Dressed-state approach to quantum systems

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Using the nonperturbative method of *dressed* states introduced in a previous publication [N. P. Andion, A. P. C. Malbouisson, and A. Mattos Neto, J. Phys. A **34**, 3735 (2001)], we study effects of the environment on a quantum-mechanical system, in the case in which the environment is modeled by an ensemble of noninteracting harmonic oscillators. This method makes it possible to separate the whole system into the *dressed* mechanical system and the *dressed* environment, in terms of which a nonperturbative approach is possible. When applied to the Brownian motion, we give explicit nonperturbative formulas for the classical path of the particle in the weak and strong coupling regimes. When applied to studying atomic behaviors in cavities, the method accounts for experimentally observed inhibition of atomic decay in small cavities.

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I. INTRODUCTION

Quantum-mechanical systems remain stable in the absence of interaction. When interacting with an environment they lose stability as a consequence of the interaction. A material body, for instance, an excited atom or molecule, or an excited nucleon, changes its state due to its interaction with the environment, e.g., the atom–electromagnetic field coupling in the case of an atom, or the quark-gluon interaction for a nucleon inside a nucleus. The understanding of the nature of the destabilization mechanism is important but is, in general, not an easy task, due to the fact that it is to a large extent modeled by the method, in general approximate, used to study the system. A very complete account on the subject, in particular as it applies to the study of the Brownian motion, can be found in Refs. [1,2]. From a general point of view, in modern physics, apart from computer calculations in lattice field theory, the only available method to treat the physics of interacting bodies, except for a few special cases, is perturbation theory. The perturbative solution to the problem is obtained by means of the introduction of bare, noninteracting fields, to which are associated bare quanta, the interaction being introduced order by order in powers of the coupling constant in the perturbative expansion for the observables. The perturbative method gives remarkably accurate results in quantum electrodynamics and in weak interactions. In high-energy physics, asymptotic freedom makes it possible to apply quantum chromodynamics in its perturbative form and very important results have been obtained in this way in the last decades [3]. However, in spite of its wide applicability, there are situations where the use of perturbation theory is not possible, as in the low-energy domain of quantum chromodynamics where confinement of quarks and gluons takes place, or are of little usefulness, as, for instance, in atomic physics, in resonant effects associated with the coupling of atoms with strong radio-frequency fields. These situations have led a long time to attempt to circumvect the limitations of perturbation theory, in particular, in situations where strong effective couplings are involved. In some nonperturbative approachs in statistical physics and constructive field theory, general theorems can be derived using cluster like expansions and other related methods [4]. In some cases, these methods lead to the rigorous construction of field-theoretical models (see, for instance, Ref. [5] and references therein), but, in spite of the rigor and in some cases the beauty of demonstrations, they are not of great usefulness in calculations of a predictive character. In another mathematical framework, there are a large number of successful attempts in the literature to circumvent the limitations of perturbation theory. In particular, there are methods to perform resummations of perturbative series (even if they are divergent), which amounts in some cases to analytically continue weak-coupling series to a strong-coupling domain [6–13]. For instance, starting from a function of a coupling constant g defined formally by means of a series (not necessarily convergent),

$$f(g)=\sum_{n=0}^{\infty} a_n g^n, \tag{1.1}$$

we can, under certain analyticity assumptions (the validity of the Watson-Nevanlinna-Sokal theorem, see, for instance, Ref. [14] and other references therein), define its Borel transform as the associated series,

$$B(b)=\sum_{n=0}^{\infty} \frac{a_n}{n!} b^n, \tag{1.2}$$

which has an analytic continuation on a strip along the real b axis from zero to infinity. It can be easily verified that the inverse Borel transform

$$\tilde{B}(g)=\frac{1}{g}\int_0^{\infty} db \;\; e^{-b/g} B(b), \tag{1.3}$$

reproduces formally the original series (1.1). From a physical point of view, the important remark is that the series $B(b)$ can be convergent and summed up even if the series (1.1) diverges. In this case, the inverse Borel tranform (1.3) de-

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fines a *function* of g, $\tilde{B}(g)$, which we can think of as the ''sum'' of the divergent series (1.1). This function $\tilde{B}(g)$ can be defined for values of g not necessarily small and, in this sense, we can perform an analytic continuation from a weak to a strong-coupling regime. Techniques of this type are of a predictive character and have been largely employed in recent years in quantum field theory literature.

Nevertheless, as a matter of principle, due to the nonvanishing of the coupling constant, the idea of a bare particle associated with a bare matter field is actually an artifact of perturbation theory and is physically meaningless. A charged physical particle is always coupled to the gauge field; in other words, it is always ''dressed'' by a cloud of quanta of the gauge field (photons, in the case of electrodynamics). In fact, as mentioned above, from a phenomenological point of view there are situations even within the scope of QED where perturbation methods are of little usefulness, for instance, resonant effects associated with the coupling of atoms with strong radio-frequency fields Ref. [15]. As remarked in Ref. [16], the theoretical understanding of these effects using perturbative methods requires the calculation of very high order terms in perturbation theory, what makes the standard Feynman diagrams technique practically unreliable. The trials of treating systems of this type nonperturbatively have led to the idea of the ''dressed atom,'' introduced originally in Refs. [17] and [18]. Since then this concept has been used to investigate several situations involving the interaction of atoms and electromagnetic fields, as, for instance, atoms embedded in a strong radio-frequency field background [19,20], and atoms in intense resonant laser beams [21]. In order to give a precise mathematical definition and a clear physical meaning to the idea of dressed atom, a crucial aspect is the nonlinear character of the problem involved in realistic situations, which, in genaral, does not allow that a rigorous definition of ''dressed atom'' could be given. A way to circumvent these mathematical difficulties is to assume that under certain conditions the coupled atom–electromagnetic field system may be approximated by the system composed of a harmonic oscillator coupled *linearly* to the field through some effective coupling constant g. This is the case in the context of the general QED linear response theory, where the electric dipole interaction gives the leading contribution to the radiation process (Refs. [22–24]). Also, in a slightly different context, recently a significant number of works have been devoted to the study of cavity QED, in particular to the theoretical investigation of higher-generation Schrödinger cat states in high-Q cavities, as has been done, for instance, in Ref. [25]. Linear approximations of this type have been applied along the last years in condensed-matter physics for studies of Brownian motion and in quantum optics to study decoherence, by assuming a linear coupling between a cavity harmonic mode and a thermal bath of oscillators at zero temperature, as has been done in Refs. [26] and [27].

In this paper we adopt a general physicist's point of view. We do not intend to describe all the specific features of a real nonlinear physical situation. Instead, we analyze a simplified linear version of the atom-field or particle-environment system and try to extract the most detailed information we can from this model. We will introduce *dressed* states by means of a precise and rigorous definition to solve our problem. Our *dressed* states can be viewed as a rigorous version of the semiqualitative idea of dressed atom mentioned above, which can be constructed in view of the linear character of our problem. We take a linear model in order to have a clearer understanding of what is one of the essential points, namely, the need for nonperturbative analytical treatments of coupled systems, which is the basic problem underlying the idea of a *dressed* quantum-mechanical system. Of course, such an approach to a realistic nonlinear system is an extremely hard task, and here we achieve what we think is a good agreement between physical reality and mathematical reliability.

