Exact calculation of the skyrmion lifetime in a ferromagnetic Bose-Einstein condensate

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The tunneling rate of a skyrmion in ferromagnetic spin-1/2 Bose-Einstein condensates through an off-centered potential barrier is calculated exactly with the periodic instanton method. The prefactor is shown to depend on the chemical potential of the core atoms, the level at which the atom tunnels. Our results can be readily extended to estimate the lifetime of other topological excitations in the condensate, such as vortices and monopoles.

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I. INTRODUCTION

Macroscopic quantum tunneling, the tunneling of a macroscopic variable of a macroscopic system, has recently received much attention in studies of Bose-Einstein condensation (BEC). The tunneling of a condensate through an optical lattice potential [1,2] provides an atomic physics analog of a Josephson-junction array, while in principle, the analog of a single junction can be realized by two condensates confined in a double-well potential [3,4]. The recent experimental success in all-optical trapping of an atomic condensate [5] opens the prospect of studies into the internal structure of spinor BECs, including the possibility of creating some topological excitations [6], such as skyrmions, monopoles, merons or axis-symmetric or non-axis-symmetric vortices both for antiferromagnetic and ferromagnetic condensates. Among various topological structures, the Mermin-Ho [7] and Anderson-Toulouse [8] coreless nonsingular vortices are demonstrated to be thermodynamically stable in ferromagnetic spinor Bose-Einstein condensates with the hyperfine state $F=1$ [9]. Skyrmions, which do not have an ordinary vortex core due to the spin degree of freedom, are also proposed in the spinor BEC [10–12] and are shown to be thermodynamically unstable objects without rotation [13,14]. Once created, the radius of such a skyrmion shrinks to zero, so that one must detect and manipulate it in the duration of its lifetime.

The skyrmion texture in a ferromagnetic spinor condensate can be described conveniently by a position-dependent spinor [14],

$$\zeta(\mathbf{r})=\exp\left\{-\frac{i}{S}\frac{\omega(r)}{r}\mathbf{r}\cdot\mathbf{S}\right\}\zeta^{Z}. \tag{1}$$

The constant spinor ζ^{Z} is the usual basis that diagonalizes the S_z component of the spin matrices $\mathbf{S}$, and $\omega(r)$ is a real function of radius r satisfying the boundary conditions $\omega(0)=2\pi$ and $\lim_{r\to\infty}\omega(r)=0$. For the skyrmion with size of the order of the correlation length $\xi=1/\sqrt{8\pi a n_\infty}$ or less, where n_∞ is the average atomic density and a the s-wave interatomic scattering length, the problem can be reduced to a nonlinear Schrödinger equation by an *Ansätz* for $\omega(r)$ with the gradient term $|\nabla\zeta(\mathbf{r})|^2$ in the Gross-Pitaevskii energy functional regarded as some external potential $V(r)=\hbar^2|\nabla\zeta(\mathbf{r})|^2/2m$. In the spin-1/2 case, for example, the latter takes the form [13]

$$V(r)=\frac{\hbar^2}{2m}\frac{32}{\lambda^2}\frac{(r/\lambda)^2[3+2(r/\lambda)^4+3(r/\lambda)^8]}{[1+(r/\lambda)^4]^4} \tag{2}$$

for an *Ansätz* $\omega(r)=4\cot^{-1}[(r/\lambda)^2]$, where the variational parameter λ corresponds physically to the size of the skyrmion. The lifetime of the skyrmion is estimated by employing a WKB expression for the tunneling rate $\Gamma=(\omega_0/2\pi)e^{-S_c/\hbar}$, S_c being the action through the barrier and ω_0 the characteristic frequency of the harmonic potential that was used to approximate the potential $V(r)$. Due to the inaccuracy of the prefactor in the tunneling rate, which makes it difficult to give a reliable result for the decay rate, more efficient methods are needed for the investigation of this problem.

The instanton method as a powerful tool for dealing with quantum tunneling phenomena has generally been used in the evaluation of the splitting of degenerate ground states or the escape rate from metastable ground states [15]. A method of evaluating quantum-mechanical tunneling at excited energy states has been developed recently by means of periodic instantons and bounces [16], which are characterized by nonzero energy and satisfy nonvacuum boundary conditions. Solvable models include level splittings for the double-well and sinh-Gordon potentials, decay rates for the inverted double-well and cubic potentials, and energy-band structures of the sine-Gordon and Lamé potentials [16,17]. The off-centered potential barriers serve as another class of physical systems, which permit analytical evaluation.

