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Lepton pair production in heavy-ion collisions in perturbation theory

E. Bartoš,* S. R. Gevorkyan,† and E. A. Kuraev
Joint Institute for Nuclear Research, 141980 Dubna, Russia

N. N. Nikolaev‡
L. D. Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia
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We derive the first terms in the amplitude of lepton pair production in the Coulomb fields of two relativistic heavy ions. Using the Sudakov technique, which simplifies the calculations in momentum space for the processes at high energies, we get compact analytical expressions for the differential cross section of the process under consideration in the lowest order in the fine-structure constant (Born approximation) valid for any momentum transfer and in a wide kinematic region for produced particles. Exploiting the same technique, we consider the next terms of the perturbation series (up to the fourth order in the fine-structure constant) and investigate their energy dependence and limiting cases. It has been shown that taking into account all relevant terms in the corresponding order, one obtains expressions that are gauge invariant and finite. We estimate the contribution of the Coulomb corrections to the total cross section and discuss the cancellations of the different terms in the total cross section.

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I. INTRODUCTION

The process of lepton pair production in collision of relativistic heavy ions has attracted increased interest in the past years, mainly connected with the operation of RHIC (relativistic heavy ion collider) (Lorentz factor $\gamma = E/M = 100$) and LHC (large hadron collider) ($\gamma = 3000$). The total cross section of lepton pair production in the collision of two relativistic particles

$$A + B \rightarrow e^+ e^- + A + B \tag{1}$$

in its lowest order in the fine-structure constant $\alpha = e^2/4\pi = \frac{1}{137}$ has been known long ago [1,2]. Over the last decade a lot of work has been done to investigate this process for the case when A and B are heavy ions (see, e.g., Ref. [3] and references therein). When heavy ions collide at relativistic velocities, their electromagnetic fields are Lorentz contracted and the process (1) can be viewed as pair production in two external fields which can be represented by classical electromagnetic potentials. Such an approach allows one [4,5] to investigate the impact parameter dependence of this process and calculate its cross section with higher accuracy than predicted by the widely exploited Weizsäcker-Williams (WW) approximation. These works are based on the Born approximation (see Fig. 1), but even in this case the obtained expressions are complex, so that certain ‘‘state of art’’ efforts in their computing are required [6].

The topical problem is to take into account the Coulomb corrections (CC) to the amplitude of this process (see Figs. 2 and 3). In spite of the fact that CC are determined by the powers of the fine-structure constant α, it enters the amplitude in combination $(\alpha Z_1)^{n_1} (\alpha Z_2)^{n_2} (\alpha Z_1 Z_2)^{n_3}$, where Z_1, Z_2 are the colliding ion charge numbers, $n_1 + n_2 \geqslant 2$ is the number of exchanged photons between nuclei and the produced pair, and $n_3 \geqslant 0$ is the number of photons exchanged between the colliding nuclei. This general case was investigated in detail in series of papers (see Ref. [7] and references therein). For the heavy-ion collisions, the parameter αZ is of the order of unity and the problem of taking into account the effect of CC arises. The exact knowledge of the total yield of lepton pairs in heavy-ion collisions is an important issue because the pair production has a huge background in experiments with relativistic heavy ions. For example, the total cross section for the process (1) at RHIC energies is tens of kilobarns for heavy-ion collisions. At LHC energies, this quantity becomes hundreds of kilobarns according to the Racah formula [2]. Moreover, the intensive pair production can destroy the ion beams circulating in the accelerator as a result of electron capture by a heavy ion (see, e.g., Refs. [8,9]).

Recently, the problem of CC in the process (1) has been investigated in a number of papers [10–13] with the unex-

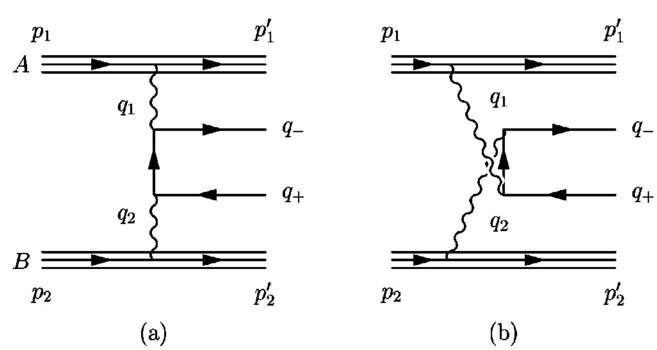

FIG. 1. The lowest-order Feynman diagrams for the two-photon pair production in the process $A + B \rightarrow A + B + e^+ e^-$.

*Present address: Department of Theoretical Physics, Comenius University, 84248 Bratislava, Slovakia.

†Present address: Yerevan Physics Institute, 375036 Yerevan, Armenia.

‡Present address: Institut fur Kernphysik, Forshchungszentrum Jülich, D-52425 Jülich, Germany.

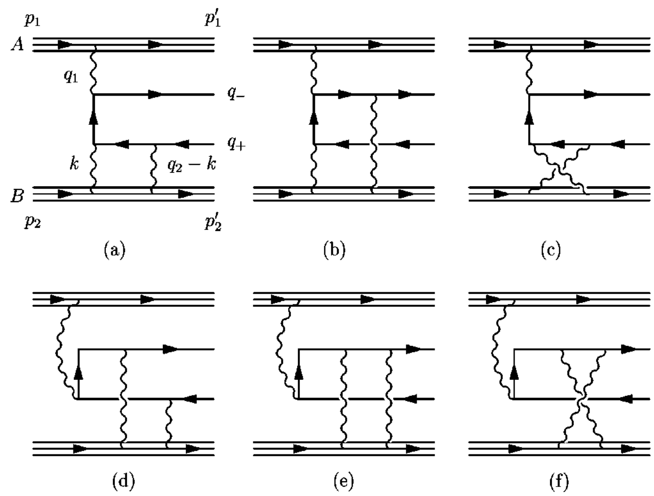

FIG. 2. Gauge invariant set of Feynman diagrams for spin-triplet production which determines the matrix element $M_{(2)}^{(1)}$.

pected result. The authors [11,12] solved the Dirac equation for an electron in the Coulomb field of two highly relativistic nuclei. Having used the crossing symmetry property, they connected the amplitude for electron scattering in the Coulomb fields of two heavy ions with the amplitude for pair production [28]. As a result the total cross section of the process (1) turns out to coincide with its Born approximation, so that CC do not have any impact on the total lepton pair yield. The same result has been obtained in Ref. [13], in which the authors used the eikonal approximation for electron scattering at high energies. They took advantage of the common wisdom that the interaction of lepton pair with Coulomb fields of two highly relativistic nuclei can be represented as a product of the eikonal amplitudes for interaction of electron and positron with the nucleus *A* and *B* separately. Such an approach allows one to take into account the contribution of CC in the amplitude of the process (1) with the above quoted result: the total cross section exactly coincides with the Born term. These works induced a series of critical papers [15,16], where it was explained why this result was so unexpected and how assumptions made through its derivation could affect this result.

As has been observed in Ref. [15], the main contribution to the CC in the process (1) comes from the case when one of the colliding ions radiates a single photon and the produced pair interacts with another nucleus by means of an arbitrary number of photons. In fact, the contributions of such type can be obtained with the use of the common WW approximation that leads to the appearance of large logarithms in energy for such terms that cannot be canceled by the next order of CC where these logarithms are absent. On the other hand, it has been shown [16] that the change in the order of integration in regularized integrals (the Coulomb phase for electron scattering diverges, so it needs regularization) completely changes the final result.

