d) $\Delta=25$.

$$\hat{\Gamma}_5^{(\alpha)}(Z)=\left(\hat{N}_{3,3}^{(\alpha)}(Z)[1-\psi_4(\infty)-\psi_8(\infty)]+\hat{N}_{3,4}^{(\alpha)}(Z)\psi_1(\infty)+\hat{N}_{3,6}^{(\alpha)}\psi_7(\infty)-\sum_{j=1}^{8}\hat{N}_{3,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_1(\infty)\right),\quad(38)$$

$$\hat{\Gamma}_6(Z)=\left[\hat{N}_{6,3}^{(\alpha)}\psi_5(\infty)+\hat{N}_{6,4}^{(\alpha)}\psi_2(\infty)+\hat{N}_{6,6}^{(\alpha)}\psi_8(\infty)-\sum_{j=1}^{8}\hat{N}_{6,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_2(\infty)\right],\quad(39)$$

$$\hat{\Gamma}_7^{(\alpha)}(Z)=p\left[\hat{N}_{3,3}^{(\alpha)}\psi_5(\infty)+\hat{N}_{3,4}^{(\alpha)}\psi_2(\infty)+\hat{N}_{3,6}\psi_8(\infty)-\sum_{j=1}^{8}\hat{N}_{3,j}^{(\alpha)}(Z)\psi_j(\infty)\psi_2(\infty)\right],\quad(40)$$

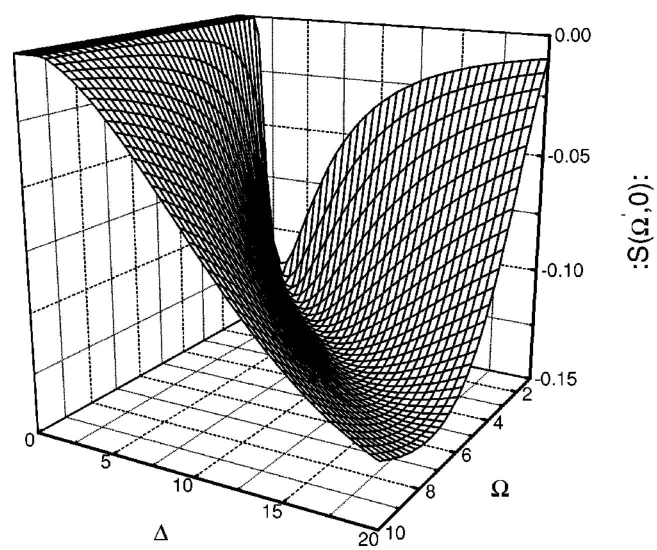

FIG. 5. The maximum squeezing of the in-phase quadrature at the Rabi sideband frequencies $\omega=\pm\Omega'$ as a function of Δ and Ω for $\omega_{21}=0.0$ and $p=0.999$.