Where the Brownian motion is concerned, there are usually two equivalent ways of modeling the environment (the thermal bath) to which the particle is coupled: to represent the thermal bath by a free field, as is done in the classical work of Ref. [1], or to consider the thermal bath as a reservoir composed of a large number of noninteracting harmonic oscillators (see, for instance, Refs. [28–31]). In both cases, exactly the same type of argument given above in the case of a charged particle applies *mutatis mutandis* to this system, we may speak of a ''dressing'' of the Brownian particle by the ensemble of the particles in the thermal bath. The Brownian particle should be always ''dressed'' by a cloud of quanta of the thermal bath. This should be true, in general, for any system in which a material body is coupled to an environment, no matter the specific nature of the environment and interaction involved.

In what follows we use the term ''particle'' or ''material body'' in a general manner, a particle may refer, for instance, to an atom coupled to a field, or to a Brownian particle coupled to a thermal bath, the two situations where we apply our formalism in this paper.

In recent publications [32,33] a method (*dressed* coordinates and *dressed* states) has been introduced that allows a nonperturbative approach to situations of the type described above, provided that the interaction between the parts of the system can be approximated by a linear coupling. More precisely, the method applies for all systems that can be described by a Hamiltonian of the form

$$H=\frac{1}{2}\left[p_0^2+\omega_0^2 q_0^2+\sum_{k=1}^{N}\left(p_k^2+\omega_k^2 q_k^2\right)\right]-q_0\sum_{k=1}^{N}c_k q_k, \tag{1.4}$$

where the subscript 0 refers to the ''material body'' and $k=1,2,\ldots,N$ refer to the harmonic environment modes. A Hamiltonian of this type, describing a linear coupling of a particle with an environment, has been used in Ref. [2] to study the quantum Brownian motion of a particle with the path-integral formalism. The limit $N\to\infty$ in Eq. (1.4) is understood. In the case of the coupled atom field system, this formalism recovers the experimental observation that excited states of atoms in sufficiently small cavities are stable. It allows to give formulas for the probability of an atom to remain excited for an infinitely long time, provided that it is

placed in a cavity of appropriate size [33]. For an emission frequency in the visible red, the size of such a cavity is in agreement with experimental observations [34,35]. We give a nonperturbative treatment to the system introducing some *dressed* coordinates that make it possible to divide the coupled system into two parts, the *dressed* material body and the *dressed* environment, which makes it unnecessary to work directly with the concepts of bare material body, bare environment, and interaction between them. In terms of these new coordinates *dressed* states are defined, and a nonperturbative approach to the time evolution of the system is possible. We investigate also the behavior of the system as a function of the strength of the coupling between the particle and the bath. In particular, we give explicitly nonperturbative formulas for the decay probability and for the classical path of the particle in the weak and strong coupling regimes.

II. THE EIGENFREQUENCY SPECTRUM AND THE DIAGONALIZING MATRIX

We consider for a moment, as in Ref. [32], the problem of a harmonic oscillator q_0 of (bare) frequency ω_0 coupled to N other oscillators q_i of frequencies ω_i, $i=1,2,\ldots,N$. In the limit $N\to\infty$ we recover our original situation of the coupling particle-bath after redefinition of divergent quantities, in a manner analogous to the naive mass renormalization done in field theories. The bilinear Hamiltonian (1.4) can be turned to principal axis by means of a point transformation,

$$q_\mu = t^r_\mu Q_r, \quad p_\mu = t^r_\mu P_r, \quad \mu=(0,\{k\}),$$
$$k=1,2,\ldots,N; \quad r=0,\ldots N, \tag{2.1}$$

performed by an orthonormal matrix $T=(t^r_\mu)$. The subscript $\mu=0$ and $\mu=k$ refer, respectively, to the particle and the harmonic modes of the bath and r refers to the normal modes. In terms of normal momenta and coordinates, the transformed Hamiltonian in principal axis reads

$$H=\frac{1}{2}\sum_{r=0}^{N}(P_r^2+\Omega_r^2 Q_r^2), \tag{2.2}$$

where the Ω_r's are the normal frequencies corresponding to the possible collective stable oscillation modes of the coupled system. The matrix elements t^r_μ are given by [32]

$$t^r_k=\frac{c_k}{(\omega_k^2-\Omega_r^2)}t^r_0, \quad t^r_0=\left[1+\sum_{k=1}^{N}\frac{c_k^2}{(\omega_k^2-\Omega_r^2)^2}\right]^{-1/2}, \tag{2.3}$$

with the condition

$$\omega_0^2-\Omega_r^2=\sum_{k=1}^{N}\frac{c_k^2}{\omega_k^2-\Omega_r^2}. \tag{2.4}$$

We take $c_k=\eta(\omega_k)^n$. In this case the environment is classified according to $n>1$, $n=1$, or $n<1$, respectively, as *supraohmic*, *ohmic*, or *subohmic*. For a subohmic environment the sum in Eq. (2.4) is convergent and the frequency ω_0 is well defined. For ohmic and supraohmic environments the sum on the right-hand side of Eq. (2.4) diverges, which makes the equation meaningless as it stands, a renormalization procedure being needed. In this case, as a first step we add and subtract the quantity $\Omega_r^2(\omega_k^2)^{n-1}$ to the numerator of the right-hand side of Eq. (2.4). After changing the term corresponding to the subtraction of $\Omega_r^2(\omega_k^2)^{n-1}$ to the left-hand side, Eq. (2.4) can be rewritten in the form

$$\omega_0^2-\eta^2\sum_{k=1}^{N}\frac{(\omega_k^2)^n-\Omega_r^2(\omega_k^2)^{n-1}}{\omega_k^2-\Omega_r^2}-\Omega_r^2=\sum_{k=1}^{N}\frac{\Omega_r^2(\omega_k^2)^{n-1}}{\omega_k^2-\Omega_r^2}. \tag{2.5}$$

If $n=1$ (*ohmic* system), this step is sufficient, the right-hand side of Eq. (2.5) is convergent, and we define from the left-hand side of Eq. (2.5) the renormalized frequency by Eq. (2.13) below. If $n\geqslant 1$ further steps are necessary, we add to and subtract from the numerator of the right-hand side of Eq. (2.5) the quantity $(\Omega_r^2)^2(\omega_k^2)^{n-2}$ and we change the term corresponding to its subtraction to the left-hand side. The process is continued until the series in right-hand member of the resulting equation is convergent. This is attained after a number of steps, and the result can be rewritten in the form

$$\omega_0^2-\delta\omega^2-\Omega_r^2=\eta^2\Omega_r^{2[[n]]}\sum_{k=1}^{N}\frac{1}{\omega_k^2-\Omega_r^2}, \tag{2.6}$$

where we have defined the counterterm

$$\delta\omega^2=\frac{\eta^2}{4}\sum_{k=1}^{N}\sum_{\alpha=1}^{[[n]]}\Omega_r^{2\alpha}\omega_k^{2(n-\alpha)}, \tag{2.7}$$

with the notation $[[n]]$ standing for the smallest integer containing n. Note that all the k-dependence characteristic of the numerator of the right-hand side of Eq. (2.4) has moved to the counterterm (2.7). From an analysis of Eq. (2.6) it can be seen that if $\omega_0^2>\delta\omega^2$, Eq. (2.6) yields only positive solutions for Ω^2, while if $\omega_0^2<\delta\omega^2$, Eq. (2.6) has a negative solution Ω_-^2. This means that in this case there is a damped collective normal mode that does not allow stationary configurations. Nevertheless, it should be remarked that in a different context, it is precisely this runaway solution that is related to the existence of a bound state in the Lee-Friedrechs model [36]. This solution is considered in Ref. [37] in the framework of a model to describe qualitatively the existence of bound states in particle physics. We will not consider this situation. We consider the situation in which all normal modes are harmonic, which corresponds to taking $\omega_0^2>\delta\omega^2$ and defining the *renormalized* frequency

$$\overline{\omega^2}=\omega_0^2-\delta\omega^2, \tag{2.8}$$

in terms of which Eq. (2.6) becomes

$$\overline{\omega^2}-\Omega_r^2=\eta^2\sum_{k=1}^{N}\frac{\Omega_r^{2[[n]]}}{\omega_k^2-\Omega_r^2}. \tag{2.9}$$