In Ref. [12], the two approaches employed in investigation of this problem were analyzed in detail, with the conclusion that the perturbation approach in fact corresponds to the exclusive process of the coherent production of lepton pairs in the collisions of relativistic ions [i.e., reaction (1)]. As to the approach based on the retarded solutions of the Dirac equation, it gives the inclusive (in fact, it is the average multiplicity in the terms of definitions [12]) cross section for production of an arbitrary number of lepton pairs, i.e., in this way one obtains the contribution of inclusive pair production. The reason for such an unexpected result, from our point of view, is the inapplicability of the crossing symmetry property to the higher-order (beyond the Born approximation) terms, the fact that is known long ago (see, e.g., Ref. [17]) Thus, the issue of calculation of the CC in the correct way in the process (1) is an open task up to now, and it turns out to be a more complicated problem than it seems at a glance. All this stimulated us to investigate the CC contribution in the perturbative expansion of the amplitude for exclusive process under discussion in more detail with an aim to understand how one can take into account the CC in heavy-ion collisions in the framework of perturbative QED. Our approach in the investigating of exclusive process differs from the one used in Ref. [12] where the summation over multiple pairs was implied.

Our paper is organized as follows. In the following section, we present the amplitude for lepton pair creation in the collision of relativistic particles in the Born approximation. We are restricted by the kinematics in which the pair production does not influence the kinematics of the colliding particles. Moreover, we supposed that transfer momenta between the produced pair and initial particles are much smaller in comparison with the energy of the initial particles, e.g., we consider the case of their quasielastic scattering. We use the powerful Sudakov technique for the evaluation of the Feynman diagrams in the momentum representation, which allows one to obtain the results with power accuracy in energy. Compact analytic expressions have been obtained for the Born amplitude and differential cross section of the process (1) which allows one to calculate it in a wide interval of kinematic variables. In the limiting case of quasireal photons, the well known results of the WW approximation can be easily obtained from these expressions.

In Sec. III, we consider all possibilities for the pair production by three-photon exchange, the so-called amplitudes of the type $M_{(2)}^{(1)}$. We present the general formula for its contribution to the pair production amplitude and the relevant cross section. Our result appears to be the generalization of the known result to the amplitude of the process of pair photoproduction in the odd charge state derived in Ref. [18]. We have carried out the numerical estimate for the rel-

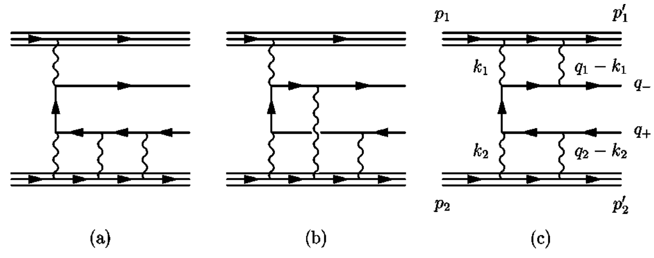

FIG. 3. Some Feynman diagrams for the four-photon pair production in the process $A+B\rightarrow A+B+e^+e^-$.

evant contribution to the total cross section in the main (double logarithmic) approximation.

In Sec. IV, we present the results of the amplitude computation and the corresponding contributions to the cross section from processes of the two-photon exchange between each ion and the created pair (fourth-order terms in the fine-structure constant), and the case when one of the ions is attached to the pair by a single photon and another ion is connected with a pair by three photons in all possible ways. In the last case, our result appears to be generalization of the known result in the case of photoproduction of the pair in an even charged state [19]. We show that in any order the obtained expressions are infrared finite. We analyze the main contributions from the obtained CC to the total cross section and show that a number of remarkable cancellations between the different CC and their interference with the Born term take place. In Sec. V, we shortly listed the main results obtained in the paper. In Appendix A, we give the results of analytic integration on loop momenta for spin-singlet and spin-triplet parts of CC, which allows one to obtain the first CC in the analytic form. In Appendix B, we present the spin structures that determine the fourth-order terms in the amplitude of the process under consideration.

II. THE BORN APPROXIMATION

We are interested in the exclusive coherent process of lepton pair production in the collision of two relativistic nuclei with charge numbers Z_1, Z_2,

$$A(p_1)+B(p_2)\to e^+(q_+)+e^-(q_-)+A(p_1')+B(p_2'). \tag{2}$$

We define the usual center-of-mass system (c.m.s.) total energy of colliding nuclei $s=(p_1+p_2)^2=4\gamma_1\gamma_2 m_1 m_2$, where m_i and $\gamma_i=E_i/m_i$ are the masses and Lorentz factors of the colliding nuclei. Keeping in mind the fast decrease of the $\gamma^*\gamma^*\to e^+e^-$ cross section with the transfer momentum, i.e., the photon virtuality, and with the invariant mass of the pair $s_p=(q_+ + q_-)^2$, we will work in the kinematics,

$$s=(p_1+p_2)^2 \gg |q_1^2|,\quad |q_2^2|,\quad s_p .$$

Later on we calculate the amplitudes neglecting the pieces of the order m_i^2/s, so our results are valid for relativistic particles with power accuracy. The total cross section of the reaction (2) in the lowest order in the fine-structure constant (Born approximation) is known long time ago [1,2]. Nevertheless, we carry out the derivation of the amplitude and differential cross section in this approximation using the Sudakov technique that is very useful for the calculation of Feynman diagrams (FD) for the processes at high energies. This allow us to illustrate the method and approximations to be used later on in the CC derivation. Moreover, we will obtain compact analytic expressions that are valid not only for heavy-ion collisions, but for lepton pair production in the Coulomb fields of any relativistic particles (hadrons, leptons).

The lowest-order FD describing the process (2) are depicted in Fig. 1. The corresponding amplitude reads

$$M_{(1)}^{(1)}=-i\frac{(4\pi\alpha)^2 Z_1 Z_2}{q_1^2 q_2^2}[\overline{u(p_1')}\gamma_\mu u(p_1)\overline{u(p_2')}\gamma_\nu u(p_2) \times g_{\mu\alpha}g_{\nu\beta}\overline{u(q_-)}T_{\alpha\beta}v(q_+)], \tag{3}$$

$$T_{\alpha\beta}=\gamma_\beta\frac{\hat{q}_1-\hat{q}_+ +m}{(q_1-q_+)^2-m^2}\gamma_\alpha+\gamma_\alpha\frac{\hat{q}_2-\hat{q}_+ +m}{(q_2-q_+)^2-m^2}\gamma_\beta ,$$

$$\hat{q}_i=\gamma_\mu q_i^\mu .$$

Using the Gribov decomposition of the metric tensor $g_{\mu\nu} = g_{\mu\nu}^\perp + (2/s)(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu)$, it is easy to show that at high energies the main contribution is given by its longitudinal part $g_{\mu\alpha}g_{\nu\beta}\to(2/s)^2 p_1^\nu p_2^\mu p_1^\alpha p_2^\beta$. As a result, expression (3) can be cast in the form

$$M_{(1)}^{(1)}=-is\frac{(8\pi\alpha)^2 Z_1 Z_2}{q_1^2 q_2^2}\overline{u}(q_-)R_{(1)}^{(1)}v(q_+)N_1 N_2 ,$$

$$R_{(1)}^{(1)}=\frac{1}{s}p_1^\mu p_2^\nu T_{\mu\nu},\quad N_1=\frac{1}{s}\overline{u}(p_1')\hat{p}_2 u(p_1), \tag{4}$$

$$N_2=\frac{1}{s}\overline{u}(p_2')\hat{p}_1 u(p_2).$$

For a spinless projectile and a target, as well as for colliding fermions, at definite chirality state $|N_1|=|N_2|=1$, so that later on [29] we will set $N_1=N_2=1$. It should be noted that if we consider the heavy-ion dimensions, it is sufficient to multiply the amplitude (4) by the colliding ion form factors $F(q_i^2)$.