In this paper, we investigate the tunneling behavior of the skyrmion from the core to the outer region through an off-centered barrier. We first solve the equation of motion in the Euclidean version to find the classical configuration, which in our case is a bounce. In Sec. III we present the formalism of the periodic instanton theory for tunneling and calculate the decay rate exactly. The results obtained are applied to estimate the rate of shrinking of the skyrmion in the two-component ferromagnetic Bose-Einstein condensate. Finally, we summarize the main results.

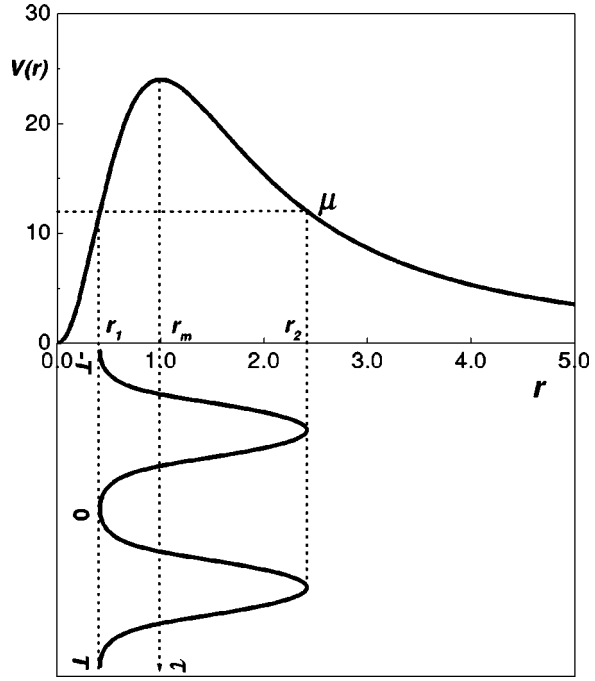

FIG. 1. The off-centered potential and the bounce configuration in two imaginary time periods. For the spin-1/2 ^{87}Rb condensate, the potential and the radius are in units of $\hbar^2/2m\xi^2$ and ξ, respectively, and the parameters are chosen as $A=96$ and $B=1$, where the size of the skyrmion (λ) used is approximately the corelation length ξ corresponding to 20 core atoms.

II. INSTANTONS FOR OFF-CENTERED POTENTIAL BARRIER

In this section we consider the instanton solution for the tunneling in an off-centered potential barrier as depicted in Fig. 1 in which the potential

$$V(r)=\frac{Ar^2}{(1+Br^2)^2} \tag{3}$$

takes into account all the main features of the real barriers in the skyrmion excitation in the condensate, both in spin-1/2 and spin-1 condensates, for any reasonable *Ansatz* $\omega(r)$. We observe here some essential conditions for this simplified model. First, as a function the *Ansatz* for ω should decrease monotonically from 2π to 0, since this will correspond to the smallest gradient energy for the spin deformations; correspondingly, this excludes any oscillation in the decrease of the potential $V(r)$ when r tends to $+\infty$. Furthermore, $V(r)$ should be an off-centered potential barrier with a maximum height $V(r_m)$ at $r=r_m$, and $V(0)=V(+\infty)=0$. Finally, the potential should be an even function of r, and to avoid the point $r=0$ becoming a singularity, we have $V''(0)>0$ so that the harmonic-oscillation frequency ω_0 can be well defined as $\sqrt{V''(0)/m}$. The barrier (3) is just the simplest form fulfilling the above requirements, with parameter A determining the barrier height and parameter B determining the position of the barrier:

$$r_m=\sqrt{1/B},\quad V_m=\frac{A}{4B},\quad \omega_0=\sqrt{2A/m}. \tag{4}$$

To estimate the lifetime of the skyrmion, we calculate the tunneling rate from the core to the outer region through a barrier, the core atoms having a chemical potential μ_{core} (hereafter abbreviated as μ). The first step of the instanton method is the so-called Wick rotation of a phase space corresponding to a transformation to imaginary time $\tau=it$. After the transformation, the Lagrangian is replaced by its Euclidean counterpart,

$$\mathcal{L}=\frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2+V(r). \tag{5}$$

