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# Conservation of orbital angular momentum in stimulated down-conversion

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We report on an experiment demonstrating the conservation of the orbital angular momentum in stimulated down-conversion. It has been demonstrated that the orbital angular momentum is not transferred to the individual beams of the spontaneous down-conversion. It is also known that it is conserved when twin photons are taken individually. We observe the conservation law for an individual beam of the down-conversion through cavity-free stimulated emission.

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The cavity-free stimulated parametric down-conversion was first studied by Mandel and co-workers [1,2] and more recently it has been explored by other authors [3–5]. One important aspect of this process is its connection with the spontaneous parametric down-conversion, where entangled states for two photons can be easily prepared. Signals obtained in stimulated down-conversion are much larger than those obtained in the spontaneous process and carry information about the details of the parametric interaction, such as phase matching conditions. This information is preserved thanks to the stimulation without optical cavities, where the optical mode properties are determined mainly by the cavity configuration. Therefore, stimulated down-conversion is a useful tool for understanding entanglement properties of the twin photons from the parametric down-conversion. We have recently demonstrated the transfer of coherence and images from the pump and auxiliary lasers to the stimulated down-conversion field [5], in direct connection with the analogous process in the context of the quantum correlations observed in coincidence measurements [6].

The possibility of preparing entangled photons in different degrees of freedom has also become a subject of interest. In particular, the orbital angular momentum (OAM) of the light has been studied in the context of classical [7] and quantum optics [8]. Conservation of OAM in the up-conversion process [9,10], optical pumping of cold atoms [11], and quantum entanglement [8] have been observed experimentally for this degree of freedom. However, in the spontaneous parametric down-conversion process, the OAM is not transferred from the pump to each individual signal or idler beam [12]. This is a consequence of the fact that signal and idler beams are incoherent when considered individually [13].

In this work, we observe experimentally the manifestation of the conservation law for the OAM in the stimulated down-conversion process, for the idler beam. In this case, besides the pump, a second auxiliary laser is aligned with one of the down-conversion modes, inducing emission. Conservation of the topological charge can be written as $m_p = m_s + m_i$, where *p*, *s*, and *i* stands for pump, signal, and idler, respectively.

Light beams with OAM can be described by Laguerre-Gauss $LG_{l,m}$ modes, where $l$ and $m$ are radial and azimuthal mode numbers. Under the paraxial approximation the angular momentum of a light beam can be separated into orbital and spin contributions [14] and the OAM is given by $m\hbar$ per photon [7]. Therefore, the conservation of topological charge may be read as a statement of OAM conservation. In our experiment, $LG_{0,1}$ modes are produced by diffraction on computer-generated holograms as in Ref. [15], for example. The identification of the modes was made in our experiment by the passage of each beam through a Michelson interferometer, operating with a small misalignment [15]. The resulting interference pattern shows the sign and the absolute value of the topological charge in the mode. This method is very simple and presents some advantages compared to other most common ones, where a coherent reference field is needed [16], or where a Dove prism is inserted inside a Mach-Zhender interferometer [17].

The spatial intensity distribution of the idler beam in the stimulated down-conversion for thin crystals can be predicted by Eq. (10) of Ref. [4],

$$I(\mathbf{r}_i)\propto\left\{\int d\boldsymbol{\rho}|\mathcal{W}_p(\boldsymbol{\rho})|^2+\left|\int d\boldsymbol{\rho}\mathcal{W}_p(\boldsymbol{\rho})\mathcal{W}_s^{\star}(\boldsymbol{\rho})\times\exp\left[i|\boldsymbol{\rho}_i-\boldsymbol{\rho}|^2\frac{k_i}{2z}\right]\right|^2\right\}, \tag{1}$$

where $\mathbf{r}_i=(\boldsymbol{\rho}_i,\ z)$ is the position in the plane transverse to the idler beam propagation at a distance $z$ from the crystal. $\mathcal{W}_p$ and $\mathcal{W}_s$ are, respectively, the pump and the auxiliary lasers transverse field distributions at the crystal and $k_i$ is the idler wave number. Pump and auxiliary lasers can be prepared in $LG_{l,m}$ modes, so that the above equation can be written in terms of them,

$$I(\mathbf{r}_i)\propto\left\{\int d\boldsymbol{\rho}|(LG_{l,m})_p(\boldsymbol{\rho})|^2+\left|\int d\boldsymbol{\rho}(LG_{l,m})_p(\boldsymbol{\rho})\times(LG_{l',m'})_s^{\star}(\boldsymbol{\rho})\exp\left[i|\boldsymbol{\rho}_i-\boldsymbol{\rho}|^2\frac{k_i}{2z}\right]\right|^2\right\}. \tag{2}$$

One special, but important case, is the one where spontaneous emission is negligible. We can do it experimentally, by

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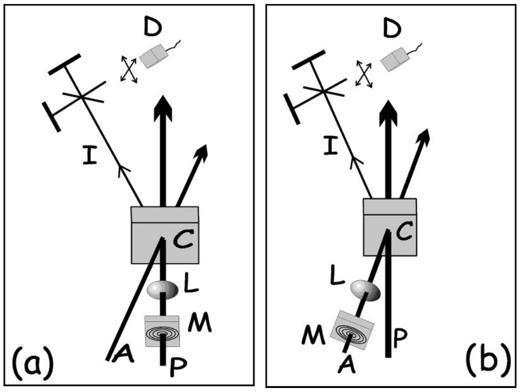