In what follows, we will use the standard Sudakov expansion of all the momenta in the two almost light-light vectors $\tilde{p}_1$, $\tilde{p}_2$ and in the two-dimensional transverse component $q_\perp p_1 = q_\perp p_2 = 0$,

$$q_1=\alpha_1\tilde{p}_2+\beta_1\tilde{p}_1+q_{1\perp},\quad q_2=\alpha_2\tilde{p}_2+\beta_2\tilde{p}_1+q_{2\perp},$$

$$q_\pm=\alpha_\pm\tilde{p}_2+\beta_\pm\tilde{p}_1+q_{\pm\perp}, \tag{5}$$

$$\tilde{p}_1=p_1-p_2\frac{p_1^2}{s},\quad \tilde{p}_2=p_2-p_1\frac{p_2^2}{s},\quad \tilde{p}_1^2=\tilde{p}_2^2=O\left(\frac{m^6}{s^2}\right),$$

$$s\approx 2p_1p_2\approx 2\tilde{p}_1\tilde{p}_2 .$$

From the on-mass shell condition, for instance $p_1'^2=(p_1-q_1)^2=m_1^2$, $p_2'^2=(p_2-q_2)^2=m_2^2$, one can easily see that $\beta_1\sim\alpha_2\sim(m/\sqrt{s})$, $\alpha_1\sim\beta_2\sim(q^2/s)$, which allows one to neglect, where it is possible, the values of α_1, β_2 which greatly simplify calculations and allow the results to be obtained with power accuracy. In this approximation, we have

$$q_1\approx\beta_1\tilde{p}_1+q_{1\perp},\quad q_2\approx\alpha_2\tilde{p}_2+q_{2\perp}, \tag{6}$$

$$-q_1^2=\frac{\mathbf{q}_1{}^2+\beta_1^2m_1^2}{1-\beta_1},\quad -q_2^2=\frac{\mathbf{q}_2{}^2+\alpha_2^2m_2^2}{1-\alpha_2},$$

$$s_p=(q_++q_-)^2=(q_1+q_2)^2\approx\alpha_2\beta_1 s-(\mathbf{q}_1+\mathbf{q}_2)^2.$$

Hereafter $\mathbf{q}_i$ always stands for $q_{i\perp}$, etc. The conservation laws provide the relations among the introduced variables and thus the limits on the β_1, α_2 variation,

$$\beta_1=\beta_-+\beta_+,\quad \alpha_2=\alpha_-+\alpha_+,\quad \mathbf{q}_1+\mathbf{q}_2=\mathbf{q}_-+\mathbf{q}_+,\tag{7}$$

$$\frac{s_p+(\mathbf{q}_1+\mathbf{q}_2)^2}{s}<\beta_1\sim\alpha_2<1.$$

As we shall see later, the production amplitudes take a simple form in terms of the following variables:

$$z_\pm=\beta_\pm/\beta_1,\quad z_-=1-z_+,\quad \mathbf{k}_\pm=\mathbf{q}_\pm-z_\pm\mathbf{q}_1.\tag{8}$$

In what follows, we will work in the kinematics, where

$$\frac{m^2}{s}\ll\alpha_2\approx\beta_1\ll 1,$$

which gives the main contribution to the total cross section (see, e.g., Ref. [7]). In the cms frame of colliding beams, this corresponds to the production of low-energy pairs emitted at arbitrary angles. As a result, expression (4) can be represented as

$$M_{(1)}^{(1)}=-is\frac{(8\pi\alpha)^2}{(\mathbf{q}_1{}^2+\beta_1^2m_1^2)(\mathbf{q}_2{}^2+\alpha_2^2m_2^2)}\times Z_1Z_2\overline{u}(q_-)R_{(1)}^{(1)}v(q_+).\tag{9}$$

It is convenient to use the gauge invariant condition $q_1^\mu T_{\mu\nu}=0$ which, in view of Eq. (5), entails

$$p_1^\nu T_{\nu\mu}=-\frac{q_{1\perp}^\nu}{\beta_1}T_{\nu\mu}\equiv-\frac{|\mathbf{q}_1|}{\beta_1}e_1^\nu T_{\nu\mu},\quad e_1^\nu=\frac{q_1^\nu}{|\mathbf{q}_1|},\tag{10}$$

providing the important property $R_{(1)}^{(1)}\to 0$ at $\mathbf{q}_1\to 0$. Substituting this relation into Eq. (4) and after a bit of Dirac algebra, we find

$$R_{(1)}^{(1)}=\frac{|\mathbf{q}_1|}{s\beta_1}[m\hat{e}_1R_1+\hat{e}_1\hat{Q}_1+2z_+\mathbf{Q}_1\mathbf{e}_1+2|\mathbf{q}_1|z_+z_-R_1]\hat{p}_2,\tag{11a}$$

$$R_1=S(\mathbf{k}_-)-S(\mathbf{k}_+),\quad \mathbf{Q}_1=\mathbf{k}_-S(\mathbf{k}_-)+\mathbf{k}_+S(\mathbf{k}_+),$$

$$S(\mathbf{k})=\frac{1}{\mathbf{k}^2+z_+z_-\mathbf{q}_1{}^2+m^2}.\tag{11b}$$

In Eq. (11a), replacing $\mathbf{q}_1=\mathbf{q}_++\mathbf{q}_--\mathbf{q}_2$, it is easy to see that it also vanishes when $\mathbf{q}_2\to 0$.

For completeness, we cite here an alternative expression for the amplitude that can be obtained if one uses the gauge invariant constraint by another photon $q_2^\mu T_{\mu\nu}=0$,

$$R_{(1)}^{(1)}=\frac{|\mathbf{q}_2|}{s\alpha_2}[m\hat{e}_2R_1'+\hat{e}_2\hat{Q}_1'+2y_+\mathbf{Q}_1'\mathbf{e}_2+2|\mathbf{q}_2|y_+y_-R_1']\hat{p}_1,\tag{12}$$

$$R_1'=S'(\mathbf{l}_-)-S'(\mathbf{l}_+),\quad \mathbf{Q}_1'=\mathbf{l}_+S'(\mathbf{l}_+)+\mathbf{l}_-S'(\mathbf{l}_-),$$

$$S'(\mathbf{l})=\frac{1}{\mathbf{l}^2+y_+y_-\mathbf{q}_2^2+m^2},\quad \mathbf{l}_\pm=\mathbf{q}_\pm-y_\pm\mathbf{q}_2,$$

$$y_\pm=\frac{\alpha_\pm}{\alpha_2},\quad y_++y_-=1.$$

The two factors

$$\frac{|\mathbf{q}_1|}{\mathbf{q}_1{}^2+m_1^2\beta_1^2},\quad \frac{|\mathbf{q}_2|}{\mathbf{q}_2{}^2+m_2^2\alpha_2^2}$$

are the familiar bremsstrahlung amplitudes for the photons with transverse momenta $\mathbf{q}_{1,2}$, which determine the luminosity of $\gamma\gamma$ collisions [8]. The square of $\overline{u}(q_-)R_{(1)}^{(1)}v(q_+)$ (up to the well-known kinematic factors) gives the cross section of the process $\gamma^*\gamma^*\to e^+e^-$.