The classical solution r_c which minimizes the corresponding Euclidean action satisfies the equation

$$\frac{1}{2}m\left(\frac{dr_c}{d\tau}\right)^2-V(r)=-E_{cl}, \tag{6}$$

which can be viewed as the equation of motion for a particle of mass m with energy $-E_{cl}$ in a potential $-V$. For the tunneling process in the condensate, we assume that the skyrmion has decreased to a size for which the barrier is so high that the overlap between the core atoms and the external atoms is exponentially small. The classical turning points on both sides of the barrier can be determined by the relation $V(r_{1,2})=\mu$, as suggested in Ref. [13]

$$r_{1,2}=\frac{\sqrt{A/\mu}}{2B}(1\mp\sqrt{1-4\mu B/A}),\quad 0<\mu<V_m. \tag{7}$$

The reason why we can handle a nonlinear problem by means of a linear equation of motion is that we discuss the tunneling behavior in the barrier region where the nonlinear interaction is negligibly small. Furthermore, the condensate at the ground state can be well described by a macroscopic wave function with unique phase just as in the single-particle case. However, there are obvious differences between the BEC tunneling system and the usual one-body problem, i.e., the nonlinear interaction contributes a finite chemical potential μ, which replaces the integration constant E_{cl} on the right-hand side of Eq. (6).

The classical configuration is a bounce that is the solution of Eq. (6) and can be expressed in an implicit form,

$$f(r_c)=\omega_c\tau. \tag{8}$$

Here we have assigned a characteristic frequency

$$\omega_c=\sqrt{\frac{2\mu B^2}{m}}, \tag{9}$$

and the function f takes the form

$$f(r_c)=\frac{1}{r_2}u_1+Br_2[E(u_1)-k^2\,\mathrm{sn}\,u_1\,\mathrm{cd}\,u_1], \tag{10}$$

where sn and cd are two Jacobian elliptic functions, $u_1 = F(\varphi,k)$ and $E(u_1)$ are the first and second kind of incomplete elliptic integrals with modulus $k=\sqrt{1-r_1^2/r_2^2}$, respectively [19], and

$$\varphi = \sin^{-1}\sqrt{\frac{r_2^2(r_c^2-r_1^2)}{r_c^2(r_2^2-r_1^2)}}. \tag{11}$$

The solution is subject to the following boundary conditions:

$$\tau=0,\quad r=r_1,$$

$$\tau=\pm T,\quad r=r_1, \tag{12}$$

$$\tau=\pm T/2,\quad r=r_2,$$

and exhibits periodic oscillation with imaginary time period

$$T=\frac{2}{\omega_c}\left(\frac{1}{r_2}K(k)+Br_2E(k)\right), \tag{13}$$

where $K(k)$ and $E(k)$ are the first and second kind of complete elliptic integrals with modulus k, respectively. In Fig. 1 we depict the periodic oscillation of this pseudoparticle in the two periods. A remarkable feature of this bounce configuration is that there is no vacuum analog as in the case of the simple cubic metastable potential, the latter describing the tunneling behavior of a particle located at the ground state. As the energy E_{cl} (or the chemical potential μ) approaches zero, the barrier will become infinitely thick and the particle confined in the core region will be stable, with no possibility to tunnel to the outer region.

III. EXACT CALCULATION OF THE DECAY RATE

The tunneling rate of the condensate core atoms was given by a simple expression of the form $\Gamma = Pe^{-W/\hbar}$, where P and W are coefficients that depend on the detailed form of the metastable potential. The quantity W appearing in the exponential is the Euclidean action of the bounce solution and gives the dominant contribution to the tunneling rate, while the prefactor P originates from the fluctuation around the classical configuration. For a rather rough estimate, P is often taken to be the attempt frequency $\omega_0/2\pi$, as was done in Ref. [13]. However, as we will show below, this simple evaluation is not accurate. This paper provides a powerful instanton tool to obtain this prefactor.

