Continuous-variable quantum teleportation of entanglement

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Entangled coherent states can be used to determine the entanglement fidelity for a device that is designed to teleport coherent states. This entanglement fidelity is universal in that the calculation is independent of the use of entangled coherent states and applies generally to the teleportation of entanglement using coherent states. The average fidelity is shown to be a poor indicator of the capability of teleporting entanglement; i.e., very high average fidelity for the quantum teleportation apparatus can still result in low entanglement fidelity for one-mode of the two-mode entangled coherent states.

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I. INTRODUCTION

Quantum teleportation [1], whereby a quantum state of a system can be transferred by a sender Alice to a remote receiver Bob through the use of classical communication and a shared entanglement resource, is a remarkable demonstration of how nonlocal correlations in quantum mechanics can be used to advantage. More than simply a novelty, quantum teleportation is useful for quantum information processing; for example, it can be used as a universal primitive for quantum computation [2], as a fundamental component to quantum computation with linear optics [3], and as a means to implement nonlocal quantum transformations [4]. One realization is the quantum teleportation of continuous variables (CV) [5], which teleports states of dynamical variables with continuous spectra; such a realization allows for the teleportation of quantum states of light.

Any realistic (imperfect) quantum teleportation device can be characterized by a figure of merit to quantify its ability to perform successful teleportation. The *fidelity* F is useful as a measure of the distinguishability of the output state from the input state, although the threshold for demonstrating genuine quantum teleportation is debatable. One such threshold demonstrates that an entanglement resource was used during the experiment [5]. Another threshold [1,6] is that Alice teleports the state to Bob without learning about the state in question. For teleportation of a distribution of states in a set $\mathcal{C}$, one can define the average fidelity $\overline{F}$ of the device as the average of the fidelity F over $\mathcal{C}$. The average fidelity provides a figure of merit for the device to teleport states in this set.

However, any quality measure for a useful quantum teleportation apparatus must be based on its intended application. One important application is that the device allows for the teleportation of entanglement, i.e., that entanglement is preserved between the teleported system and another (unteleported) one [7]. Consider a user, Victor, who acts to verify that the device functions as advertised. Alice and Bob claim that their device teleports coherent states with a particular average fidelity, as in experimental quantum teleportation [8]. Victor wishes to test the ability of this device to teleport entanglement, but is restricted to supplying Alice with coherent states according to the advertised capability (that is, Alice must receive states in the specified set $\mathcal{C}$).

One option is for Victor to employ a two-mode entangled coherent state (ECS) [9]. By supplying Alice with only one mode, Victor can conceal from her and Bob that they are teleporting only one portion of a two-mode entangled state. Bob returns the state to Victor so that he can test whether or not the ECS is reconstructed with high fidelity, and thus whether or not the teleportation has preserved the entanglement. Alice and Bob specified that the advertised $\overline{F}$ applies only to coherent states. However, if they decide to check, they will indeed see that they are receiving a mixture of coherent states from Victor, so the supply of states from Victor does not violate the specification that these states are drawn from a distribution of coherent states. Thus, Victor uses these ECSs to quantify the capability of this quantum teleportation to preserve entanglement. Provided that these states are entanglements of coherent states that are nearly orthogonal, Alice and Bob cannot detect that Victor is using ECSs to verify the efficacy of the scheme.

We show in this paper that ECSs are useful for testing the ability of a device to teleport entanglement. Moreover, we show that this entanglement fidelity does not depend on using these ECSs, but applies generally to the teleportation of entanglement for a device that teleports coherent states. We note that quantum teleportation of ECSs has been studied, but in entirely different contexts. One such investigation is the teleportation of ECSs in their entirety [10], and another consideration has been to use an ECS as a substitute for the standard entanglement resource provided by the two-mode squeezed vacuum state [11]. Our study is quite different from these two cases; in our investigation, Victor employs ECSs to replicate the conditions that Alice and Bob experience in the experiment of Furusawa *et al.* [8], and Victor uses a second entangled mode to verify that quantum teleportation of entanglement is taking place.

The paper is constructed as follows. In Sec. II, we develop the theory of quantum teleportation of coherent states according to a formalism that is useful for subsequent sections. In Sec. III, we discuss the quantum teleportation using ECSs as a means of verifying the capability of teleporting entanglement; we include a discussion of the entanglement fidelity as a measure of this capability. We define a noisy quantum teleportation scheme in Sec. IV, and present the result that the entanglement fidelity for the noisy quantum teleportation of ECSs is extremely sensitive to very small errors in Alice's measurement. We conclude with Sec. V.