After a lengthy but straightforward calculation, we obtain

$$\frac{1}{4}\sum|\overline{u}(q_-)R_{(1)}^{(1)}v(q_+)|^2 =\frac{1}{4}\mathrm{Sp}(\hat{q}_-+m)R_{(1)}^{(1)*}(\hat{q}_+-m)R_{(1)}^{(1)}$$
$$=\frac{1}{2}z_+z_-\mathbf{q}_1{}^2[(m^2+4z_+z_-\mathbf{q}_1{}^2)R_1^2+(z_+^2+z_-^2)\mathbf{Q}_1{}^2+4z_+z_-(z_+-z_-)R_1\mathbf{q}_1\mathbf{Q}_1].\tag{13}$$

One should note the rapid decrease of right-hand side of Eq. (13) and cross section for $|\mathbf{q}_1{}^2|$, which reflect the nature of $R_1(k)$ and $\mathbf{Q}_1(k)$. The different terms in this expression have a transparent physical meaning in terms of the transverse and scalar (often called longitudinal) polarizations of the virtual photon γ_1^*; and there is a one-to-one correspondence with the discussion of the helicity structure function in diffractive DIS (deep inelastic scattering) and diffractive production of vector mesons [20]. Namely, in expression (13) the first term proportional to m^2 corresponds to the pair production by the transverse photon with the sum of lepton helicities $\lambda+\overline{\lambda}=\pm 1$; the term $(z_+^2+z_-^2)\mathbf{Q}_1{}^2$ provided the excitation of the pair with $\lambda+\overline{\lambda}=0$ by the transverse photons. The term $\propto 4z_+z_-\mathbf{q}_1{}^2R_1^2$ in Eq. (13) describes the pair production by scalar photons, whereas the last term in Eq. (13) is the ST (LT) interference that vanishes upon the phase-space integration by z_+ and z_-.

Due to the exact conservation of the helicity of relativistic electrons in the Coulomb scattering, the above structure of the amplitude (11a) and its square (13) will also be retained in higher order. This important property will be widely used later on in constructing the terms of higher orders in the fine-structure constant.

The differential cross section of the process (2) is connected with the full amplitude by the relation

$$d\sigma=\frac{1}{8s}\sum |M|^2 d\Gamma. \tag{14}$$

The summation is carried over the final particle polarizations. The phase-space volume for four particles in the final state can be expressed through the variables introduced above and phase volume of the final leptons,

$$d\Gamma_{\pm}=\frac{d^3q_+}{2E_+}\frac{d^3q_-}{2E_-}\delta^4(q_1+q_2-q_+-q_-), \tag{15}$$

in the following way:

$$d\Gamma=\frac{1}{(2\pi)^8}\frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'}\frac{d^3q_-}{2E_-}\frac{d^3q_+}{2E_+}\delta^4(p_1+p_2-p_1'-p_2' -q_+-q_-)$$
$$=\frac{ds_p}{4(2\pi)^8 s(1-\alpha_2)(1-\beta_1)}\frac{d\beta_1}{\beta_1}d^2\mathbf{q}_1 d^2\mathbf{q}_2 d\Gamma_{\pm}. \tag{16}$$

In obtaining this expression, we used the following relations for the final particles momenta:

$$\frac{d^3q}{E}=2\delta(q^2-m^2)d^4q=s\,d\alpha\, d\beta\, d^2\mathbf{q}\,\delta(s\alpha\beta-\mathbf{q}^2-m^2).$$

The above expressions allow one to calculate the differential cross section of the process under consideration in the Born approximation with power accuracy in energy.

The total cross section of the pair production in the elementary process $\gamma_1^*\gamma_2^*\rightarrow e^+e^-$ drops rapidly as soon as one of the photons is off-mass shell, which imposes the upper limit of integration over the photon transfer momenta $\mathbf{q}_1{}^2,\mathbf{q}_2{}^2\lesssim m^2$. This limits are much smaller than the characteristic momentum where the form factors are at work,

$$|\mathbf{q}_{1,2}|^2\lesssim m_e^2\ll\frac{6}{\langle R_{ch}^2\rangle_A}\approx\frac{10}{R_A^2}\approx\frac{0.25}{A^{2/3}}\ \text{GeV}^2,$$

where $R_A\sim1.2A^{1/3}$ fm is the nuclear radius, so that the effect of nuclear form factors can be safely neglected in correspondence with Ref. [21]. (It is not the case for the production of muon pairs or hadronic states though.)

For this reason the total cross section in the case of electron pair production can be evaluated like the nucleus approximation, i.e., using the above cited expressions. It can be shown, see details in Ref. [22], that integrating the expression for the differential cross section (14) using the relations (9), (13), and (16), one obtains the famous Racah formula [2] for the total cross section

$$\sigma=\frac{\alpha^4Z_1^2Z_2^2}{\pi m^2}\frac{28}{27}[L^3-2.2L^2+3.84L-1.636], \tag{17}$$

where $L=\ln(\gamma_1\gamma_2)$.

The total cross section of the process under consideration in the Born approximation is a growing function of the invariant energy (as the third power of logarithm in energy). This is the direct consequence of the fact that both photons are quasireal, which brings to second power of logarithm in Eq. (17) and one power comes from the common, for all orders integration by the rapidity variable β_1.

Before proceeding to calculation of proper Coulomb corrections, let us make the following remark. Lepton pair created by two photons is produced in even charge, i.e., spin-singlet state. The exchange of the odd number of photons between the Coulomb fields of heavy ions and the produced leptons will lead to a pair creation in odd charge, i.e., spin-triplet state that does not interfere with the Born amplitude. Nevertheless, later on we have to take into account only the interference of the Born amplitude with the amplitude where the four-photon exchanges take place between leptons and colliding particles (see Fig. 3). Now we can go to the main topic of the present work, calculations of CC. We begin with the simplest case that is provided by three-photon exchange between the produced lepton pair and colliding particles.

III. THE COULOMB CORRECTIONS TO THE PAIR PRODUCTION IN *C*-ODD STATE

We start with the discussion of the pair production which proceeds via the exchange of the odd number of photons between the colliding particles and produced pair. The amplitude relevant to the first CC of this type consists of two parts $M_{(12)}=M_{(2)}^{(1)}+M_{(1)}^{(2)}$ where the amplitude $M_{(2)}^{(1)}$ corresponds to the case when lepton pair is attached to the colliding particle A by one photon and connected with B through two photons in all possible ways. The case when two photons are attached to A (one to B) is described by the term $M_{(1)}^{(2)}$. As will be shown later, this term can be obtained from another one by a simple substitution; so it is enough to consider one of them.