We recall for the sake of convenience the main ideas of the periodic instanton approach. Let us first denote the wave function of the core atom condensate with chemical potential μ by $\zeta(\mathbf{r})|\psi_\mu\rangle$, where $|\psi_\mu\rangle=\sqrt{n(\mathbf{r})}$ originates from the density and satisfies the Gross-Pitaevskii equation for single component,

$$H|\psi_\mu\rangle=\mu|\psi_\mu\rangle,\quad H=-\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{r})+g|\psi_\mu|^2. \tag{14}$$

The effective external potential $V(\mathbf{r})$ [14] comes from the gradient term of the spinor $|\nabla\zeta(\mathbf{r})|^2$, and the term with coupling constant g represents the strength of the interatomic interactions.

The tunneling effect leads to the decay of the metastable state. In the case under discussion, the nonconservation of an exponentially small probability current through the barrier requires that the chemical potential has an imaginary part proportional to the decay rate [18], $\Gamma=(2/\hbar)\mathrm{Im}\,\mu$. Consider the transition amplitude from the state $|\psi_\mu\rangle$ to itself due to quantum tunneling in Euclidean time period T. The amplitude is simply

$$A=\langle\psi_\mu|e^{-HT/\hbar}|\psi_\mu\rangle=e^{-\mu T/\hbar}. \tag{15}$$

In general, the transition amplitude is calculated with the help of the path-integral method as

$$A=\int\psi_\mu^*(r_f)\psi_\mu(r_i)\mathcal{K}(r_f,T;r_i,0)dr_i dr_f, \tag{16}$$

where $r_f=r_c(T), r_i=r_c(0)$ denote the end points of the bounce motion, which tend to the turning points r_1 [see the boundary condition, Eq. (12)]. The wave functions $\psi_\mu(r_i),\psi_\mu(r_f)$ in the barrier region are specified in the WKB approximation as [21]

$$\psi_\mu(r)=\frac{C}{\sqrt{|p|}}\exp\left(-\frac{1}{\hbar}\int_{r_1}^{r}p\,dr\right), \tag{17}$$

$$p=\sqrt{2m[\mu-V(r)]}, \tag{18}$$

with C a normalization constant to be determined below. The Feynman kernel is defined as the summation over all possible classical paths $r(\tau)$,

$$\mathcal{K}(r_f,T;r_i,0)=\int_{r_i}^{r_f}\mathcal{D}\{r\}\exp(-S/\hbar). \tag{19}$$

We know that the classical solution (8) that minimizes the action S gives rise to the major contribution to the above kernel integral, while the quantum fluctuation around it results in a prefactor P. In the period T the bounce, Eq. (8), completes one oscillation and crosses the barrier region twice, back and forth. The Euclidean action is thus calculated in this period as

$$S_E=\int\mathcal{L}(r,\dot{r})d\tau=\int_0^T\left[m\left(\frac{dr_c}{d\tau}\right)^2+\mu\right]d\tau=W+\mu T, \tag{20}$$

while the so-called abbreviated Euclidean action [20]

$$W=2\int_{r_1}^{r_2}dr\sqrt{2m[V(r)-\mu]} \tag{21}$$

can be expressed in terms of elliptic integrals,

$$W=\frac{4}{\omega_c}\left[\frac{Ar_1^2\Pi(\alpha^2,k)}{r_2(1+Br_1^2)}-\frac{\mu}{r_2}K(k)-\mu Br_2E(k)\right], \quad (22)$$

with $\Pi(\alpha^2,k)$ the complete elliptic integral with the parameter

$$\alpha^2=\frac{k^2}{1+Br_1^2}. \quad (23)$$

It is obvious from the potential that $A,B>0$, so $0<\alpha^2<k^2$, the third elliptic integral is complete and belongs to the case III [19].

The imaginary part of the chemical potential can be derived by considering the amplitude A as the sum of contributions from any number of bounces [16]. The zero-bounce contribution results in the real part of the chemical potential,

$$A^{(0)}=e^{-\mu T/\hbar}. \quad (24)$$

The one-bounce contribution comes from the classical configuration with period T, and can be obtained by expanding the kernel (19) around the bounce (8)

$$A^{(1)}=-iT\frac{C^2}{m}e^{-W/\hbar}e^{-\mu T/\hbar}. \quad (25)$$

Generalizing to the case of n bounces straightforwardly, i.e., assuming the pseudoparticle completing n oscillations in the period T, one has

$$A^{(n)}=(-i)^n\frac{T^n}{n!}\left(\frac{C^2}{m}\right)^n e^{-nW/\hbar}e^{-\mu T/\hbar}. \quad (26)$$