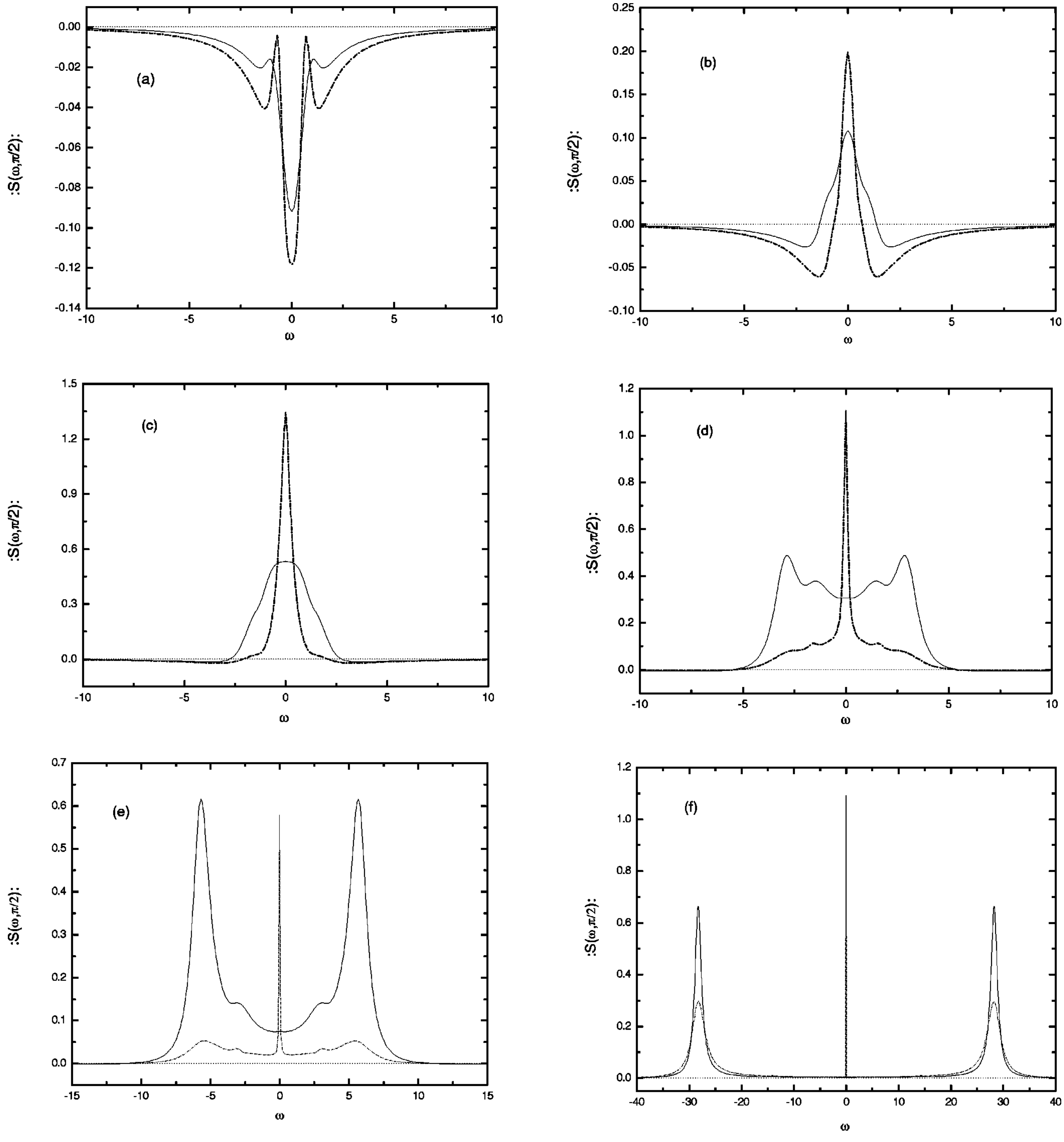

FIG. 6. Squeezing spectra of the out-of-phase quadrature for $\omega_{21}=1,\Delta=0$, $p=1.0$(dot-dashed lines) and 0.0 (solid lines).(a) $\Omega=0.15$; (b) $\Omega=0.25$; (c) $\Omega=0.5$; (d) $\Omega=1.0$; (e) $\Omega=2.0$; (f) $\Omega=10$.

$$\hat{\Gamma}_8(Z)=p\Bigg(\hat{N}^{(\alpha)}_{6,3}(Z)[1-\psi_4(\infty)-\psi_8(\infty)]+\hat{N}^{(\alpha)}_{6,4}\psi_1(\infty)+\hat{N}^{(\alpha)}_{6,6}\psi_7(\infty)-\sum_{j=1}^{8}\hat{N}^{(\alpha)}_{6,j}(Z)\psi_j(\infty)\psi_1(\infty)\Bigg). \quad (41)$$

In the above equations, the matrix function $\hat{N}^{(\pm)}(Z)=(\pm Z+\hat{M})^{-1}$. Using Eqs. (33) and (34)–(41), we can numerically calculate the squeezing spectrum (14).