The amplitude $M_{(2)}^{(1)}$ is described by six Feynman diagrams that are depicted in Fig. 2. The only essential difference from the Born approximation is the additional photon exchange with four-momentum $k=\alpha\tilde{p}_2+\beta\tilde{p}_1+k_\perp$, in which the integration has been carried out in the amplitude. To integrate over the Sudakov variables α and β, we use 12 FD instead of six, which are relevant to this amplitude, and introduce the statistical factor 1/2!. This trick permits us to provide the eikonal-type integration over β using the identity

$$\int\frac{ds\beta}{2\pi i}\left(\frac{1}{s\beta+i0}+\frac{1}{-s\beta+i0}\right)=1. \tag{18}$$

The convergence of the integration over α is provided by all six FDs obtained by permutation of all absorption points of exchanged photons by the pair. Closing the α integration contour to the upper half plane, one can see that only four of them are relevant. The result is (the details can be found in appendixes of Ref. [18])

$$M^{(1)}_{(2)}=s\frac{(4\pi\alpha)^3 Z_1 Z_2^2}{2\pi(\mathbf{q}_1{}^2+m_1^2\beta_1^2)}\overline{u}(q_-)\mathfrak{R}^{(1)}_{(2)}v(q_+), \tag{19}$$

$$\mathfrak{R}^{(1)}_{(2)}=\frac{|\mathbf{q}_1|}{s\beta_1}\int\frac{d^2\mathbf{k}}{\pi}\frac{1}{\mathbf{k}^2(\mathbf{q}_2-\mathbf{k})^2}[m\hat{e}_1R_2(k)+\hat{e}_1\hat{Q}_2(k)+2z_+\mathbf{Q}_2(k)\mathbf{e}_1+2|\mathbf{q}_1|z_+z_-R_2(k)]\hat{p}_2$$
$$\equiv\frac{|\mathbf{q}_1|}{s\beta_1}[m\hat{e}_1R^{(1)}_{(2)}+\hat{e}_1\hat{Q}^{(1)}_{(2)}+2z_+\mathbf{Q}^{(1)}_{(2)}\mathbf{e}_1+2|\mathbf{q}_1|z_+z_-R^{(1)}_{(2)}]\hat{p}_2,$$

with

$$R_2(\mathbf{k})=S(\mathbf{k}_-)+S(\mathbf{k}_+)-S(\mathbf{k}_--\mathbf{k})-S(\mathbf{k}_+-\mathbf{k}), \tag{20}$$

$$\mathbf{Q}_2(\mathbf{k})=\mathbf{k}_-S(\mathbf{k}_-)-\mathbf{k}_+S(\mathbf{k}_+)-(\mathbf{k}_--\mathbf{k})S(\mathbf{k}_--\mathbf{k})+(\mathbf{k}_+-\mathbf{k})S(\mathbf{k}_+-\mathbf{k}).$$

It is easily seen that these structures vanish when any of the photon momenta tends to zero. This property leads to infrared finiteness of expression (19).

Next, one can see that the spin structure of the first CC is precisely the same as in the Born case. This is the direct result of lepton helicity conservation and, as was shown in Refs. [19,23], is valid to all orders of perturbation series for the case when one considers the single-photon exchange between the pair and projectile A with any number of exchanges with target B and vice versa. Using the relations from Appendix A, one can perform the integration in Eq. (19) and obtain the analytical expression for $\mathfrak{R}^{(1)}_{(2)}$. In the general case, this expression is rather cumbersome; therefore, we will present in Appendix A only the result of integration for the case when the single photon, which is attached to projectile A, is real $(\mathbf{q}_1\to 0)$. The square of amplitude the $M^{(1)}_{(2)}$ can be obtained by analogy with the Born case,

$$\frac{1}{4}\sum|\overline{u}(q_-)\mathfrak{R}^{(1)}_{(2)}v(q_+)|^2=\frac{1}{2}z_+z_-\mathbf{q}_1{}^2\{(m^2+4z_+z_-\mathbf{q}_1{}^2)\times(R^{(1)}_{(2)})^2+(z_+^2+z_-^2)[(\mathbf{Q}^{(1)}_{(2)})^2+4z_+z_-(z_+-z_-)\times R^{(1)}_{(2)}\mathbf{q}_1\mathbf{Q}^{(1)}_{(2)}]\}. \tag{21}$$

The amplitude $M^{(2)}_{(1)}$ (two-photon exchange between lepton pair and the nucleus A_1) is readily obtained from $M^{(1)}_{(2)}$ by the following substitutions:

$$\beta_1\leftrightarrow\alpha_2,\quad z_\pm\leftrightarrow y_\pm,\quad \mathbf{q}_1\leftrightarrow\mathbf{q}_2,\quad \mathbf{e}_1\leftrightarrow\mathbf{e}_2,\quad \hat{p}_2\leftrightarrow\hat{p}_1. \tag{22}$$

Let us estimate the considered CC contribution to the total cross section. This can be easily done if one restricts himself to the terms growing with energy as the main power of the logarithm,

$$\int_{\beta_{1min}}^{1}\frac{d\beta_1}{\beta_1}\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mathbf{q}_1{}^2f(\mathbf{q}_1{}^2)}{(\mathbf{q}_1{}^2+m_1^2\beta_1^2)^2}\approx L^2f(0)+O(L), \tag{23}$$
$$\beta_{1min}=\frac{1}{\gamma_1\gamma_2}.$$

So that to the leading logarithmical accuracy

$$\sigma^{tot}_{odd}=\frac{\alpha^6Z_1^2Z_2^2(Z_1^2+Z_2^2)L^2}{\pi m^2}P, \tag{24}$$
$$P=\int\frac{d^2\mathbf{q}_-}{\pi}\int\frac{d^2\mathbf{q}_+}{\pi}\left[m^4(R^{(1)}_{(2)})^2+\frac{2}{3}m^2(\mathbf{Q}^{(1)}_{(2)})^2\right]_{\mathbf{q}_1=0}.$$

Using the expressions for $R^{(1)}_{(2)}$ and $\mathbf{Q}^{(1)}_{(2)}$ from Appendix A, we calculated this quantity with the result $P\approx 3{,}6$. We want to emphasize that the general expression (19) contains the leading logarithm contribution as well as the nonleading ones, omitting only those terms that are suppressed by the factor m^2/s and vanish in the high-energy limit.

The considered case $\mathbf{q}_1\to 0$ corresponds to the WW approximation, i.e., to the case when the scattered ion A is not detected and will scatter at very small angles. We see that for the case with a semi-inclusive setup (when the scattered ion is fixed), we have a much more complicated picture that nevertheless can be described by the above cited expressions.

The contribution from the interference of the amplitudes $M^{(1)}_{(2)}$ and $M^{(2)}_{(1)}$ is enhanced only by the first power of large logarithm L due to the boost effect and its impact on the total cross section will be considered further. Here we only cite this interference contribution to the odd charge part of the total cross section in the general form

$$\sigma^{int}_{odd}=\frac{\alpha^6Z_1^3Z_2^3L}{\pi^2}\int\frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1{}^2}\frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2{}^2}\int ds_p\int d\Gamma_\pm\frac{1}{4}\mathrm{Sp}(\hat{q}_-+m)\times\mathfrak{R}^{(2)}_{(1)}(\hat{q}_+-m)\mathfrak{R}^{(1)}_{(2)}. \tag{25}$$

IV. THE COULOMB CORRECTIONS TO THE PAIR PRODUCTION IN *C*-EVEN STATE

The lowest CC to the pair production in the C-even state are provided by the four-photon exchange between the colliding ions and the produced pair in all possible ways. Conventionally, one can divide them into two sets. The first one is determined by FD, some of which are depicted in Figs. 3(a),3(b). The corresponding amplitude denoted by $M_{13}=M^{(1)}_{(3)}+M^{(3)}_{(1)}$ represents the sum of all possible single attachments to one of the colliding ions with three-photon exchanges between lepton pair and another ion. The second set is determined by FD depicted in Fig. 3(c) and includes all double photon exchanges between lepton pair and every colliding ion. We write this amplitude as $M^{(2)}_{(2)}$. Using the same approximations and technique as above for the amplitude $M^{(1)}_{(3)}$, we obtain

$$M_{(3)}^{(1)}=\frac{is}{3!}\frac{(4\pi\alpha)^4Z_1Z_2^3}{(2\pi)^2(\mathbf{q}_1{}^2+\beta_1^2m_1^2)}\overline{u}(q_-)\mathfrak{R}_{(3)}^{(1)}v(q_+),\quad (26)$$