The total transition amplitude is given by the sum over all bounce contributions,

$$A=\sum_n A^{(n)}=e^{-\mu T/\hbar}\exp\left(-iT\frac{C^2}{m}e^{-W/\hbar}\right). \quad (27)$$

The imaginary part of the chemical potential is obtained by comparing Eq. (27) with Eq. (15),

$$\mathrm{Im}\,\mu=\frac{\hbar C^2}{m}e^{-W/\hbar}, \quad (28)$$

which results in the decay rate

$$\Gamma=\frac{1}{2}\frac{2}{\hbar}\mathrm{Im}\,\mu=\frac{C^2}{m}e^{-W/\hbar}, \quad (29)$$

where the factor 1/2 comes from the analytical continuation. Physically, this results from the assumption in the decay problem (and not in the macroscopic quantum coherence problem) that the wave that has tunneled will never return. Mathematically, it is due to the fact that the deformed contour runs from 0 to $i\infty$, and not from $-i\infty$ to $i\infty$ [15].

The constant C can be determined from the normalization of the wave function in the classically accessible region, which is connected with those in the barrier region through [21]

$$\frac{C}{\sqrt{|p|}}\exp\left(-\frac{1}{\hbar}\int_{r_1}^{r}pdr\right)\rightarrow\frac{2C}{\sqrt{|p|}}\cos\left(-\frac{1}{\hbar}\int_{r_1}^{r}pdr-\frac{\pi}{4}\right). \quad (30)$$

We restrict the integration in the classically accessible region, i.e., in the potential well, $r<r_1$, since outside of this range ψ decreases exponentially. Because the argument of the cosine in the wave function is a rapidly varying function, we can, with sufficient accuracy, replace the squared cosine by its mean value 1/2. This gives

$$C^2=\frac{m\omega}{2\pi}. \quad (31)$$

Inserting this into the decay rate we have

$$\Gamma=\frac{\omega}{2\pi}\exp\left[-\frac{W}{\hbar}\right], \quad (32)$$

where ω is the frequency of the classical periodic motion,

$$\omega(\mu)=\frac{2\pi}{2m\int\frac{dx}{p}}=\frac{\pi}{\sqrt{2m}\int_0^{r_1}\frac{dr}{\sqrt{\mu-V(r)}}} \quad (33)$$

and can be calculated as

$$\omega(\mu)=\omega_c\frac{\pi}{2}\left[\left(\frac{1}{r_2}+Br_2\right)K(k')-Br_2E(k')\right]^{-1} \quad (34)$$

with the complementary modulus $k'=\sqrt{1-k^2}$. It must be recalled that the frequency ω is in general different for different levels, being a function of the chemical potential. We find that our expression for the decay rate equation (32) is more accurate than that of Refs. [13,14], i.e., in the prefactor a chemical potential dependent frequency replaces the constant attempt frequency ω_0. In Fig. 2 we show the dependence of this frequency on the chemical potential; it decreases from ω_0 as the chemical potential increases from 0. This factor suppresses the tunneling rate greatly when the chemical potential approaches the barrier top, as shown in the figure, which would be expected to increase the lifetime of the skyrmion.

IV. NUMERICAL RESULTS FOR SKYRMIONS

The skyrmions in a ferromagnetic condensate are energetically unstable as shown in Ref. [14]. The time scale on which the skyrmion shrinks may be evaluated for two cases: For a large skyrmion, its size decreases at a rate $\Gamma_{large}\approx 18\mathrm{sec}^{-1}\xi/\lambda$ for ^{87}Rb spin-1/2 condensate of central density 10^{11} cm^{-3} and realistic experimental conditions. For skyrmions with sizes of the order or less than the correlation length ξ, the shrinking rate is determined by the tunneling