We see that in the limit $N\to\infty$ the above procedure is exactly analogous to the naive mass renormalization in quantum field theory: the addition of a counterterm $-\delta\omega^2 q_0^2$ allows one to compensate the infinity of ω_0^2 in such a way as to leave a finite, physically meaningful renormalized frequency $\bar{\omega}$. This simple renormalization scheme was originally introduced in Ref. [38].

To proceed, we take the constant η as $\eta=\sqrt{2g\Delta\omega}$, $\Delta\omega$ being the interval between two neighboring bath frequencies (supposed uniform) and where g is some constant [with dimension of (frequency)$^{2-\eta}$]. For reasons that will become apparent later, we restrict ourselves to the physical situations in which the environment frequencies ω_k can be written in the form

$$\omega_k=2k\pi/L, \quad k=1,2,\ldots. \tag{2.10}$$

Then, using the formula

$$\sum_{k=1}^{N}\frac{1}{(k^2-u^2)}=\left[\frac{1}{2u^2}-\frac{\pi}{u}\cot(\pi u)\right], \tag{2.11}$$

Eq. (2.9) can be written in closed form,

$$\cot\left(\frac{L\Omega}{2c}\right)=\frac{\Omega^3}{\pi g\Omega^{2[[n]]}}+\frac{c}{L\Omega}\left(1-\frac{\bar{\omega}^2L\Omega^2}{\pi gc\Omega^{2[[n]]}}\right). \tag{2.12}$$

For an ohmic environment we have $c_k=\eta\omega_k$ and $\delta\omega^2=N\eta^2$. Taking in Eq. (2.6) $\omega_0^2>N\eta^2$, the *renormalized* oscillator frequency $\bar{\omega}$ is given by

$$\bar{\omega}=\sqrt{\omega_0^2-N\eta^2} \tag{2.13}$$

and the eigenfrequency spectrum for an *ohmic* environment is given by the equation

$$\cot\left(\frac{L\Omega}{2c}\right)=\frac{\Omega}{\pi g}+\frac{c}{L\Omega}\left(1-\frac{\bar{\omega}^2L}{\pi gc}\right). \tag{2.14}$$

The solutions of Eq. (2.14) and Eq. (2.12) with respect to Ω give the spectrum of eigenfrequencies Ω_r corresponding to the collective normal modes.

The transformation matrix elements turning the material body-bath system to principal axis is obtained in terms of the physically meaningful quantities Ω_r, $\bar{\omega}$, after some rather long but straightforward manipulations analogous to the procedure in Ref. [32]. They read

$$t_0^r=\frac{\eta\Omega_r}{\sqrt{(\Omega_r^2-\bar{\omega}^2)^2+\frac{\eta^2}{2}(3\Omega_r^2-\bar{\omega}^2)+\pi^2g^2\Omega_r^2}},$$

$$t_k^r=\frac{\eta\omega_k}{\omega_k^2-\Omega_r^2}t_0^r. \tag{2.15}$$

III. THE DRESSED PARTICLE IN AN OHMIC ENVIRONMENT

To fix our framework and to give precise applications of our formalism, we study in this paper an *ohmic* environment. The normalized eigenstates of our system (eigenstates of the Hamiltonian in principal axis) can be written in terms of normal coordinates,

$$\begin{aligned}\langle Q|n_0,n_1,\ldots\rangle&\equiv\phi_{n_0n_1n_2\cdots}(Q,t)\\&=\prod_s\left[\sqrt{\frac{2^{n_s}}{n_s!}}H_{n_s}\left(\sqrt{\frac{\Omega_s}{\hbar}}Q_s\right)\right]\Gamma_0(Q)\\&\quad\times\exp\left(-i\sum_s n_s\Omega_s t\right),\end{aligned} \tag{3.1}$$

where H_{n_s} stands for the n_sth Hermite polynomial and Γ_0 is the normalized vacuum eigenfunction.

Next we intend to divide the system into the *dressed* particle and the *dressed* environment by means of some conveniently chosen *dressed* coordinates, q_0' and q_j' associated, respectively, with the *dressed* particle and the *dressed* oscillators composing the environment. These coordinates will allow a natural division of the system into the dressed (physically observed) particle and the *dressed* environment. The dressed particle will contain automatically all the effects of the environment on it. Clearly, these dressed coordinates should not be introduced arbitrarily. Since our problem is linear, we will require a linear transformation between the normal and *dressed* coordinates [different from the transformation (2.1) linking the normal to the bare coordinates]. Also, we demand the physical condition of vacuum stability. We assume that at some given time ($t=0$) the system is described by *dressed* states, whose wave functions are defined by

$$\begin{aligned}\psi_{\kappa_0\kappa_1,\ldots}(q')&=\prod_\mu\left[(2^{-\kappa_\mu}\kappa_\mu!)^{-1/2}H_{\kappa_\mu}\left(\sqrt{\frac{\bar{\omega}_\mu}{\hbar}}q_\mu'\right)\right]\\&\quad\times\Gamma_0(q'),\end{aligned} \tag{3.2}$$

where $q_\mu'=q_0'$, q_i', $\bar{\omega}_\mu=(\bar{\omega},\omega_i)$ and Γ_0 is the invariant ground-state eigenfunction introduced in Eq. (3.1). Note that the above wave functions will evolve in time in a more complicated form than the unitary evolution of the eigenstates (3.1), since these wave functions are not eigenstates of the diagonal Hamiltonian (1.4). It is precisely the nonunitary evolution of these wave functions which will allow (see below) a nonperturbative study of the radiation and dissipation processes of the particle.