II. QUANTUM TELEPORTATION OF COHERENT STATES

Quantum teleportation was proposed [1] as a means by which a quantum state can be transferred from a system A (Alice) to a remote system B (Bob) by employing only classical communication and a shared entanglement resource. Let Alice hold an arbitrary quantum state $|\psi\rangle_1 \in \mathcal{H}_1$ that she wishes to send to Bob. In CV quantum teleportation, $\mathcal{H}_1$ is an infinite-dimensional Hilbert space; typically, the Hilbert space for a harmonic oscillator is used. In addition to this quantum system, Alice holds a second quantum system with Hilbert space $\mathcal{H}_2$, and Bob holds a third with Hilbert space $\mathcal{H}_3$. These two systems are in the two-mode squeezed state [12]

$$|\eta\rangle_{23} = \sqrt{1-\eta^2}\sum_{n=0}^{\infty} \eta^n |n\rangle_2 |n\rangle_3, \tag{1}$$

with $|n\rangle$ being the n-boson Fock state. In the limit $\eta \rightarrow 1$, Alice and Bob share a maximally entangled Einstein-Podolsky-Rosen (EPR) state.

To perform quantum teleportation of the unknown state $|\psi\rangle_1$, Alice begins by performing a joint projective measurement on $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the form

$$\Pi_\alpha = D_1(\alpha)|\eta\rangle_{12}\langle\eta|D_1^\dagger(\alpha), \tag{2}$$

where $D_1(\alpha)$ is the displacement operator on system 1, defined by

$$D_1(\alpha) = \exp(\alpha\hat{a}_1^\dagger - \alpha^*\hat{a}_1), \quad \alpha \in \mathbb{C}. \tag{3}$$

A measurement result is a complex number $\alpha_0 = x_0 + ip_0$. Such a measurement can be implemented by mixing the two states on a beam splitter and performing balanced homodyne detection on each of the two output modes [5]. Alice sends this measurement result via a classical channel to Bob, who performs a displacement operation $D_3(\alpha_0)$ (realizable by mixing with a strong coherent field with adjustable amplitude and phase) on his system $\mathcal{H}_3$. Bob's system is now in the state $|\psi\rangle_3$, which is identical to the initial state $|\psi\rangle_1$ received by Alice in the limit $\eta \rightarrow 1$.

In an experiment [8], various constraints (including finite squeezing and imperfect projective measurements) limit the performance of the quantum teleportation. One must then define a figure of merit to describe how well a physical quantum teleportation device approximates the ideal case. One such measure is the *average fidelity* of the process. Consider a pure input state $|\psi\rangle_1$, which is imperfectly teleported such that Bob receives the generally mixed state ρ_3. The fidelity of the output state compared to the input state is given by

$$F(|\psi\rangle_1, \rho_3) = {}_1\langle\psi|\rho_3|\psi\rangle_1. \tag{4}$$

(Note that some authors define the fidelity to be the square root of this quantity.)

For a given distribution $\mathcal{C}$ of input states to be teleported, the average fidelity $\overline{F}$ is defined to be the weighted average of the fidelity over $\mathcal{C}$. In the experiment by Furusawa *et al.* [8], a distribution of coherent states with fixed amplitude varied over all phases was chosen to test the teleportation. It is also possible to employ a distribution over both amplitude and phase [13].

III. QUANTUM TELEPORTATION OF ENTANGLED COHERENT STATES

The average fidelity serves well as a figure of merit for certain applications. However, any measure quantifying the performance of a quantum teleportation device must be placed in the context of its intended use. In particular, one may ask how well a device performs the important task of teleporting entanglement.

Victor, who wishes to test Alice and Bob's quantum teleportation device, supplies Alice with a (possibly mixed) quantum state ρ, and after the teleportation, Bob returns a state ρ' to Victor. Victor can then perform measurements on ρ' to determine the success or failure of the quantum teleportation.

In offering their services, Alice and Bob can be expected to quote Victor a measure of performance of their quantum teleportation device (such as the average fidelity), as well as a restriction on the type of states that they can quantum teleport. For example, they may advertise an average fidelity $\overline{F}$ for quantum teleportation of coherent states sampled from some distribution. Depending on his intended use of the quantum teleportation device, these particular measures of performance may not be adequate.