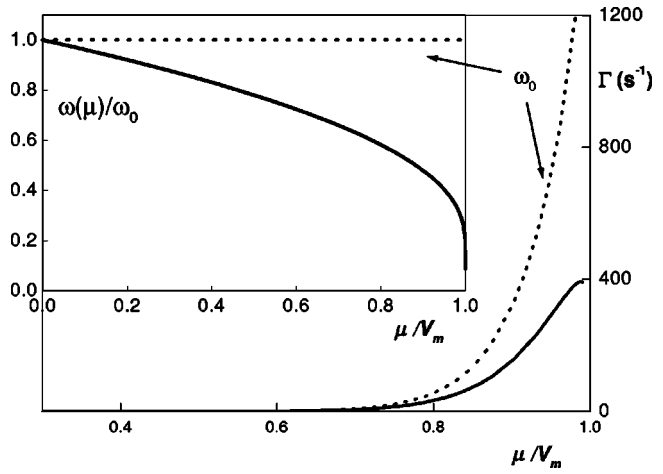

FIG. 2. The decay rate as a function of the chemical potential. Inset represents the chemical-potential-dependent frequency ω. The solid curves represent our exact result while the dotted curves correspond to the case for constant attempt frequency ω_0. All curves are calculated for the parameter of a ^{87}Rb spinor condensate, and the chemical potential μ is given in units of the barrier height V_m.

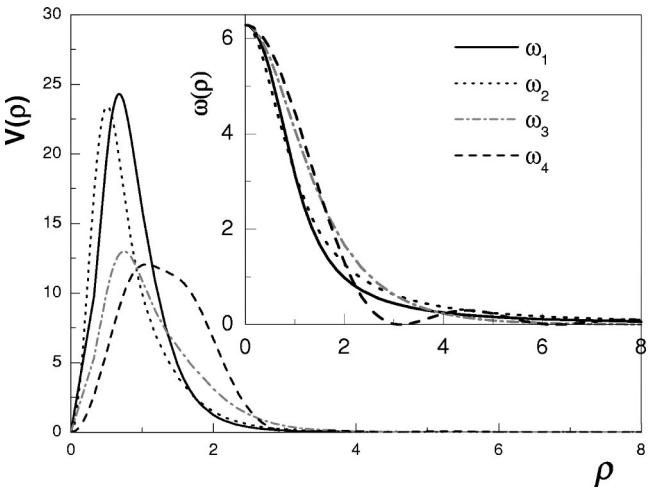

FIG. 3. *Ansätze* for $\omega(\rho)$ and the corresponding potential barriers $V(\rho)$. The potential and the radius are again in units of $\hbar^2/2m\xi^2$ and ξ, respectively.

rate from the core of the skyrmion to the outer region. In previous studies [13,14] the authors estimated roughly the lifetime of this small skyrmion due to the tunneling process employing a WKB expression for the tunneling rate. In this section we thus reconsider the lifetime using the result derived above, with the modification originating from the prefactor included.

With an *Ansatz* for $\omega(\rho)$, the problem is simplified to a nonlinear Schrödinger equation with the external potential of the off-centered form. For a different functional behavior of $\omega(\rho)$, it turns out that the effective potential will not be very different, as long as the *Ansatz* satisfies the boundary conditions $\omega(0)=2\pi$ and $\lim_{\rho\to\infty}\omega(\rho)=0$ and falls off monotonically. Here we compute the decay rates of different skyrmion textures with the same size, by taking into account two *Ansätze* that were proposed in Ref. [14] as trial functions for simplifying the pair of nonlinear and coupled equations [Eqs. (8) and (9) in Ref. [14]], namely,

$$\omega_1(\rho)=4\cot^{-1}[(\rho/\lambda)^2], \tag{35}$$

$$\omega_2(\rho)=\frac{2\pi}{1+(\rho/\lambda)^2}, \tag{36}$$

where $\rho=r/\xi$, and λ corresponds to the size of the skyrmion and is also given in units of ξ. Considering the large-distance behavior of the coupling equations for $n(\rho)$ and $\omega(\rho)$, we see that for large skyrmions, the density fluctuations scale as $1/\rho^2$, and so should the *Ansätze* for $\omega(\rho)$. This is the reason why we would arrive at a nonphysical result for a seemingly reasonable *Ansatz* $\omega_3(\rho)=2\pi\,\mathrm{sech}(\rho/\lambda)$. We also check that the nonmonotonic behavior, i.e., oscillations in the falling of $\omega(\rho)$, for example, $\omega_4(\rho)=2\pi[\sin(\rho/\lambda)/\rho/\lambda]^2$, will inevitably lead to singularities in the density profile, though the effective potential holds an off-centered form. We show in Fig. 3 these *Ansätze* for $\omega(\rho)$ and their corresponding effective potentials $V(\rho)$.