$$\mathfrak{R}_{(3)}^{(1)}=\frac{|\mathbf{q}_1|}{s\beta_1}\int\frac{d^2\mathbf{k}_1}{\pi}\frac{d^2\mathbf{k}_2}{\pi}\frac{1}{\mathbf{k}_1{}^2\mathbf{k}_2{}^2(\mathbf{q}_2-\mathbf{k}_1-\mathbf{k}_2)^2}[m\hat{e}_1R_3(k)+\hat{e}_1\hat{Q}_3(k)+2z_+\mathbf{Q}_3(k)\mathbf{e}_1+2|\mathbf{q}_1|z_+z_-R_3(k)]\hat{p}_2$$
$$\equiv\frac{|\mathbf{q}_1|}{s\beta_1}[m\hat{e}_1R_{(3)}^{(1)}+\hat{e}_1\hat{Q}_{(3)}^{(1)}+2z_+\mathbf{Q}_{(3)}^{(1)}\mathbf{e}_1+2|\mathbf{q}_1|z_+z_-R_{(3)}^{(1)}]\hat{p}_2,$$

which are in close analogy with the amplitudes of diffractive DIS

$$R_3(\mathbf{k}_1,\mathbf{k}_2)=-S(\mathbf{k}_-)+S(\mathbf{k}_--\mathbf{k}_1)+S(\mathbf{k}_--\mathbf{k}_2)+S(\mathbf{k}_--\mathbf{q}_2+\mathbf{k}_1+\mathbf{k}_2)-[\mathbf{k}_-\leftrightarrow\mathbf{k}_+],\quad (27)$$

$$\mathbf{Q}_3(\mathbf{k}_1,\mathbf{k}_2)=\mathbf{k}_-S(\mathbf{k}_-)-(\mathbf{k}_--\mathbf{k}_1)S(\mathbf{k}_--\mathbf{k}_1)-(\mathbf{k}_--\mathbf{k}_2)S(\mathbf{k}_--\mathbf{k}_2)-(\mathbf{k}_--\mathbf{q}_2+\mathbf{k}_1+\mathbf{k}_2)\times S(\mathbf{k}_--\mathbf{q}_2+\mathbf{k}_1+\mathbf{k}_2)-[\mathbf{k}_-\leftrightarrow\mathbf{k}_+].$$

One can see that as in the case of the third-order CC considered above, the fourth-order amplitude of first kind (i.e., with one single attachment) has the same spin structure as the Born one and can be obtained from it by a simple replacement $(R_1,\mathbf{Q}_1)\leftrightarrow(R_3,\mathbf{Q}_3)$. To get the amplitude $M_{(1)}^{(3)}$, it is sufficient to make the replacement (22) in the above expressions. It is easy to check that the amplitude M_{12} vanishes when any of the photon momenta q_1 or q_2 goes to zero. Finally, in the WW limit $q_1,q_2\to 0$, one obtains from Eqs. (26),(27) the expression that coincides with the respective one from Ref. [24].

The main contribution to the total cross section comes from the interference of M_{13} with the Born amplitude $M_{(1)}^{(1)}$. To the leading logarithmic accuracy this interference can be read as

$$\sigma_{(1)}^{(3)}+\sigma_{(3)}^{(1)}=-\frac{4\alpha^6(Z_1Z_2)^2(Z_1^2+Z_2^2)L^2}{3\pi m^2}P_1,\quad (28)$$

$$P_1=\int\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_-}{\pi}\frac{m^2}{\mathbf{q}_2}\left[m^2R_1R_{(3)}^{(1)}+\frac{2}{3}\mathbf{Q}_1\mathbf{Q}_{(3)}^{(1)}\right]_{\mathbf{q}_1=0}.$$

Comparing this expression with the cross section in the case of C-odd CC (24), one can see that both the terms are of the same order in the fine-structure constant and have the same energy dependence. Moreover, we remark that $P_1=P$. Indeed, the product $R_1R_{(3)}^{(1)}$ contains 16 terms and making the proper shifts of the integration variables, one can establish a one-to-one correspondence to 16 terms in $(R_{(2)}^{(1)})^2$. Similarly, the same can be done with the vector products $\mathbf{Q}_1\mathbf{Q}_{(3)}^{(1)}$ and $(\mathbf{Q}_{(2)}^{(1)})^2$. This can be regarded as a manifestation of the well-known Abramovski, Grivov, and Kancheli rules [25]. Really, the two-photon exchange production of lepton pair can be viewed as a diffraction excitation of the photon, whereas the three-photon exchange can be viewed as absorption correction to the one-photon exchange. Furthermore, the total cross section for pair photoproduction in the Coulomb field reads [26]

$$\sigma^{\gamma Z\to e^+e^-Z}(s)=\frac{28Z^2\alpha^3}{9m^2}\left[\ln\frac{s}{m^2}-\frac{109}{42}-(Z\alpha)^2\times\sum_{n=1}^{\infty}\frac{1}{n(n^2+(Z\alpha)^2)}\right],\quad (29)$$

which for the relevant order gives

$$P\approx P_1=\frac{28}{9}\zeta(3)\approx 3.74,\quad \zeta(3)=\sum_{n=1}^{\infty}\frac{1}{n^3}=1.202.$$

By means of numerical calculations, we were convinced that the equality $P\approx P_1$ is fulfilled within several percent accuracy, which is presumably the artifact of numerical calculation.

Now we can estimate the relevant contribution of the first CC to the total lepton pair yield (17). Using expressions (24) and (28), one can write the total lowest CC to the pair creation to the leading logarithmic accuracy,

$$\Delta\sigma_{odd}=\sigma_{odd}^{tot}+\sigma_{(1)}^{(3)}+\sigma_{(3)}^{(1)}=-\frac{\alpha^6(Z_1Z_2)^2(Z_1^2+Z_2^2)L^2}{3\pi m^2}P.\quad (30)$$

In the case of Au-Au collisions at RHIC, the considered contribution is approximately 10%.

Returning to the second set of the fourth-order CC (two-photon exchange between the pair and every heavy ion), we calculated all respective terms in accordance with the FD [see Fig. 3(c)] and obtained the matrix element $M_{(2)}^{(2)}$ in the following form:

$$M_{(2)}^{(2)}=is\alpha^4 2^4(Z_1Z_2\pi)^2\overline{u}(q_-)\mathfrak{R}_{(2)}^{(2)}v(q_+),\quad (31)$$

$$\mathfrak{R}_{(2)}^{(2)}=\int\frac{d^2\mathbf{k}_1}{\pi}\int\frac{d^2\mathbf{k}_2}{\pi}\frac{1}{\mathbf{k}_1{}^2(\mathbf{q}_1-\mathbf{k}_1)^2}\frac{1}{\mathbf{k}_2{}^2(\mathbf{q}_2-\mathbf{k}_2)^2}\times(1+P_{ud})R_{(2)}^{(2)}.$$

The structure $\mathfrak{R}_{(2)}^{(2)}$ has the so-called up-down symmetry that is decoded in the permutation operator P_{ud} acting as follows:

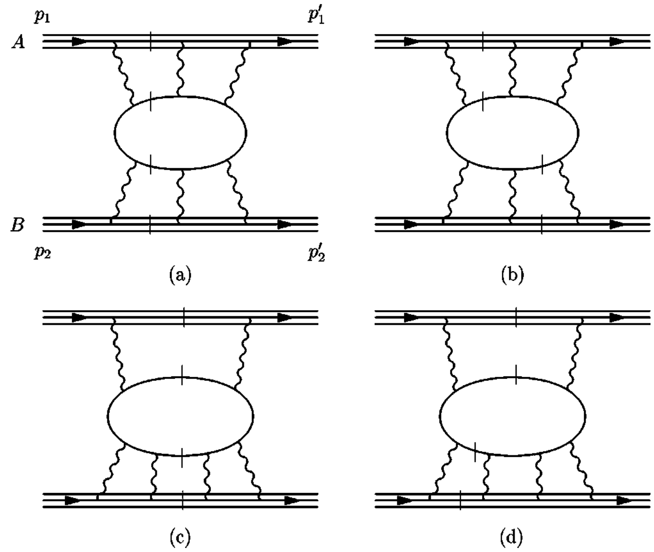

FIG. 4. Different *s*-channel cuts for forward elastic scattering in sixth order of perturbation theory.

$$P_{ud}F(p_1,p_2,\alpha,\beta,q_1,k_1,q_2,k_2)$$
$$=F(p_2,p_1,\beta,\alpha,q_2,k_2,q_1,k_1). \quad (32)$$

The quantity $R^{(2)}_{(2)}$ is the sum of all terms corresponding to all possible FD. Its explicit expression is given in Appendix B.

It can be shown that the quantity $(1+P_{ud})R^{(2)}_{(2)}$ tends to zero in four kinematic limits $\mathbf{k}_1\to\mathbf{0}$, $\mathbf{k}_2\to\mathbf{0}$, $(\mathbf{q}_1-\mathbf{k}_1)\to\mathbf{0}$, and $(\mathbf{q}_2-\mathbf{k}_2)\to\mathbf{0}$, which is the consequence of the gauge invariance. This property provides its convergence. Unfortunately, we did not succeed in trying to write down the result for spin structures in this case as it has be done above for the terms where there is a single attachment of the exchanged photon to one of the colliding ions.

Nevertheless, we cite here the leading contribution to the *C*-even state pair creation cross section, which comes from the interference of this amplitude and the Born term,

$$\sigma^{int}_{even}=\frac{\alpha^6(Z_1Z_2)^3L}{\pi^2}\int\frac{d^2\mathbf{q}_1}{\pi\mathbf{q}_1{}^2}\frac{d^2\mathbf{q}_2}{\pi\mathbf{q}_2{}^2}\int ds_p$$
$$\times\int d\Gamma_\pm\frac{1}{4}Sp(q_-+m)\mathfrak{R}^{(2)}_{(2)}(q_+-m)\tilde{\mathfrak{R}}^{(1)}_{(1)}. \quad (33)$$

This expression is of the same order in the fine-structure constant as the considered above interference between the Born amplitude and the first set of the fourth-order term [see Eq. (28)], but it has one power less in its energy dependence.

On the other hand, the first power dependence on L and the same order in α has a contribution to the total cross section arising from the interference between the amplitudes $M^{(1)}_{(2)}$ and $M^{(2)}_{(1)}$ [see Eq. (25)], which corresponds to the pair production in the *C*-odd state. From very general arguments these contributions have to cancel each other within the logarithmical accuracy. The relevant (sixth order in α) elastic amplitude F for AB scattering at zero angle (see Fig. 4) has the following structure:

$$F(AB\to AB)\sim\ln s-i\pi. \quad (34)$$

The logarithmic energy dependence of the real part of this amplitude is due to the boost freedom of a light-light scattering block, whereas the *s*-channel discontinuity of this amplitude has no logarithmic enhancement at all. Therefore, in the total cross section of the pair production, the terms proportional to $(Z_1Z_2\alpha^2)^3L$ are absent. It is interesting to check this statement by straightforward calculations, that will be done elsewhere. Moreover, the contributions of the form $(Z_1\alpha)^{2k+1}(Z_2\alpha)^{2l+1}$ are absent in the total cross section in any orders of perturbation theory by the same reason. This statement is in the complete agreement with the results recently obtained for the Coulomb corrections [27].

V. CONCLUDING REMARKS

In this work, we have considered lepton pair production in the Coulomb field of two highly relativistic heavy ions. Using the powerful technique of Sudakov variables which allows one to obtain the results that are correct with the power accuracy in energy, we get the first terms in perturbation expansion of the amplitude (up to the fourth order in the fine-structure constant) and show that they are finite and gauge invariant. We have obtained simple analytic expressions for the Born amplitude and relevant cross section which allow one to calculate lepton pair yield in a wide interval of kinematic variables. It is shown that the terms in the amplitude, which correspond to FD with at least one single exchange between the created pair and one of the ion, have the same spin structure in all orders in α, and we proposed simple rules to build them.

In every order of perturbation theory, we have analyzed the obtained CC isolating the leading energy terms, and showed that the remarkable cancellations among the different terms in the total cross section take place. Finally, the numerical estimates of the main contribution from the CC to the total cross section have been carried out.

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APPENDIX A

We give here the result of two-dimensional integration by loop momenta for scalar and vector structures determining the first CC,

$$I(\mathbf{q}_-)=\int\frac{d^2\mathbf{k}}{\pi}\frac{1}{(\mathbf{k}^2+\lambda^2)((\mathbf{q}-\mathbf{k})^2+\lambda^2)((\mathbf{q}_--\mathbf{k})^2+m^2)}$$

$$=\frac{1}{\mathbf{q}^2}\left[\frac{1}{\langle q_-\rangle}+\frac{1}{\langle q_+\rangle}\right]\ln\frac{\mathbf{q}^2}{\lambda^2}+\frac{1}{\Delta}\left[\frac{\mathbf{q}_-^{\,2}}{\langle q_-\rangle}+\frac{\mathbf{q}_+^{\,2}}{\langle q_+\rangle}-1\right]\ln\frac{m^2}{\mathbf{q}^2}$$

$$+A_1\ln\frac{\langle q_-\rangle}{m^2}+A_2\ln\frac{\langle q_+\rangle}{m^2},\tag{A1}$$

$$A_1=-\frac{\mathbf{q}\cdot\mathbf{q}_-}{\mathbf{q}^2}\left[\frac{1}{\Delta}+\frac{1}{\langle q_-\rangle\langle q_+\rangle}\right]+\frac{1}{\Delta}\left[\frac{\mathbf{q}_-^{\,2}}{\langle q_-\rangle}+\frac{\mathbf{q}_+^{\,2}}{\langle q_+\rangle}-\frac{m^2\mathbf{q}\cdot\mathbf{q}_+}{\langle q_-\rangle\langle q_+\rangle}\right],\tag{A2}$$

$$A_2=-\frac{\mathbf{q}\cdot\mathbf{q}_+}{\mathbf{q}^2}\left[\frac{1}{\Delta}+\frac{1}{\langle q_-\rangle\langle q_+\rangle}\right]+\frac{1}{\Delta}\left[\frac{\mathbf{q}_-^{\,2}}{\langle q_-\rangle}+\frac{\mathbf{q}_+^{\,2}}{\langle q_+\rangle}-\frac{m^2\mathbf{q}\cdot\mathbf{q}_-}{\langle q_-\rangle\langle q_+\rangle}\right],$$

$$\mathbf{I}(\mathbf{q}_-)=\int\frac{d^2\mathbf{k}}{\pi}\frac{\mathbf{k}}{(\mathbf{k}^2+\lambda^2)((\mathbf{q}-\mathbf{k})^2+\lambda^2)((\mathbf{q}_--\mathbf{k})^2+m^2)}$$

$$=\frac{\mathbf{q}}{\mathbf{q}^2\langle q_+\rangle}\ln\frac{\langle q_+\rangle}{\lambda^2}-\frac{1}{\Delta}\mathbf{q}_+\left[\ln\frac{\langle q_-\rangle}{\mathbf{q}^2}+\ln\frac{\langle q_+\rangle}{m^2}\right]$$

$$+\frac{1}{\Delta}\mathbf{q}\left[\frac{\mathbf{q}_+^{\,2}}{\langle q_+\rangle}\ln\frac{\langle q_+\rangle}{m^2}-\frac{\langle q_-\rangle-\mathbf{q}^2}{\mathbf{q}^2}\ln\frac{\langle q_-\rangle}{\mathbf{q}^2}\right],\tag{A3}$$

where we use the notation $\langle a\rangle=\mathbf{a}^2+m^2$ and besides

$$\Delta=\langle q_-\rangle\langle q_+\rangle-m^2\mathbf{q}^2,\quad \mathbf{q}=\mathbf{q}_++\mathbf{q}_-\,.$$