III. RESULTS AND DISCUSSIONS

In the following calculations, for simplicity, we take $\gamma_1=\gamma_2=\gamma$ and $\Omega_1=\Omega_2=\Omega$, and scale Ω, ω_{21}, and Δ by γ.

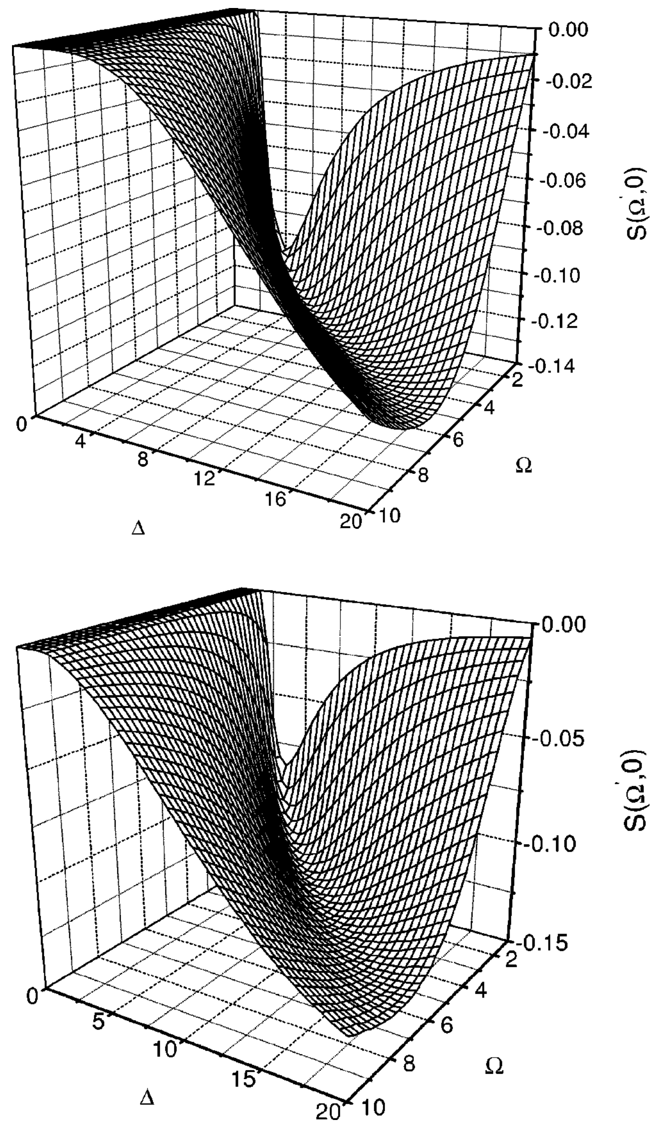

FIG. 7. Same as Fig. 5 but $\omega_{21}=1.0$, and (a) $p=1.0$; (b) $p=0.0$.