One important parameter we should determine is the chemical potential of the core atoms, because we should know at which level the atom will tunnel out. In principle, one should solve the two coupled nonlinear differential equations and derive the density profile and the spinor [or the function $\omega(\rho)$]. As already mentioned above, we employ alternatively a simple approach, i.e., by introducing the *Ansatz* for $\omega(\rho)$. From the resulting density distribution we calculate the energy for a particular value of λ, then the core chemical potential can be calculated numerically by differentiating the energy with respect to the number of core atoms. Performing the calculation within a Thomas-Fermi approximation, which means that in the expression for the energy we neglect the kinetic-energy term, we finally obtain the chemical potential of the core atom μ for different values of λ and *Ansatz*.

For the *Ansätze* ω_1 and ω_2 we calculate the corresponding chemical potential for $\lambda=\xi$. The shrinking rates of the corresponding skyrmions are calculated according to our decay rate expression Eq. (32), with the action given by Eq. (21) and the prefactor given by Eq. (33). Figure 4 gives the tunneling rates as a function of the number of core atoms. The calculation was performed for a ^{87}Rb spin-1/2 condensate with a scattering length of $a=5.4$ nm.

We observe that [22] the correction resulting from the accurate prefactor $\omega(\mu)$ for ω_1 is minor but significant for ω_2. In Fig. 2 we could generally take the range of the chemical potential from 0 to V_m. Unlike the situation in a harmonic trap where the chemical potential μ increases with the number of condensed atoms as $N^{2/5}$ in the Thomas-Fermi approximation, in our case μ decreases with N instead. This is because of the fact that the trap frequency (for ω_1) ω_0 is inversely proportional to the equilibrium skyrmion width λ_0, which in turn increases with N (apparently faster than $N^{1/5}$). Here λ_0 is determined from minimizing the total energy, tak-

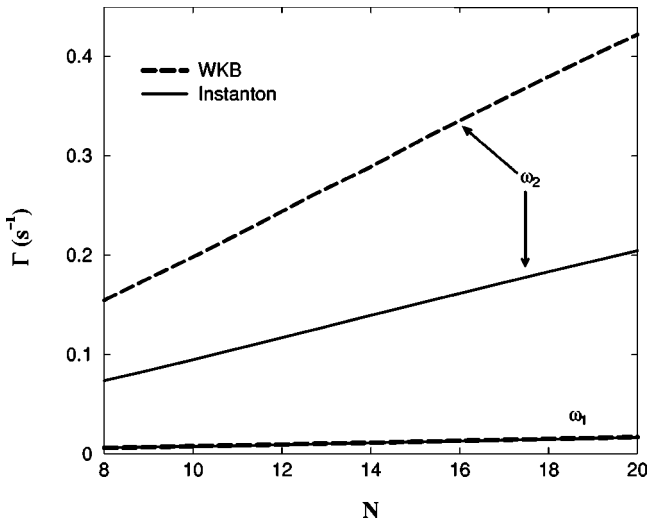

FIG. 4. The shrinking rate of skyrmions as a function of the number of core atoms for ω_1 and ω_2. The calculation was performed for a ^{87}Rb spin-1/2 condensate with a scattering length of $a=5.4$ nm. The dashed lines are the WKB calculations in Ref. [13], while the solid lines show our results from the periodic instanton method.

ing into account the outer region of the skyrmion. This restricts us to a special domain of μ. For numbers of core atoms ranging from 1 to 20, μ/V_m ranges roughly from 0.16 to 0.04. In this interval, $\omega(\mu)/\omega_0$ starts from 0.985 for $N_{core}=1$ and ends at 0.998 for $N_{core}=20$, which are almost indistinguishable in Fig. 4. However, for *Ansatz* ω_2, the chemical potential (in units of $\hbar^2/2m\xi^2$) starts at 42 for one core atom and ends at 9.3 for 20 core atoms. The corresponding correction is shown in Fig. 4.

Till now there is still no clear experimental evidence for the skyrmions in the condensate. From the above calculations we see that the result for the decay rate depends crucially on the detailed form of the *Ansatz* ω. It remains a challenging task to solve the coupled nonlinear equations numerically, and to compare the results with those above.