In order to satisfy the physical condition of vacuum stability (invariance under a tranformation from normal to *dressed* coordinates) we remember that the the ground-state eigenfunction of the system has the form

$$\Gamma_0(Q)\propto\exp\left(-\frac{1}{2\hbar}\sum_{r=0}^{N}\Omega_rQ_r^2\right), \tag{3.3}$$

and we require that the ground state in terms of the *dressed* coordinates should have the form

$$\Gamma_0(q') \propto \exp\left(-\frac{1}{2\hbar} \sum_{\mu=0}^{N} \overline{\omega}_\mu (q'_\mu)^2 \right). \tag{3.4}$$

From Eqs. (3.3) and (3.4) it can be seen that the vacuum invariance requirement is satisfied if we define *dressed* coordinates by

$$\sqrt{\overline{\omega}_\mu} q'_\mu = \sum_{r=0}^{N} t^r_\mu \sqrt{\Omega_r} Q_r . \tag{3.5}$$

These *dressed* coordinates are new *collective* coordinates, different from the bare coordinates q_0, $\{q_i\}$ describing the bare particle and the free field modes, and also from the normal (collective) coordinates $\{Q_r\}$. Indeed these *dressed* coordinates are related to the bare coordinates by [32]

$$q'_\mu = \sum_\nu \alpha_{\mu\nu} q_\nu, \quad \alpha_{\mu\nu} = \frac{1}{\sqrt{\overline{\omega}_\mu}} \sum_r t^r_\mu t^r_\nu \sqrt{\Omega_r}. \tag{3.6}$$

As we have already mentioned above, our *dressed* states, given by Eq. (3.2), are *collective* but *nonstable* states, linear combinations of the (stable) eigensatates (3.1) defined in terms of the normal modes. The coefficients of these combinations are given in Eq. (4.5) below and explicit formulas for these coefficients for an interesting physical situation are given in Eq. (4.11). This gives a complete and rigorous definition of our dressed states. Moreover, our dressed states have the interesting property of distributing the energy initially in a particular dressed state, among itself and all other dressed states with precise and well-defined probability amplitudes [32]. We *choose* these dressed states as physically meaningful and we test successfully this hypothesis by studying the radiation process by an atom in a cavity. In both cases, of a very large or a very small cavity, our results are in agreement with experimental observations.

Having introduced *dressed* coordinates and *dressed states*, in the following section we will apply these concepts to study the time evolution of the expectation value of the particle coordinate.

IV. BROWNIAN MOTION AT ZERO TEMPERATURE

As a first application of our formalism we consider the study of Brownian motion. The Brownian particle is modeled by a harmonic oscillator coupled to an *ohmic* environment, the whole system being described by the Hamiltonian (1.4). This model for the Brownian motion is in fact not new and has been implemented using the path integral formalism in, for instance, Refs. [2,30,31]. In this approach, an effective action for the Brownian particle is obtained, which, in general, is very complicated and nonlocal in time. From this effective action, an equation for the classical path of the Brownian particle can be derived. However, this equation obtained from the effective action is a very complicated integro-differential equation that cannot be solved analytically. But in general terms it describes the expected damped behavior of the particle.

We will approach this problem using the *dressed* states introduced in the preceding section, and we will treat in detail the case in which the environment is at zero temperature (which corresponds to considering the environment initially in its ground state). Our method will account for the expected behavior in a simpler way than the usual path integral approach.

We assume, as usual, that initially the Brownian particle and the environment are decoupled and that the coupling is turned on suddenly at some given time that we choose at $t=0$. Since we treat here the case in which the environment is at zero temperature, our assumption is that the initial system can be described by a pure *dressed* state. The environment state at zero temperature should be described by its *dressed* ground state. Thus we can write the initial state of the system particle-environment in the form [in the following, primed labels refer to dressed states $\psi(q')$ and unprimed labels refer to the eigenstates $\phi(Q)$],

$$|\lambda, n'_1, n'_2, \ldots; t=0\rangle = |\lambda\rangle |n'_1, n'_2, \ldots\rangle. \tag{4.1}$$

In the above equation $|\lambda\rangle$ is the initial *dressed state* of the particle [see Eq. (4.2) below] and $|n'_1, n'_2, \ldots\rangle$ is the initial *dressed* state of the environment, whose wave functions $\psi_{n'_0 n'_1 \ldots}(q')$ are given by Eq. (3.2) (in the following we will take $n'_1 = n'_2 = \cdots = 0$, corresponding to the environment at zero temperature). To proceed, we recall that the classical path in the case of the quantum harmonic oscillator is given by the mean value of the operator position in a coherent state. In our formalism, we define $|\lambda\rangle$ as a *dressed* coherent state given by

$$|\lambda\rangle = e^{-|\lambda|^2/2} \sum_{n'_0=0}^{\infty} \frac{(\lambda)^{n'_0}}{\sqrt{n'_0!}} |n'_0\rangle, \tag{4.2}$$

and accordingly the classical path of the Brownian particle should be given by the time evolution of the *dressed* particle position operator in the dressed coherent state (4.1). It is useful to examinate first the time evolution of the initial coherent dressed state as given by Eq. (4.1). Replacing Eq. (4.2) in Eq. (4.1) we obtain,

$$|\lambda, n'_1, n'_2, \ldots; t=0\rangle = e^{-|\lambda|^2/2} \sum_{n'_0=0}^{\infty} \frac{\lambda^{n'_0}}{\sqrt{n'_0!}} |n'_0 n'_1, \ldots\rangle. \tag{4.3}$$

Now, since the eigenstates $|n_0, n_1, \ldots\rangle$ form a complete basis [stable states having eigenfunctions given by Eq. (3.1)], we can write Eq. (4.1) as

$$|\lambda,n_1',n_2',\ldots;t=0\rangle = e^{-|\lambda|^2/2}\sum_{n_0'=0}^{\infty}\sum_{\{n_r\}}\frac{\lambda^{n_0'}}{\sqrt{n_0'!}}T^{n_0,n_1,\ldots}_{n_0',n_1',\ldots}|n_0,n_1,\ldots\rangle, \tag{4.4}$$

where $\{n_r\}=(n_0,n_1,n_2,\ldots)$ and

$$T^{n_0,n_1,\ldots}_{n_0',n_1',\ldots} = \langle n_0,n_1,\ldots|n_0',n_1',\ldots\rangle = \int dQ\,\phi_{n_0,n_1,\ldots}(Q)\psi_{n_0',n_1',\ldots}(q'). \tag{4.5}$$

Since $|n_0,n_1,\ldots\rangle$ are eigenvectors of the Hamiltonian (2.2), the time evolution of Eq. (4.4) is given by

$$|\lambda,n_1',n_2',\ldots;t\rangle = e^{-|\lambda|^2/2}\sum_{n_0'=0}^{\infty}\sum_{\{n_r\}}\frac{\lambda^{n_0'}}{\sqrt{n_0'!}}T^{n_0,n_1,\ldots}_{n_0',n_1',\ldots}\exp\left(-i\sum_r \Omega_r(n_r+1/2)t\right)|n_0,n_1,\ldots\rangle. \tag{4.6}$$

Now we can compute $q_\lambda'(t)$, the time-dependent mean value for the dressed oscillator position operator, i.e., the mean value of the *dressed* particle position operator taken in the *dressed* coherent state (4.1),

$$q_\lambda'(t)=\langle\lambda,n_1',n_2',\ldots;t|q_0'|\lambda,n_1',n_2',\ldots;t\rangle = \exp^{-|\lambda|^2}\sum_{n_0',m_0'}\sum_{\{n_r,m_r\}}\frac{(\lambda^*)^{m_0'}}{\sqrt{m_0'!}}\frac{\lambda^{n_0'}}{\sqrt{n_0'!}}T^{m_0,m_1,\ldots}_{m_0',n_1',\ldots}\,T^{n_0,n_1,\ldots}_{n_0',n_1',\ldots}\exp\left(-i\sum_r \Omega_r(n_r-m_r)t\right)\langle m_0,m_1,\ldots|q_0'|n_0,n_1,\ldots\rangle. \tag{4.7}$$