A. Entangled coherent states

For example, Victor may wish to teleport one component of an entangled state, and ensure that the final state returned by Bob is still entangled with the system he kept. If Alice and Bob advertise that they can only teleport distributions of coherent states, it is important that the state ρ that Victor supplies to Alice is indeed in the allowed set. Consider the two-mode ECS [9],

$$|\Psi(\alpha,\beta)\rangle_{ab} = N(|\alpha\rangle_a|\beta\rangle_b - |\beta\rangle_a|\alpha\rangle_b), \quad \alpha \neq \beta, \tag{5}$$

with $N = [2 - 2\exp(-|\alpha-\beta|^2)]^{-1/2}$ being the normalization. This state is not separable, and thus possesses entanglement between modes a and b [14]. The reduced density matrix for mode b is

$$\begin{aligned}\rho_b &= \mathrm{Tr}_1(|\Psi(\alpha,\beta)\rangle_{ab}\langle\Psi(\alpha,\beta)|) \\ &= N^2(|\alpha\rangle_b\langle\alpha| + |\beta\rangle_b\langle\beta| - \langle\alpha|\beta\rangle|\alpha\rangle_b\langle\beta| - \langle\beta|\alpha\rangle|\beta\rangle_b\langle\alpha|).\end{aligned} \tag{6}$$

For the case where the overlap $\langle\alpha|\beta\rangle$ is negligibly small, this reduced density matrix is indistinguishable from

$$\rho_b = \frac{1}{2}(|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta|), \tag{7}$$

i.e., a mixture of two coherent states. Thus, a quantum teleportation device that functions for coherent-state inputs should function equally well for such a state. Note that the issues of overlaps can be avoided if Victor instead uses a two-level system for a (such as a two-level atom), and entangles coherent states of mode b to orthogonal states in a. States of this form have been experimentally realized [15].

B. Entanglement fidelity

The average fidelity may not be a good indicator of the quality of quantum teleportation. In particular, it overstates the capability to teleport entanglement. *Entanglement fidelity* [5,16] is a superior measure to quantify the ability of a process to preserve entanglement, and reduces to the standard fidelity for the case of pure states (e.g., coherent states).

Consider a process (quantum channel) described by a superoperator $\mathcal{E}$ and a generally mixed state ρ as input. Let $\rho' = \mathcal{E}(\rho)$ be the corresponding output state. We introduce $|\Gamma\rangle$ as a purification of ρ, i.e., a pure state obtained by introducing an ancilla system A such that $\mathrm{Tr}_A(|\Gamma\rangle\langle\Gamma|) = \rho$. The entanglement fidelity

$$\mathcal{F}_e(\rho,\mathcal{E}) \equiv \langle\Gamma|(\mathcal{I}_A \otimes \mathcal{E})(|\Gamma\rangle\langle\Gamma|)|\Gamma\rangle \tag{8}$$

has many exceptional properties; see Ref. [17]. It is independent of the choice of purification, meaning that it depends only on the reduced density matrix ρ_b supplied to Alice. This property will be useful in testing quantum teleportation, as Victor can choose any purification (any choice of entanglement between the state he supplies Alice and the state he retains) and achieve the same measure. Also, if $\rho = |\psi\rangle\langle\psi|$ is a pure state, the entanglement fidelity $\mathcal{F}_e(|\psi\rangle\langle\psi|,\mathcal{E})$ reduces to the standard fidelity $F(|\psi\rangle\langle\psi|,\mathcal{E}(|\psi\rangle\langle\psi|))$. Thus, the results of tests employing the entanglement fidelity can be directly compared to the fidelity of teleporting pure coherent states.

IV. NOISY QUANTUM TELEPORTATION

In this section, we consider a noisy quantum teleportation device that includes finite squeezing of the entanglement resource, propagation errors, imperfect detectors, and other errors that introduce a stochastic error given by a Gaussian distribution. We show that, whereas the average fidelity for quantum teleportation of coherent states is quite robust against such errors, the entanglement fidelity for the quantum teleportation of distributions of coherent states of the form (7) drops off very rapidly even with highly squeezed states and small errors.

In quantum teleportation, Alice must measure a complex amplitude α_0 via a joint measurement of the form (2) and report this measurement result to Bob. Bob then performs a displacement $D(\alpha_0)$ on his state conditioned on this result. If there is a measurement error, however, Alice may send to Bob the values $\alpha_0 + z_i$, where z_i is a complex error (an error in both position and momentum) sampled from some ensemble $Z = \{z_i\}$ with corresponding probabilities $P = \{p(z_i)\}$. Bob then performs the displacement $D(\alpha_0 + z_i)$ rather than $D(\alpha_0)$. (Equivalently, Bob's displacement operation could be subject to a similar error.) The result is that the quantum teleportation process will no longer be ideal; it becomes a noisy process described by some superoperator $\mathcal{E}$.