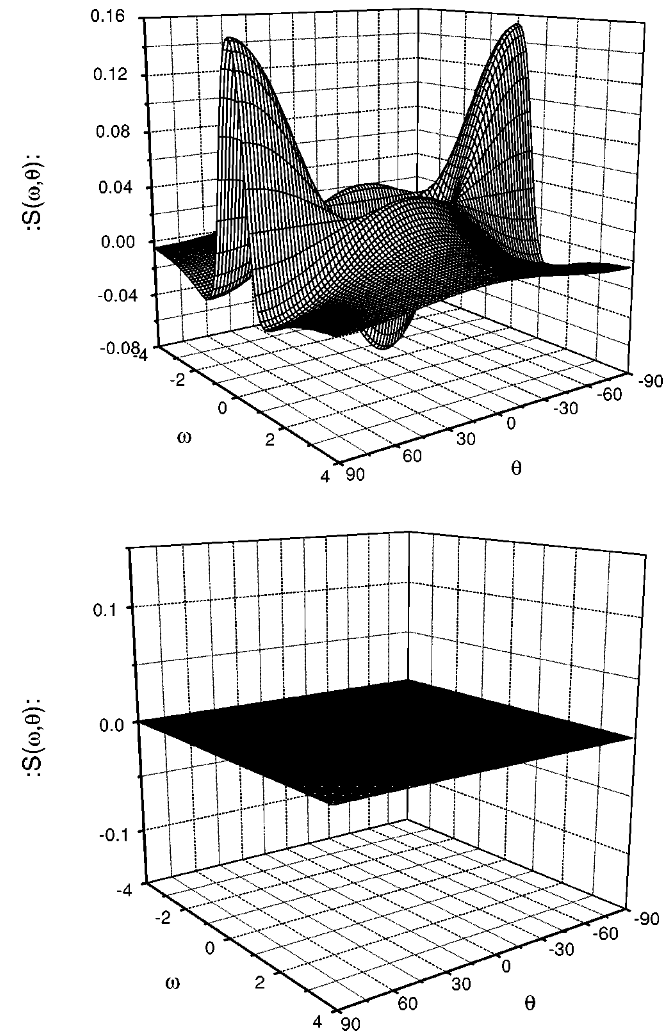

FIG. 8. Three-dimensional spectra of phase quadratures for $\Omega=0.15$, $\omega_{21}=1$, and $\Delta=0.5$. (a) $p=0$; (b) $p=1.0$.

We also set $e^{2i\omega_L r/c}=1$ and scale the squeezing spectrum by $\mu^2 f^2(r)/4\pi\gamma$.

At first, let us consider the degenerate case, i.e., $\omega_{21}=0$. As the two-level case [3,6], we also find that for weak resonance excitation the maximum squeezing always appears in the frequency component of the out-of-phase quadrature around the atomic transition frequency and the in-phase quadrature does not exhibit any squeezing. In Fig. 1, squeezing spectra of the out-of-phase quadrature are plotted for different resonance excitation intensities with and without quantum interference. Since $p=1.0$ means that quantum interference is maximum and resonance fluorescence is completely depressed [10,11], we take $p=0.999$ for nearly complete quantum interference in the present calculations. We observe that for weak excitation intensities the maximum squeezing always occurs at $\omega=0.0$, i.e., atomic transition frequency, and the squeezing can exist in all frequency components, as shown in Figs. 1(a) and 1(b). It means that for weak resonance excitation the total quantum fluctuation in the out-of-phase quadrature can be squeezed down lower than the shot-noise limit for the vacuum of the electromagnetic field. With increase in the Rabi frequency, the maximum squeezing shifts from the frequency component at $\omega=0$ to the wings of the spectrum, as shown in Figs. 1(b) and 1(c). Comparing the spectra with quantum interference to the ones without quantum interference, we find that quantum interference can greatly enhance the squeezing and extend the squeezing frequency region for appropriate excitation intensities as shown in Figs. 1(b) and 1(c). When the Rabi frequency increases further, the squeezing disappears and the sidebands of the Mollow fluorescent triplet develop, as shown in Fig. 1(d). We notice that quantum interference greatly broadens the Mollow sidebands. In order to have an overall view of the variation of the squeezing spectrum against the Rabi frequency, three-dimensional squeezing spectra of the out-of-phase quadrature are given in Fig. 2. We notice that there is a larger optimal Rabi frequency for the maximum squeezing and the squeezing in the wings of the spectra exists in a wider region when quantum interference takes place.