Using Eq. (3.5) for $\mu=0$ and the formula

$$\langle m_\alpha|Q_\alpha|n_\alpha\rangle = \sqrt{\frac{\hbar}{2\Omega_\alpha}}(\sqrt{n_\alpha}\;\delta_{m_\alpha,n_\alpha-1}+\sqrt{n_\alpha+1}\;\delta_{m_\alpha,n_\alpha+1}), \tag{4.8}$$

in Eq. (4.7) it is easy to obtain

$$q_\lambda'(t)=e^{-|\lambda|^2}\sqrt{\frac{h}{2\omega}}\sum_s\sum_{n_0',m_0'}\sum_{\{n_r\}}t_0^s\sqrt{n_s+1}\;T^{n_0,n_1,\ldots,n_\alpha\cdots}_{n_0',n_1',\ldots}\;T^{n_0,n_1,\ldots,(n_s+1)\cdots}_{n_0',n_1',\ldots}\left[\frac{(\lambda^*)^{m_0'}}{\sqrt{m_0'!}}\frac{\lambda^{n_0'}}{\sqrt{n_0'!}}e^{-\Omega_s t}+\frac{(\lambda^*)^{n_0'}}{\sqrt{n_0'!}}\frac{\lambda^{m_0'}}{\sqrt{m_0'!}}e^{\Omega_s t}\right]. \tag{4.9}$$

As we have mentioned above, the situation in which the environment is at zero temperature corresponds to $n_1'=n_2'=\ldots=0$. In this case from Eqs. (4.5), (3.2), (3.5), (3.1), and with the help of the theorem [39]

$$\frac{1}{n_0'!}\left[\sum_r (t_\mu^r)^2\right]^{m/2}H_{n_0'}\left(\frac{\sum_r t_\mu^r\sqrt{\frac{\Omega_r}{\hbar}}Q_r}{\sqrt{\sum_r (t_\mu^r)^2}}\right)=\sum_{m_0+m_1+\cdots=n_0'}\frac{(t_\mu^0)^{m_0}(t_\mu^1)^{m_1}\cdots}{m_0!m_1!\cdots}H_{m_0}\left(\sqrt{\frac{\Omega_0}{\hbar}}Q_0\right)H_{m_1}\left(\sqrt{\frac{\Omega_1}{\hbar}}Q_1\right)\cdots, \tag{4.10}$$

we get

$$T^{n_0,n_1,n_2,\ldots}_{n_0,0,0,\ldots} = \sqrt{\frac{n_0'!}{n_0!n_1!\cdots}}(t_0^0)^{n_0}(t_0^1)^{n_1}(t_0^2)^{n_2}\delta_{m_0',n_0+n_1+n_2+\cdots}. \tag{4.11}$$

Replacing Eq. (4.11) in Eq. (4.9), we obtain after some straightforward calculations

$$q_\lambda'(t)=\sqrt{\frac{h}{2\omega}}[\lambda f^{00}(t)+\lambda^* f^{00*}(t)], \tag{4.12}$$

where

$$f^{00}(t)=\sum_s (t_0^s)^2 e^{-i\Omega_s t}. \tag{4.13}$$

From Refs. [32,33] we recognize the function $f^{00}(t)$ as the probability amplitude that at time t the *dressed* particle still

be excited, if it was initially (at $t=0$) in the first excited level. We see that underlying our dressed states formalism is a unified way to study two physically different situations, the radiation process and Brownian motion. To illustrate this unifying aspect of our formalism, in the following section we perform a study of the radiation process of the dressed particle.

Returning to the study of the Brownian particle, we see that to obtain an expression for the classical path we have to perform the sum appearing in Eq. (4.13) and replace the result in Eq. (4.12). If we assume, as is currently done in studies of the Brownian motion, that the environment distributes itself over the whole free space, its frequencies ω_k should have a continuous distribution. This continuum can be realized simply taking the limit $L\to\infty$ in Eq. (2.10). In this case the matrix elements t_0^r, given by Eq. (2.15), become

$$t_0^r = \lim_{\Delta\Omega\to\infty} \frac{\sqrt{2g}\Omega\sqrt{\Delta\Omega}}{\sqrt{(\Omega^2-\bar{\omega}^2)^2+\pi^2 g^2\Omega_r^2}}, \tag{4.14}$$

and the function $f^{00}(t)$ in Eq. (4.13) can be written in the form

$$f^{00}(t) = \int_0^\infty \frac{2g\Omega^2 e^{-i\Omega t}d\Omega}{(\Omega^2-\bar{\omega}^2)^2+\pi^2 g^2\Omega^2}. \tag{4.15}$$

Before going ahead let us define a "driving parameter" κ by

$$\kappa = \sqrt{\bar{\omega}^2 - \frac{\pi^2 g^2}{4}} \tag{4.16}$$

and let us study the above integral $f^{00}(t)$ in the different cases (a) $\kappa^2>0$, (b) $\kappa^2=0$, and (c) $\kappa^2<0$. The extreme cases in (a) and (c), $\kappa^2\gg 0$ or $\kappa^2\ll 0$ correspond, respectively, to the situations of a *weak* coupling between the particle and the environment ($g\ll\bar{\omega}$) or of a *strong* coupling ($g\gg\bar{\omega}$). We get for the above situations

(a) $\kappa^2>0$,

$$f^{00}(t) = \left(1-\frac{i\pi g}{2\kappa}\right)e^{-i\kappa t-\pi g t/2}+2iJ(t). \tag{4.17}$$

(b) $\kappa^2=0$,

$$f^{00}(t) = \left(1-\frac{\pi g t}{2}\right)e^{-\pi g t/2}+2iJ(t), \tag{4.18}$$

and (c) $\kappa^2<0$,

$$f^{00}(t) = \frac{1}{2}\left\{\left(1+\frac{\pi g}{2\tilde{\omega}}\right)e^{-(\pi g/2+|\kappa|)t} + \left(1-\frac{\pi g}{2|\kappa|}\right)e^{-(\pi g/2-|\kappa|)t}\right\}+2iJ(t), \tag{4.19}$$

where

$$J(t) = 2g\int_0^\infty dy \frac{y^2 e^{-yt}}{(y^2+\bar{\omega}^2)^2-\pi^2 g^2 y^2}. \tag{4.20}$$

Replacing the above equations in Eq. (4.12) we obtain for the classical path of the Brownian particle at zero temperature the following expressions:

$$q'_\lambda(t) = \sqrt{\frac{\hbar\bar{n}}{2\bar{\omega}}}\left\{\left[2\cos(\kappa t+\delta)-\frac{\pi g}{\kappa}\sin(\kappa t+\delta)\right]e^{-\pi g t/2} + 2\sin\delta J(t)\right\} \qquad (\kappa>0), \tag{4.21}$$

$$q'_\lambda(t) = \sqrt{\frac{\hbar\bar{n}}{2\bar{\omega}}}\left[2(\cos\delta)\left(1-\frac{\pi g}{2}t\right)e^{-\pi g t/2} + 2(\sin\delta)J(t)\right] \qquad (\kappa=0), \tag{4.22}$$

$$q'_\lambda(t) = \sqrt{\frac{\hbar\bar{n}}{2\bar{\omega}}}\left[2(\cos\delta)\left(\cosh|\kappa|t-\frac{\pi g}{2|\kappa|}\sinh|\kappa|t\right)e^{-\pi g t/2} + 2(\sin\delta)J(t)\right] \qquad (\kappa<0). \tag{4.23}$$