Using these integrals and expression (20), for scalar combination which enters into $M^{(1)}_{(2)}$ we obtain

$$R^{(1)}_{(2)}=\int\frac{d^2\mathbf{k}}{\pi}\frac{R_2(k)}{\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2}$$

$$=\frac{2}{\Delta}\left(1-\frac{\mathbf{q}_-^{\,2}}{\langle q_-\rangle}-\frac{\mathbf{q}_+^{\,2}}{\langle q_+\rangle}\right)\ln\frac{\langle q_-\rangle\langle q_+\rangle}{m^2\mathbf{q}^2}$$

$$+\frac{2}{\mathbf{q}^2}\left(\frac{1}{\langle q_+\rangle}-\frac{1}{\langle q_-\rangle}\right)\ln\frac{\langle q_-\rangle}{\langle q_+\rangle},\tag{A4}$$

$$R_2(k)=\frac{1}{\langle q_-\rangle}-\frac{1}{\langle k-q_-\rangle}+\frac{1}{\langle q_+\rangle}-\frac{1}{\langle k-q_+\rangle}.$$

For the case of the vector structures, we obtain

$$\mathbf{Q}^{(1)}_{(2)}=\int\frac{d^2\mathbf{k}}{\pi}\frac{\mathbf{Q}_2(k)}{\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2}$$

$$=(\mathbf{q}_--\mathbf{q}_+)\left[\frac{m^2}{\Delta}\left(\frac{1}{\langle q_-\rangle}+\frac{1}{\langle q_+\rangle}\right)\ln\frac{\langle q_-\rangle\langle q_+\rangle}{m^2\mathbf{q}^2}+\frac{1}{\mathbf{q}^2}\left(\frac{1}{\langle q_-\rangle}-\frac{1}{\langle q_+\rangle}\right)\ln\frac{\langle q_-\rangle}{\langle q_+\rangle}\right]\tag{A5}$$

$$+\frac{\mathbf{q}}{\mathbf{q}^2}\left[(\langle q_+\rangle-\langle q_-\rangle)\left(\frac{1}{\Delta}\ln\frac{\langle q_+\rangle\langle q_-\rangle}{m^2\mathbf{q}^2}-\frac{1}{\langle q_+\rangle\langle q_-\rangle}\ln\frac{m^2}{\mathbf{q}^2}\right)+2\left(\frac{1}{\langle q_+\rangle}\ln\frac{\langle q_+\rangle}{m^2}-\frac{1}{\langle q_-\rangle}\ln\frac{\langle q_-\rangle}{m^2}\right)\right],$$

$$\mathbf{Q}_2(k)=\frac{\mathbf{q}_-}{\langle q_-\rangle}+\frac{\mathbf{k}-\mathbf{q}_-}{\langle k-q_-\rangle}-\frac{\mathbf{q}_+}{\langle q_+\rangle}-\frac{\mathbf{k}-\mathbf{q}_+}{\langle k-q_+\rangle}.$$

APPENDIX B

The explicit expression for $R^{(2)}_{(2)}$ is

$$R^{(2)}_{(2)}=R_{1243}+R_{1324}+R_{2431}+R_{3124}+R_{1342}+R_{1423}\,,\tag{B1}$$

with the following spin structures that determine the fourth-order amplitude of the second class [see Eq. (33)]:

$$R_{1243}=\frac{1}{s}\frac{\hat p_1(\hat q_--\hat q_1+m)\hat p_2}{\frac{\beta_+}{\beta_-}\langle q_-\rangle+\langle q_--q_1\rangle},$$

$$R_{1324}=-\frac{1}{s^2}\frac{\hat p_1(\hat q_--\hat k_1+m)\hat p_2(\hat q_--\hat k_1-\hat k_2+m)\hat p_1(-\hat q_++\hat q_2-\hat k_2+m)\hat p_2}{\frac{\beta_+}{\beta_-}\langle q_-\rangle\langle q_--k_1-k_2\rangle+\langle q_--k_1\rangle\langle -q_++q_2-k_2\rangle},$$

$$R_{2431}=\frac{1}{s^2}\frac{\hat p_1(\hat q_--\hat q_1+\hat k_1+m)\hat p_2(-\hat q_++\hat k_1+m)\hat p_1}{\alpha_-\langle -q_++k_1\rangle+\alpha_+\langle q_--q_1+k_1\rangle},$$

$$R_{3124}=-\frac{1}{s^2}\frac{\hat{p}_2(\hat{q}_- -\hat{k}_2+m)\hat{p}_1(-\hat{q}_+\hat{q}_2-\hat{k}_2+m)\hat{p}_2}{\beta_+\langle -q_- +k_2\rangle+\beta_-\langle q_- -q_1-k_2\rangle},$$

$$R_{1423}=-\frac{1}{s^2}\frac{\hat{p}_1(\hat{q}_- -\hat{k}_1+m)\hat{p}_2(-\hat{q}_+ +\hat{q}_1+\hat{k}_2-\hat{k}_1+m)\hat{p}_1(-\hat{q}_+ +\hat{k}_2+m)\hat{p}_2}{\frac{\beta_+}{\beta_-}\langle q_-\rangle\langle -q_+ +q_1+k_1-k_2\rangle+\langle q_- -k_1\rangle\langle -q_+ +k_2\rangle},$$

$$R_{2413}=-\frac{1}{s^2}\frac{\hat{p}_1(\hat{q}_- -\hat{q}_1+\hat{k}_1+m)\hat{p}_2(-\hat{q}_+ +\hat{k}_1+\hat{k}_2+m)\hat{p}_1(-\hat{q}_+ +\hat{k}_2+m)\hat{p}_2}{\frac{\beta_+}{\beta_-}\langle q_-\rangle\langle -q_+ +k_2+k_1\rangle+\langle -q_+ +k_2\rangle\langle q_2-q_+ +k_1\rangle}, \tag{B2}$$

when one used relation $s\alpha_\pm\beta_\pm=\langle q_\pm\rangle$.

The symmetry property is caused by the charge conjugation symmetry of the amplitude. There are 24 FD contributing to $M_{(2)}^{(2)}$. Instead of them it is convenient to consider $24(2)(2)=96$ FD, which take as well the permutations of emission and absorption points of exchanged photons to the nuclei. The result must be divided by $(2!)^2$. This trick provides the convergence of α_1,β_2 integrals. All the 24 FD, describing the interaction of four virtual photons with the pair components, are relevant providing the convergence of β_1, α_2 integrations. When closing the contour of integration, say in the upper planes, only 12 of them become relevant. The algorithm of the building structures R_{ijkl} can be generalized to the case of loop number exceeding 2.

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