V. CONCLUSION

We present here an accurate calculation of the tunneling rate for a class of off-centered potentials with a periodic instanton method. Apart from its application to the study of the stability of the skyrmion excitation in the two-component ferromagnetic condensate, the bounce for the off-centered potential barrier is itself a novel configuration from the viewpoint of the scalar field theory. The exact prefactor of the decay rate has been calculated, and we found that it depends on the chemical potential at the level of the atoms tunneling to the outer region. This modifies the result for the rough estimate of the lifetime by a constant attempt frequency ω_0. One can easily find some similar off-centered potentials in other topological excitations, such as vortices, monopoles, etc. Our periodic instanton formalism can be extended to the investigation of the lifetime and tunneling behavior in these systems. Further studies should include the properties of the quantum-classical transition of the decay rate when the chemical potential increases and surpasses the barrier height.

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[1] B. P. Anderson and M. A. Kasevich, Science **282**, 1686 (1998).
[2] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293**, 843 (2001).
[3] M. R. Andrews, C. G. Townsend, H. J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **275**, 637 (1997).
[4] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).
[5] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. **80**, 2027 (1998); M. D. Barrett, J. A. Sauer, and M. S. Chapman, *ibid.* **87**, 010404 (2001).
[6] Tin-Lun Ho, Phys. Rev. Lett. **81**, 742 (1998); H. T. C. Stoof, E. Vliegen, and U. Al Khawaja, *ibid.* **87**, 120407 (2001); S.-K. Yip, *ibid.* **83**, 4677 (1999); J.-P. Martikainen, A. Collin, and K.-A. Suominen, *ibid.* **88**, 090404 (2002).
[7] N. D. Mermin and Tin-Lun Ho, Phys. Rev. Lett. **36**, 594 (1976).
[8] P. W. Anderson and G. Toulouse, Phys. Rev. Lett. **38**, 508 (1977).
[9] T. Mizushima, K. Machida, and T. Kita, Phys. Rev. Lett. **89**, 030401 (2002).
[10] J. Ruostekoski and J. R. Anglin, Phys. Rev. Lett. **86**, 3934 (2001).
[11] Richard A. Battye, N. R. Cooper, and Paul M. Sutcliffe, Phys. Rev. Lett. **88**, 080401 (2002).
[12] K.-P. Marzlin, W. Zhang, and B. C. Sanders, Phys. Rev. A **62**, 013602 (2000).
[13] U. Al. Khawaja and H. T. C. Stoof, Nature (London) **411**, 918 (2001).
[14] U. Al. Khawaja and H. T. C. Stoof, Phys. Rev. A **64**, 043612 (2001).
[15] S. Coleman, Phys. Rev. D **15**, 2929 (1977); C. G. Callan Jr. and S. Coleman, Phys. Rev. D **16**, 1762 (1977); in *The Whys of Subnuclear Physics* (Plenum, New York, 1979), p. 805; in *Aspects of Symmetry* (Cambridge University Press, Cambridge, England, 1985), Chap. 7.
[16] J.-Q. Liang and H. J. W. Müller-Kirsten, Phys. Rev. D **46**, 4685 (1992); **50**, 6519 (1994); **51**, 718 (1995).
[17] Y.-B. Zhang, Y.-H. Nie, S.-P. Kou, J.-Q. Liang, H. J. W.

Müller-Kirsten, and F.-C. Pu, Phys. Lett. A **253**, 345 (1999); Y.-B. Zhang, J.-Q. Liang, and F.-C. Pu, Acta Phys. Sin. (Overseas Ed.) **7**, 510 (1998); H. J. W. Müller-Kirsten, J.-Z. Zhang, and Y.-B. Zhang, J. High Energy Phys. **11**, 011 (2001).

[18] I. Affleck, Phys. Rev. Lett. **46**, 388 (1981).

[19] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Scientists*, 2nd ed. (Springer, Berlin, 1971).

[20] U. Weiss, *Quantum Dissipative Systems*, 2nd ed. (World Scientific, Singapore, 1999).

[21] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed. (Pergamon, New York, 1977).

[22] U. Al. Khawaja and H. T. C. Stoof (private communication).