In the above equations we have written $\lambda=\sqrt{\bar{n}}e^{-i\delta}$, with $\bar{n}$ being the mean value for the number operator in the coherent state. Equations (4.21) to (4.23) give the expected behavior for the classical path of the Brownian particle. Apart from a parcel containing the integral $J(t)$, these equations describe the behavior of a damped oscillator in the three regimes corresponding to $\kappa>0$, $\kappa=0$, and $\kappa<0$, with a damping coefficient equal to πg. The above formulas describe the exact behaviors for $\delta=0$, which corresponds to a real value of the coherence parameter λ. The integral $J(t)$ in Eqs. (4.17)–(4.19) can be evaluated for large times, $t\gg 1/\bar{\omega}$. We obtain

$$J(t)\approx\frac{4g}{\bar{\omega}^4 t^3} \quad \left(t\gg\frac{1}{\bar{\omega}}\right). \tag{4.24}$$

Using Eq. (4.24) in Eq. (4.12) and remarking that for very large times the power behavior $\sim t^{-3}$ dominates over the exponential decay, we obtain identical asymptotic behaviors in the three regimes above,

$$q'_\lambda(t)\approx\sqrt{\frac{\hbar\bar{n}}{2\bar{\omega}}}\frac{8g}{\bar{\omega}^4 t^3}\sin\delta \quad \left(\kappa>0,\kappa=0,\kappa<0;\quad t\gg\frac{1}{\bar{\omega}}\right). \tag{4.25}$$

The path behavior in the different coupling regimes can be obtained from Eqs. (4.21) to (4.23) and Eq. (4.16). In the strong coupling regime, $\kappa^2\ll 0$ ($g\gg\bar{\omega}$), we obtain

$$q'_\lambda(t) \approx \sqrt{\frac{\hbar \bar{n}}{2\bar{\omega}}} \cos\delta \left(\frac{2\bar{\omega}^2}{\pi g} \right) e^{-\bar{\omega}^2 t/\pi g} + 2(\sin\delta) J(t) \tag{4.26}$$

and in the weak coupling regime, $\kappa^2 \gg 0$ ($g \ll \bar{\omega}$), we obtain from Eqs. (4.16) and (4.21),

$$q'_\lambda(t) \approx \sqrt{\frac{\hbar \bar{n}}{2\bar{\omega}}} \left[2\cos(\bar{\omega}t + \delta) - \frac{\pi g}{\bar{\omega}} \sin(\bar{\omega}t + \delta) \right] e^{-\pi g t/2} + 2(\sin\delta) J(t). \tag{4.27}$$

We see that the behaviors are quite different in the two situations, for not very large values of the time t, for which the exponential decay dominates over the power-law decay of $J(t)$: an oscillatory damped behavior with time in the weak coupling regime, while in the strong coupling regime the expected dressed coordinate value has an exponential decay. Again, asymptotically $J(t)$ dominates and both behaviors are identical, obeying a power-law decay $\sim t^{-3}$. In the following section we apply our formalism to the study of the radiation process.

V. THE RADIATION PROCESS

In this section we study the radiation process of the *dressed* particle when it is prepared in such a way that initially it is in its first excited state. We shall consider two situations, the particle in free space and the particle confined in a cavity of diameter L.

A. The particle in free space

In this case the spectrum of the frequencies ω_k has a continuous distribution as we have seen in the preceding section, and the function $f^{00}(t)$ is given by Eqs. (4.17)–(4.19). Combining these equations with Eq. (4.24), we obtain for the probability that the dressed particle still remain in its first excited state at a time $t \gg 1/\bar{\omega}$, the following expressions:

$$|f^{00}(t)|^2 = \left(1 + \frac{\pi^2 g^2}{4\bar{\omega}^2} \right) e^{-\pi g t} - e^{-\pi g t/2} \times \left[\frac{8g}{\bar{\omega}^4 t^3} \left(\sin\kappa t + \frac{\pi g}{2\kappa} \cos\kappa t \right) \right] + \frac{16 g^2}{\bar{\omega}^8 t^6} \quad (\kappa > 0), \tag{5.1}$$

$$|f^{00}(t)|^2 = \left(1 - \frac{\pi g}{2} t \right)^2 e^{-\pi g t} + \frac{16 g^2}{\bar{\omega}^8 t^6} \quad (\kappa = 0), \tag{5.2}$$

and

$$|f^{00}(t)|^2 = \left(\cosh|\kappa| t - \frac{\pi g}{2|\kappa|} \sinh|\kappa| t \right)^2 e^{-\pi g t} + \frac{16 g^2}{\bar{\omega}^8 t^6} \quad (\kappa < 0). \tag{5.3}$$

In the *weak* coupling regime $\kappa \gg 0$, we obtain from Eq. (5.1) that the probability that the particle be still excited at time $t \gg 1/\bar{\omega}$ if it was in the first excited level at $t = 0$, obeys a modified exponential decay law,

$$|f^{00}(t)|^2 \approx e^{-\pi g t} + \frac{16 g^2}{\bar{\omega}^8 t^6}. \tag{5.4}$$

In the strong coupling regime, $\kappa \ll 0$, we obtain from Eq. (5.3)

$$|f^{00}(t)|^2 \approx \left(\frac{\bar{\omega}}{\pi^2 g^2} \right) e^{-2\bar{\omega}^2 t/\pi g} + \frac{16 g^2}{\bar{\omega}^8 t^6}. \tag{5.5}$$

In both cases we see from Eqs. (5.4) and (5.5) that asymptotically the probability that the dressed particle be still excited at a very large time t obeys a power law, $|f^{00}(t)|^2|_{t\to\infty} \approx 16 g^2/\bar{\omega}^8 t^6$. However, in the weak coupling regime we can see that this behavior is dominant only for extremely large values of t, where the probability $|f^{00}(t)|^2$ is vanishingly small. For lower values of t (but satisfying the condition $t \gg 1/\bar{\omega}$), the behavior is dominated by the exponential law. For instance, let us take $\bar{\omega} = 4.0 \times 10^{14}$, $g = \bar{\omega}/137$. In this case the condition $t \gg 1/\bar{\omega}$ corresponds to $t \gg 2.5 \times 10^{-15}$ s. A numerical analysis of Eq. (5.4) with these data shows that for t in the interval 10^{-13} s $< t < 10^{-12}$ s, the curve describing the function in Eq. (5.4) is practically indistinguishable from the pure exponential $\exp(-\pi g t)$. For those values of g and $\bar{\omega}$ the contribution from the monomial term in Eq. (5.4) is negligible. In fact, this is valid for the exact probability. In Fig. 1 plots of the probability $|f^{00}(t)|^2$ from the *exact* Eq. (4.17) and from the pure exponential decay law $\exp(-\pi g t)$ are superposed for the time interval $0 \leqslant t \leqslant 10^{-12}$ s and the same values of g and $\bar{\omega}$ as before. They are completely indistinguishable for the emission frequency and the time interval considered. Similar results hold for Eq. (5.5) in the case of strong coupling $g \gg \bar{\omega}$. We can also see that we recover with our formalism the fact that the decay of the particle is enhanced for a strong coupling as compared to the weak coupling. We emphasize that the different behaviors described in Eqs. (5.4) and (5.5) are due to the fact that in the two situations the system obey *different* decay laws and that this fact *cannot* be inferred from perturbation theory. It is a consequence of the *dressed* states approach.