The probability distribution we employ is a Gaussian distribution with variance σ, defined such that vacuum noise has a variance 1/2. (Note that σ has the units of the square of the coherent-state complex amplitude α.) Using a perfect teleportation scheme [involving ideal EPR states of the form of Eq. (1) and ideal projective measurements given by Eq. (2)], but with a Gaussian-distributed error, the teleported state ρ' will be related to the input state ρ by

$$\rho' = \int \frac{d^2z}{\pi\sigma} \exp\left(-\frac{|z|^2}{\sigma}\right) D(z)\rho D^\dagger(z) \equiv \mathcal{E}_\sigma(\rho). \tag{9}$$

One can view $\mathcal{E}_\sigma$ as the transfer superoperator for this (noisy) process.

In Ref. [5], Braunstein and Kimble considered the effects of finite squeezing ($\eta < 1$) and imperfect detectors. Both of these effects lead to Gaussian noise, described by the same superoperator as given by Eq. (9). Also, propagation losses can be compensated for by linear amplification, which introduces an associated Gaussian noise described similarly. Thus, the variance for the total error is given by the sum of the variances for the individual errors as

$$\sigma = \sigma_G + \sigma_\eta + \sigma_\nu + \sigma_{\mathrm{other}}, \tag{10}$$

where σ_G is the variance for the noise introduced by linear amplification with gain G, $\sigma_\eta = \exp(-2\tanh^{-1}\eta)$ describes the effect of finite squeezing, $\sigma_\nu = (1-\nu^2)/\nu^2$ describes the noise due to finite homodyne detection efficiency ν, and σ_{other} describes other sources of Gaussian noise [5]. Again, all of these variances are defined such that $\sigma = 1/2$ is the level of vacuum noise. Thus, an effective σ describes the cummulative effects of a wide variety of noise and errors in the quantum teleportation process.

In a classical picture, without employing a squeezed resource ($\eta \to 0$), we find that $\sigma_\eta = 1$, i.e., the output state acquires two units of vacuum noise. This noise is what Braunstein and Kimble refer to as "quantum duty," or quduty [5]; one unit of vacuum noise is acquired by each pass across the quantum/classical border (one by Alice's measurement and one by Bob's reconstruction).

For an input state given by a pure coherent state $|\alpha\rangle$, it is straightforward to calculate the entanglement fidelity of the operation. (As the state is pure, the entanglement fidelity $\mathcal{F}_e$ will equal the standard fidelity F.) The entanglement fidelity for the coherent state $|\alpha\rangle$,

$$\mathcal{F}_e(|\alpha\rangle\langle\alpha|,\mathcal{E}_\sigma)=\frac{1}{1+\sigma}, \tag{11}$$

is independent of α. The lower bounds on fidelity for quantum teleportation discussed in the Introduction are clear from this equation. The bound of $F>1/2$ by Braunstein and Kimble [5] can only be satisfied if $\sigma<1$; i.e., the variance must be less than twice the vacuum noise, verifying the use of an entangled resource. The more stringent bound $F>2/3$, set by Grosshans and Grangier [6] using an argument of no cloning, requires $\sigma<1/2$ or up to one unit of vacuum noise.

If the state to be teleported is one mode of an ECS, the calculation of the entanglement fidelity is more involved. Fortunately, a purification of the state (6) is already provided: it is the ECS itself. As noted earlier, the entanglement fidelity is independent of the choice of purification. The entanglement fidelity for the noisy quantum teleportation of the b mode is given by

$$\begin{aligned}\mathcal{F}_e(\rho_b,\mathcal{E}_\sigma)&=\langle\Psi(\alpha,\beta)|(\mathcal{I}_a\otimes\mathcal{E}_{\sigma b})\\&\quad\times(|\Psi(\alpha,\beta)\rangle\langle\Psi(\alpha,\beta)|)|\Psi(\alpha,\beta)f\rangle\\&=\int\frac{d^2z}{\pi\sigma}\exp\left(-\frac{|z|^2}{\sigma}\right)\\&\quad\times|\langle\Psi(\alpha,\beta)|D_b(z)|\Psi(\alpha,\beta)\rangle|^2.\end{aligned} \tag{12}$$

The expression is complicated by the nonorthogonality of coherent states, but the overlap drops rapidly as the coherent states are increasingly separated in phase space. To simplify this expression, we assume that $|\alpha-\beta|$ is sufficiently large that we can ignore terms bounded above by $|\langle\alpha|\beta\rangle|^2=\exp(-|\alpha-\beta|^2)$. Again, we note that these overlaps can be avoided if Victor chooses to couple coherent states in b to orthogonal modes in a, and that this choice of purification gives identical results for the entanglement fidelity.