In resonance fluorescence of a two-level atom driven by a

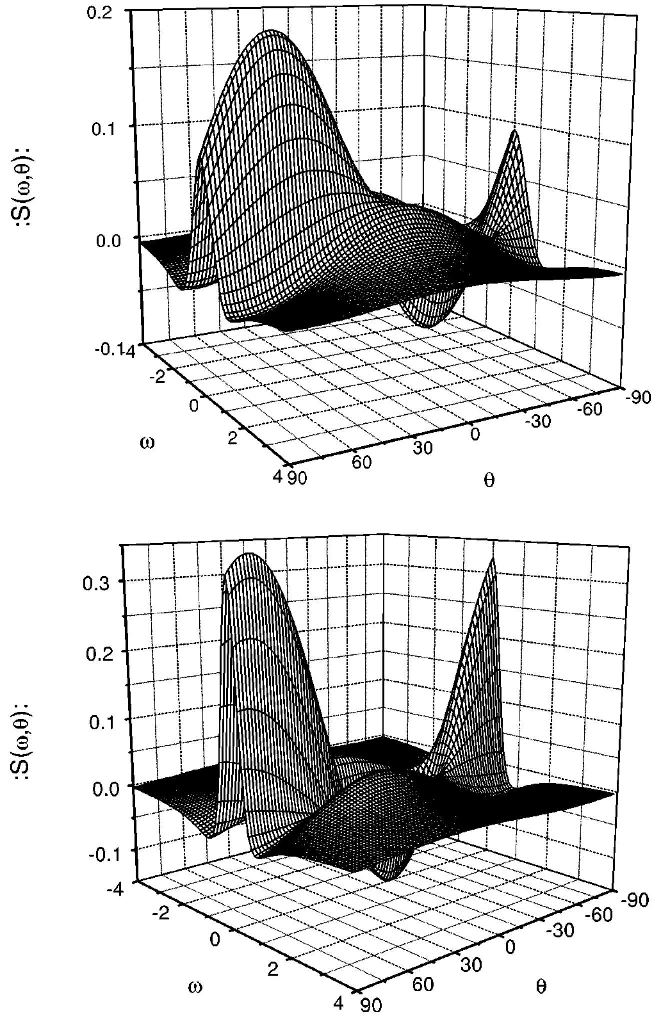

FIG. 9. Same as Fig. 8 but $\Delta = 0.3$.

monochromatic laser field and damped by a broadband squeezed vacuum, Zhou, and Swain [19] pointed out that squeezing can occur in any phase quadrature of the resonance fluorescence, depending upon the squeezing phase and the detuning. In our calculations, we notice that in either the degenerate three-level case $\omega_{21} = 0.0$ or the two-level case corresponding to $\omega_{21} \to \infty$ the squeezing spectrum is symmetrical about $\theta = 0$ when the excitation is resonant. However, the squeezing spectrum becomes nonsymmetrical about $\theta = 0$ when the excitation is off-resonant. The phase region of quadratures in which squeezing appears depends on the detuning. In Fig. 3, we present three-dimensional plots of spectra of various phase quadratures. Comparing Fig. 3(a) to Fig. 3(b), we see that quantum interference shifts the optimal squeezing phase quadrature and greatly modifies the spectra. In Ref. [8], Zhou and Swain found that for strong and far-off-resonance excitation the squeezing spectrum of the in-phase quadrature of the resonance fluorescence of a driven two-level atom exhibits two-mode squeezing at the Rabi sideband frequencies. In Fig. 4, we show squeezing spectra of the in-phase quadrature as a function of the detuning Δ. We observe that the squeezing at the Rabi sideband frequencies occurs and is enhanced with increasing of the detuning, and quantum interference obviously broadens the spectra but does not affect the peak location and the height. We also notice that the degree of squeezing does not increase monotonically with increasing detuning. As shown in Figs. 4(c) and 4(d), in fact, the squeezing does not increase but decreases when the detuning is beyond a certain value. In Fig. 5, we show the maximum squeezing in the in-phase quadrature at the Rabi sideband frequencies $\omega = \pm\Omega'$ for various values of the detuning and the Rabi frequency. Unlike the two-level case [8], we notice that the central frequencies $\pm\Omega'$ are not equal to $\sqrt{\Omega^2+\Delta^2}$ and the optimal squeezing does not lie on the line $\Omega = \Delta$.