B. Behavior of the confined system

Let us now consider the *ohmic* system in which the particle is placed in the center of a cavity of diameter L, in the case of a very small L, i.e., that satisfies the condition of being much smaller than the coherence length, $L \ll 2c/g$. We

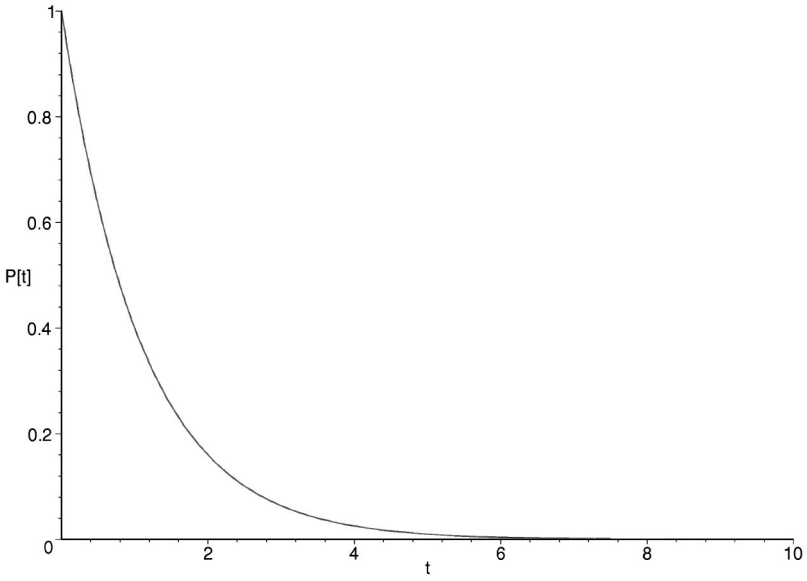

FIG. 1. Plot of the superposition of the probability of the particle being still excited at a time t, from the exact probability amplitude (4.17) and from the exponential decay law $\exp(-\pi g t)$. Both probabilities are commonly named $P[t]$. Time is in units of 10^{-13} s.

note that from a physical point of view, L stands for either the diameter of a spherical cavity or the spacing between infinite parallel mirrors. To obtain the eigenfrequency spectrum, we remark that from a graphical analysis of Eq. (2.14) it can be seen that in the case of small values of L, its solutions are very near the frequency values corresponding to the asymptotes of the curve $\cot(L\Omega/2c)$, which correspond to the environment modes $\omega_i = i2\pi c/L$, except the smallest eigenfrequency Ω_0. The larger the solutions taken, the nearer they are to the values corresponding to the asymptotes. For instance, for a value of L of the order of 2×10^{-2} m and $\bar{\omega} \sim 10^{10}$/s, only the lowest eigenfrequency Ω_0 is significantly different from the field frequency corresponding to the first asymptote, all the other eigenfrequencies Ω_k, $k=1,2,\ldots$ being very close to the field modes $k2\pi c/L$. For higher values of $\bar{\omega}$ (and lower values of L) the differences between the eigenfrequencies and the field mode frequencies are still smaller. Thus to solve Eq. (2.14) for the larger eigenfrequencies, we expand the function $\cot(L\Omega/2c)$ around the values corresponding to the asymptotes. We write

$$\Omega_k = \frac{2\pi c}{L}(k+\epsilon_k), \qquad k=1,2,\ldots, \tag{5.6}$$

with $0<\epsilon_k<1$, satisfying the equation

$$\cot(\pi\epsilon_k) = \frac{2c}{gL}(k+\epsilon_k) + \frac{1}{(k+\epsilon_k)}\left(1-\frac{\bar{\omega}^2 L}{2\pi g c}\right). \tag{5.7}$$

But since for a small value of L every ϵ_k is much smaller than 1, Eq. (5.7) can be linearized in ϵ_k, giving

$$\epsilon_k = \frac{4\pi g c L k}{2(4\pi^2 c^2 k^2 - \bar{\omega}^2 L^2}. \tag{5.8}$$

Equations (5.6) and (5.8) give approximate solutions to the eigenfrequencies Ω_k, $k=1,2,\ldots$.

To solve Eq. (2.14) with respect to the lowest eigenfrequency Ω_0, let us assume that it satisfies the condition $\Omega_0 L/2c \ll 1$ (we will see below that this condition is compatible with the condition of a small L as defined above). Inserting the condition $\Omega_0 L/2c \ll 1$ in Eq. (2.14) and keeping up to quadratic terms in Ω, we obtain the solution for the lowest eigenfrequency Ω_0,

$$\Omega_0 = \frac{\bar{\omega}}{\sqrt{1+\dfrac{\pi g L}{2c}}}. \tag{5.9}$$

Consistency between Eq. (5.9) and the condition $\Omega_0 L/2c \ll 1$ gives a condition on L,

$$L \ll \frac{2c}{g} f, \qquad f = \frac{\pi}{2}\left(\frac{g}{\bar{\omega}}\right)^2\left[1+\sqrt{1+\frac{4}{\pi^2}\left(\frac{\bar{\omega}}{g}\right)^2}\right]. \tag{5.10}$$

Let us consider, as in the preceding section, the situations of *weak* coupling, $\kappa^2 \gg 0$ $(g \ll \bar{\omega})$ and of *strong* coupling, $\kappa^2 \ll 0$ $(g \gg \bar{\omega})$, and let us consider the situation where the dressed material body is initially at its first excited level. Then from Eq. (4.13) we obtain the probability that it will still be excited after an elapsed time t,

$$|f^{00}(t)|^2 = (t_0^0)^4 + 2\sum_{k=1}^{\infty}(t_0^0)^2(t_0^k)^2\cos(\Omega_k-\Omega_0)t + \sum_{k,l=1}^{\infty}(t_0^k)^2(t_0^l)^2\cos(\Omega_k-\Omega_l)t. \tag{5.11}$$

1. Weak coupling

In the case of *weak* coupling a physically interesting situation is when interactions of electromagnetic type are involved. In this case, we take $g=\alpha\bar{\omega}$, where α is the fine-

structure constant, $\alpha = 1/137$. Then the factor f multiplying $2c/g$ in Eq. (5.10) is ~ 0.07 and the condition $L \ll 2c/g$ is replaced by a more restrictive one, $L \ll 0.07(2c/g)$. For a typical infrared frequency, for instance, $\bar{\omega} \sim 2{,}0 \times 10^{11}$/s, our calculations are valid for a value of L, $L \ll 10^{-3}$ m.