Using this assumption,

$$\begin{aligned}\mathcal{F}_e(\rho_b,\mathcal{E}_\sigma)&\simeq N^4\int\frac{d^2z}{\pi\sigma}\exp\left(-\frac{|z|^2}{\sigma}\right)\{|\langle\alpha|\alpha+z\rangle|^2+|\langle\beta|\beta+z\rangle|^2+2\,\mathrm{Re}(\exp\{i\,\mathrm{Im}[(\alpha-\beta)z^*]\}\langle\alpha+z|\alpha\rangle\langle\beta|\beta+z\rangle)\}\\&\simeq\int\frac{d^2z}{2\pi\sigma}\exp\left(-\frac{|z|^2}{\sigma}\right)\exp(-|z|^2)[1+\cos(2\,\mathrm{Im}((\alpha-\beta)z^*))]\simeq\frac{1}{2}\left(\frac{1}{1+\sigma}\right)[1+\exp(-|\alpha-\beta|^2\sigma_{\mathrm{eff}})],\end{aligned} \tag{13}$$

where $\sigma_{\mathrm{eff}}=\sigma/(1+\sigma)$.

The entanglement fidelity for the noisy quantum teleportation of one mode of an ECS differs from that of a pure coherent state [Eq. (11)] due to a term that drops exponentially in $|\alpha-\beta|^2$. As this term becomes negligibly small (for even small errors described by $\sigma\ll1$), the entanglement fidelity for the teleportation of ECSs approaches half the value for that of a pure coherent state.

In Fig. 1, we compare the standard fidelity for teleportation of a pure coherent state $|\alpha\rangle$ to the entanglement fidelity for teleporting the ECS with $\beta=-\alpha$ for two values, $\alpha=2$ and $\alpha=10$. One key feature to notice is that the entanglement fidelity for all cases is reduced significantly for a variance on the order of the vacuum noise ($\sigma\simeq1/2$). Thus, the precision of quadrature phase measurements must be very good on the scale of the standard quantum limit [18]. For the ECSs, the rapid decrease of the entanglement fidelity to approximately half that of the pure coherent states is clearly evident. Consider ECSs of the form $|\Psi(\alpha,-\alpha)\rangle$, with mean photon number $\overline{n}=|\alpha|^2$. In order to maintain a constant entanglement fidelity $\mathcal{F}_e(\rho_b,\mathcal{E}_\sigma)>1/2$, the variance of the errors must scale as $\sigma\sim1/\overline{n}$. Thus, quantum teleportation of a single mode of an ECS with high entanglement fidelity becomes increasingly difficult as the mean photon number of the state is increased.

Note that, for any distribution $\mathcal{C}$ of coherent states that leads to an average fidelity for quantum teleportation, one can also calculate an average entanglement fidelity both for pure coherent states and ECSs sampled from the same distribution. In the experiment of Furusawa *et al.*, an average fidelity of $\overline{F}=0.58$ has been obtained for the set $\mathcal{C}=\{|\alpha_0e^{i\varphi}\rangle;0\leqslant\varphi<2\pi\}$, where $|\alpha_0e^{i\varphi}\rangle$ is the coherent state with $|\alpha_0|^2\sim100$ being the mean photon flux and φ is the phase of the coherent state [8,19]. This phase is uniformly distributed over the domain $[0,2\pi)$. The experimental result [8] can be compared against our calculation for teleportation of ECSs of the form $|\Psi(\alpha_0,-\alpha_0)\rangle$ with $\alpha_0=10$, presented