Now let us consider the nondegenerate case, i.e., $\omega_{21} \neq 0.0$. When the interaction between the atomic transition from $|2\rangle$ to $|0\rangle$ and the laser is resonant, as shown above for the degenerate case, squeezing appears in the out-of-phase quadrature, and quantum interference can greatly enhance the squeezing when the excitation is weak as shown in Figs. 6(a) and 6(b), and results in an ultranarrow central peak with increasing the Rabi frequency, see Fig. 6(f). From the definition (14), we can show the following relation [13]:

$$S_{inc}(\omega_L+\omega)+S_{inc}(\omega_L-\omega) = :S(\omega,\theta): + :S\left(\omega,\theta+\frac{\pi}{2}\right):, \tag{42}$$

where $S_{inc}(\omega_L \pm \omega)$ is the incoherent part of the intensity spectrum of the resonance fluorescence. In the two-level case, Rice and Carmichael [13] showed that for weak excitation, squeezing in either the in-phase quadrature $(:S(\omega,0):<0)$ or the out-of-phase quadrature $[:S(\omega,\pi/2):<0]$ leads to the significant linewidth narrowing in the incoherent part of the fluorescent intensity spectrum. In the model under consideration, Zhou and Swain [10] showed that the incoherent fluorescent spectrum can develop an ultranarrow central line. However, this ultranarrow line results from quantum interference and is not relative to squeezing in either phase quadrature. Therefore, the ultranarrow central line in Fig. 6 is the remainder of the incoherent part of the intensity fluorescent spectrum.

For strong excitation intensities, the squeezing spectrum of the out-of-phase quadrature develops the Mollow triplet. As the degenerate case, however, two negative sideband peaks at the Rabi frequencies $\pm\Omega'$ appear in the squeezing spectrum of the in-phase quadrature. In Fig. 7, we show the maximum squeezing of the sidebands at the Rabi frequencies $\pm\Omega'$ as a function of the detuning Δ and the Rabi frequency Ω. Comparing Fig. 7(a) to Figs. 7(b) and 5, we see that the maximum squeezing points move to smaller values of the Rabi frequency because of the existence of the level splitting and quantum interference.

In the nondegenerate case, we notice that squeezing spectra are symmetrical about $\theta = 0$ when the detuning parameter $\Delta = \omega_{21}/2$, as shown in Fig. 8(a). In this case, the fluorescent light is swiched off if quantum interference takes place, as shown in Fig. 8(b). We find that squeezing spectra become nonsymmetrical when Δ departs from $\omega_{21}/2$ and the maximum squeezing occurs in the quadratures with phases $-\pi/2 \leqslant \theta \leqslant 0$ when Δ is smaller than $\omega_{21}/2$. In Fig. 9, squeezing spectra of various phase quadratures are given for $\Delta = 0.3 < \omega_{21}/2 = 0.5$. From these figures, we see that depending upon the detuning, quantum interference shifts the optimal phase quadrature for squeezing and greatly modifies the spectra.

IV. SUMMARY

The squeezing spectrum of resonance fluorescence of a three-level V-type atom, which is driven by a coherent field and coupled to vacuum modes of the electromagnetic field, is investigated. The influence of quantum interference between two decay pathways from the two excited levels to the common ground state on the spectra is emphasized. We find that for weak resonance excitation, squeezing always occurs in the out-of-phase quadrature of the resonance fluorescent field and the maximum squeezing appears in the component of the atomic transition frequency either with or without quantum interference. We show that quantum interference can greatly enhance the squeezing for some appropriate values of the Rabi frequency of the driving field. For weak off-resonance excitation, we find that quantum interference can change the optimal squeezing phase quadrature. For strong and far-off-resonance excitation, two sideband peaks of the spectra of the in-phase quadrature display strong squeezing, and quantum interference broadens the squeezing peaks. In the nondegenerate case, we also find that quantum interference can lead to an ultranarrow line in the squeezing spectrum of the out-of-phase quadrature, shift the optimal phase quadruture for squeezing, and greatly modify the shape of the spectra.

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