From Eqs. (2.15) and using the above expressions for the eigenfrequencies for small L, we obtain the matrix elements,

$$(t_0^0)^2 \approx 1 - \frac{\pi g L}{2c}, \quad (t_0^k)^2 \approx \frac{gL}{\pi c k^2}. \tag{5.12}$$

To obtain the above equations we have neglected the corrective term ϵ_k, from the expressions for the eigenfrequencies Ω_k. Nevertheless, corrections in ϵ_k should be included in the expressions for the matrix elements t_k^k, in order to avoid spurious singularities due to our approximation.

Using Eqs. (5.12) in Eq. (5.11), we obtain

$$|f^{00}(t)|^2 \approx 1 - \pi\delta + 4\left(\frac{\delta}{\pi} - \delta^2\right)\sum_{k=1}^{\infty} \frac{1}{k^2}\cos(\Omega_k - \Omega_0)t + \pi^2\delta^2 + \frac{4}{\pi^2}\delta^2 \sum_{k,l=1}^{\infty} \frac{1}{k^2 l^2}\cos(\Omega_k - \Omega_l)t, \tag{5.13}$$

where we have introduced the dimensionless parameter $\delta = Lg/2c \ll 1$, corresponding to a small value of L and we remember that the eigenfrequencies are given by Eqs. (5.6) and (5.8). As time goes on, the probability that the material body be excited attains periodically a minimum value which has a lower bound given by

$$\min(|f^{00}(t)|^2) = 1 - \frac{5\pi}{3}\delta + \frac{14\pi^2}{9}\delta^2. \tag{5.14}$$

For a frequency $\bar{\omega}$ of the order $\bar{\omega} \sim 4.00 \times 10^{14}$/s (in the red visible range), which corresponds to $\delta \sim 0.005$ and $L \sim 1.0 \times 10^{-6}$ m, we see from Eq. (5.14) that the probability that the material body be at any time excited will never fall below a value ~ 0.97, or a decay probability that is never higher that a value ~ 0.03. It is interesting to compare this result with experimental observations in Refs. [34,35], where stability is found for atoms emitting in the visible range placed between two parallel mirrors a distance $L = 1.1 \times 10^{-6}$ m apart from one another. For lower frequencies the value of the spacing L ensuring quasistability of the same order as above, for the excited particle may be considerably larger. For instance, for $\bar{\omega}$ in a typical microwave value, $\bar{\omega} \sim 2.00 \times 10^{10}$/s, and taking also $\delta \sim 0.005$, the probability that the material body will remain at the first excited level at any time would be larger than a value of the order of 97%, for a value of L of $L \sim 2.0 \times 10^{-2}$ m. The probability that the material body will remain excited as time goes on oscillates with time between a maximum and a minimum value and never departs significantly from the situation of stability in the excited state.

2. Strong coupling

In this case we see from Eqs. (4.16), (5.9), and (5.10) that $\Omega_0 \approx \bar{\omega}$ and obtain from Eq. (2.15)

$$(t_0^0)^2 \approx \frac{1}{1 + \pi\delta/2}, \quad (t_0^k)^2 \approx \frac{gL}{\pi c k^2}. \tag{5.15}$$

Using Eqs. (5.15) in Eq. (5.11), we obtain for the probability that the excited system will still be at the first energy level at time t, the expression

$$|f^{00}(t)|^2 \approx \left(\frac{2}{2 + \pi\delta}\right)^2 + \frac{2}{2 + \pi\delta}\sum_{k=1}^{\infty} \frac{2\delta}{\pi k^2}\cos(\Omega_k - \Omega_0) \times t + \frac{4}{\pi^2}\delta^2 \sum_{k,l=1}^{\infty} \frac{1}{k^2 l^2}\cos(\Omega_k - \Omega_l)t. \tag{5.16}$$

We see from Eq. (5.16) that as the system evolves in time, the probability that the material body will be excited attains periodically a minimum value that has a lower bound given by

$$\min(|f^{00}(t)|^2) = \left(\frac{2}{2 + \pi\delta}\right)^2 - \left(\frac{2}{2 + \pi\delta}\right)\frac{\pi\delta}{3} - \frac{\pi^2\delta^2}{9}. \tag{5.17}$$

The condition of positivity of Eq. (5.17) imposes for *fixed* values of g and $\bar{\omega}$ an upper bound for the quantity δ, δ_{max}, which corresponds to an upper bound to the diameter L of the cavity, L_{max} (remember that $\delta = Lg/2c$). Values of δ larger than δ_{max} or, equivalently, values of L larger than L_{max} are unphysical and should not be considered. These upper bounds are obtained from the solution of the inequality $\min(|f^{00}(t)|^2) \geq 0$. We have $\min(|f^{00}(t)|^2) > 0$ or $\min(|f^{00}(t)|^2) = 0$, for, respectively, $\delta < \delta_{max}$ or $\delta = \delta_{max}$. For a frequency $\bar{\omega}$ of the order $\bar{\omega} \sim 4.00 \times 10^{14}$/s (in the red visible range), with $g = 10\bar{\omega}$ the lower bound (5.17) above attains zero for a cavity of size $L \sim 1.1 \times 10^{-7}$m. For a typical microwave frequency $\bar{\omega} \sim 2.00 \times 10^{10}$/s, the same vanishing lower bound is attained for a cavity of size $L \sim 1.2 \times 10^{-3}$m. We see, comparing with the results of the preceding subsection, that the behavior of the system for *strong* coupling is rather different from its behavior in the weak coupling regime. For appropriate cavity sizes, which are of order 10^{-1} of those ensuring stability in the weak coupling regime, we ensure for strong coupling the complete decay of the particle to the ground state in a small elapsed time.

VI. CONCLUDING REMARKS

We have presented in this paper a nonperturbative treatment of a quantum system consisting of a particle (in the larger sense of a ''material body,'' an atom or a Brownian particle) coupled to an environment modeled by noninteracting oscillators. We have used *dressed* states that allow one to divide the system into the *dressed* particle and the *dressed* environment by means of some conveniently chosen *dressed*

coordinates, q_0' and q_j', associated, respectively, with the *dressed* particle and the *dressed* oscillators composing the environment. In terms of these coordinates a division of the system into the dressed (physically observed) particle and the *dressed* environment arises naturally. The dressed particle will contain automatically all the effects of the environment on it. This formalism allows a nonperturbative approach to the time evolution of a system that may be approximated by a particle coupled linearly to its environment, in rather different situations as confinement of atoms in cavities or the Brownian motion. In other words, underlying our dressed state formalism is a unified way to study two physically different situations, the radiation process and the Brownian motion. We have approached these situations using the *dressed* states, and in both cases we have obtained results in good agreement with experimental observations or with expected behaviors. In the Brownian motion we have treated in detail the case in which the environment is at zero temperature (which corresponds to considering the environment initially in its ground state). Our method accounts for the expected damped behavior of the particle in a simpler way than the usual path integral approach. For atomic systems we recover with our formalism the experimental observation that excited states of atoms in sufficiently small cavities are stable. We are able to give formulas for the probability of an atom to remain excited for an infinitely long time, provided it is placed in a cavity of appropriate size. For an emission frequency in the visible red range, the size of such a cavity is in agreement with experimental observations [33]. The generalization of the work presented in this paper to the case of a generic (supraohmic or subohmic) environment and finite temperature is in progress and will be presented elsewhere.

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