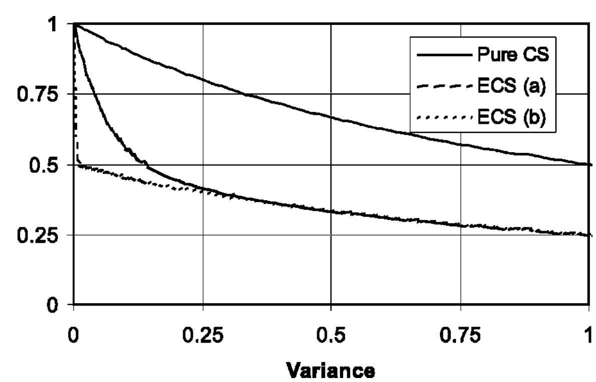

FIG. 1. Entanglement fidelity as a function of the standard deviation σ for the errors in noisy quantum teleportation. The entanglement fidelity is plotted for a pure coherent state (CS) $|\alpha\rangle$, and for entangled coherent states (ECS) of the form $|\Psi(\alpha,-\alpha)\rangle$ for (a) $\alpha=2$ and (b) $\alpha=10$. Note that a variance of 1 corresponds to two units of vacuum noise, i.e., two "quduties."

in Fig. 1. Whereas the entanglement fidelity for the pure coherent state is very high for small σ, the entanglement fidelity for the ECS is less than 0.5 even for $\sigma = 0.01$ (corresponding to a total error around 2% of the vacuum noise). Note that with otherwise perfect conditions (no propagation loss, detector noise, etc.), at least 8.5 dB of squeezing is required to achieve an entanglement fidelity of greater than 0.5 for this state; this amount of squeezing represents a target for high entanglement fidelity quantum teleportation of distributions of coherent states.

V. CONCLUSIONS

We have shown that the average fidelity is not necessarily a good measure of successful quantum teleportation, and in particular is shown to be a poor indicator for the capability of quantum teleportation to preserve entanglement. On the other hand, the entanglement fidelity provides a useful figure of merit for the quantum teleportation of entanglement. We demonstrate that entanglement with other systems can be used to test claims of quantum teleportation, even for a restricted set of allowed input states. In particular, ECSs can be used to test quantum teleportation devices that advertise only teleportation of mixtures of coherent states. We show that the entanglement fidelity of distributions of coherent states is extremely fragile, and can be drastically reduced from the fidelity of the pure coherent states by the effects of finite squeezing, imperfect detection, propagation errors, or small stochastic errors in Alice's measurements (or Bob's transformations).

An important application of teleporting coherent states is in a distributed quantum network that employs only Gaussian states and Gaussian-preserving operations, i.e., linear optics [20]. In such a network, the appropriate figure of merit for the teleportation of entanglement between nodes is clearly the entanglement fidelity.

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[1] C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] D. Gottesman and I.L. Chuang, Nature (London) **402**, 390 (1999).
[3] E. Knill, R. Laflamme, and G.J. Milburn, Nature (London) **409**, 46 (2001).
[4] J. Eisert, K. Jacobs, P. Papadopoulos, and M.B. Plenio, Phys. Rev. A **62**, 052317 (2000).
[5] S.L. Braunstein and H.J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[6] F. Grosshans and P. Grangier, Phys. Rev. A **64**, 010301(R) (2001).
[7] S.M. Tan, Phys. Rev. A **60**, 2752 (1999).
[8] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, Science **282**, 706 (1998).
[9] B.C. Sanders, Phys. Rev. A **45**, 7746 (1992); **46**, 2966 (1992); B.C. Sanders and D.A. Rice, *ibid.* **61**, 013805 (2000).
[10] X. Wang, Phys. Rev. A **64**, 022302 (2001). H. Jeong, M.S. Kim, and J. Lee, *ibid.* **64**, 052308 (2001).
[11] S.J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).
[12] S.J. van Enk, Phys. Rev. A **60**, 5095 (1999).
[13] W.P. Bowen, N. Treps, B.C. Buchler, R. Schnabel, T.C. Ralph, H.-A. Bachor, T. Symul, and P.-K. Lam, e-print quant-ph/0207179.
[14] A. Mann, B.C. Sanders, and W.J. Munro, Phys. Rev. A **51**, 989 (1995).
[15] M. Brune, E. Hagley, J. Dreyer, X. Matre, A. Maali, C. Wunderlich, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).
[16] B. Schumacher, Phys. Rev. A **54**, 2614 (1996).
[17] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[18] C.M. Caves, Phys. Rev. D **23**, 1693 (1981).
[19] T. C. Ralph (private communication).
[20] S.D. Bartlett, B.C. Sanders, S.L. Braunstein, and K. Nemoto, Phys. Rev. Lett. **88**, 097904 (2002).