FIG. 1. Schematic representation of the experiment. *P* labels the pump beam, *A* is the auxiliary beam, *L* is the lens, *C* is the nonlinear crystal, *M* is a diffraction mask, *I* is the idler beam, and *D* is the detector. The idler beam is directed to the Michelson interferometer and the measurements are performed in its output. (a) The pump beam is prepared in the $LG_{0,1}$ mode. (b) The auxiliary beam is prepared in the $LG_{0,1}$ mode.

decreasing the pump intensity compared to the auxiliary lasers'. Thus, the first term of the above equations can be neglected. The transverse amplitude of one of the laser fields can also be approximated by a constant, if we expand one of the laser beams. If $(LG_{l,m})_p$ is constant, we have

$$I(\mathbf{r}_i)\propto\left|\int d\boldsymbol{\rho}(LG_{l,m})_p(\boldsymbol{\rho})\exp\left[i|\boldsymbol{\rho}_i-\boldsymbol{\rho}|^2\frac{k_i}{2z}\right]\right|^2. \qquad (3)$$

and if $(LG_{l,m})_s$ is constant, we have

$$I(\mathbf{r}_i)\propto\left|\int d\boldsymbol{\rho}(LG_{l,m})_s^*(\boldsymbol{\rho})\exp\left[i|\boldsymbol{\rho}_i-\boldsymbol{\rho}|^2\frac{k_i}{2z}\right]\right|^2. \qquad (4)$$

Equation (3) [(4)] tells us that the intensity profile of the idler beam will look like a doughnut if the pump (auxiliary) laser is prepared in an $LG_{0,m}$ $(m\neq 0)$ mode. They also show that the idler beam propagates as a $LG_{0,m}$ mode.

In Refs. [4,5], the quantum treatment used has shown to be useful in describing the stimulated down-conversion process and it would be interesting to derive the state of the idler field when either the pump or the auxiliary laser is prepared in *LG* modes, within the same formalism. However, this calculation is not straightforward and is beyond the scope of the present work. In the following, we will present experimental results supporting the predictions in Eqs. (3) and (4), concerning the intensity distributions and supporting the intuition that if the idler presents a doughnut shape it ''should'' possess some OAM.

The experimental setup is sketched in Fig. 1. A 200 mW He-Cd laser pumps a type II, 3 mm long BBO nonlinear crystal, with a cw 442 nm wavelength beam. Nondegenerate twin beams are generated with signal and idler wavelengths around 845 nm and 925 nm, respectively. An auxiliary beam is obtained from a diode laser oscillating around 845 nm. The diode laser power is about 150 mW. It is aligned with the signal beam, so that their modes have good overlap and emission is stimulated in this down-conversion mode by the laser. As a result, the idler beam is completely changed with respect to its intensity and spectral properties, as described in Refs. [1–5]. The goal of the experiment is to prepare the pump beam in an $LG_{0,1}$ mode and to measure the OAM of the idler beam. The same procedure is repeated, preparing the auxiliary beam in an $LG_{0,1}$ mode and measuring the OAM of the idler beam. The idler beam is directed onto a Michelson interferometer, before it is detected by an avalanche photodiode single-photon counting module. The Michelson interferometer is slightly misaligned along the horizontal axis, so that for a plane-wave input, the resulting interference pattern presents vertical parallel stripes. The larger the misalignment, the narrower the stripes. When an $LG_{0,m}$ $(m\neq 0)$ mode enters the interferometer, the beam with doughnut shape is divided in two and the misalignment works to make the side of one beam interfere with the center of the other and vice versa. Two opposed bifurcations appear in the interference pattern. The orientation of the bifurcations is related to the sign of the topological charge and the number of derivations in the fork is related to the absolute value of the charge. For example, the fork on the left pointing up and fork on the right pointing down means that the topological charge is negative. A rotation of the pattern does not change this configuration, only a reflection would change it, but a reflection implies in changing the sense of propagation. Those are the signatures of the *LG* mode, which can be easily identified by comparing the measured interference patterns with calculated ones.

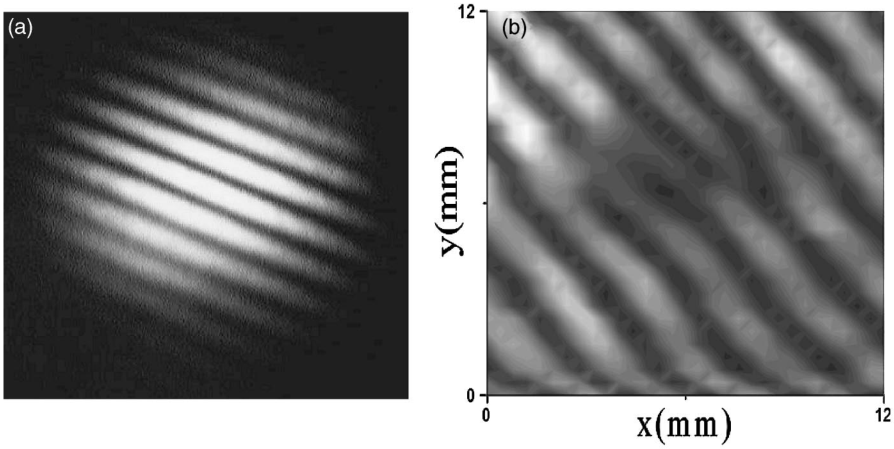

FIG. 2. (a) Interference pattern for the pump in an $LG_{0,0}$ mode. CCD camera picture. (b) Gray scale bitmap plotted from a 30 $\times 30$ matrix with the transverse interference pattern of the pump beam in an $LG_{0,1}$ mode. Detection with a photon counting detector.

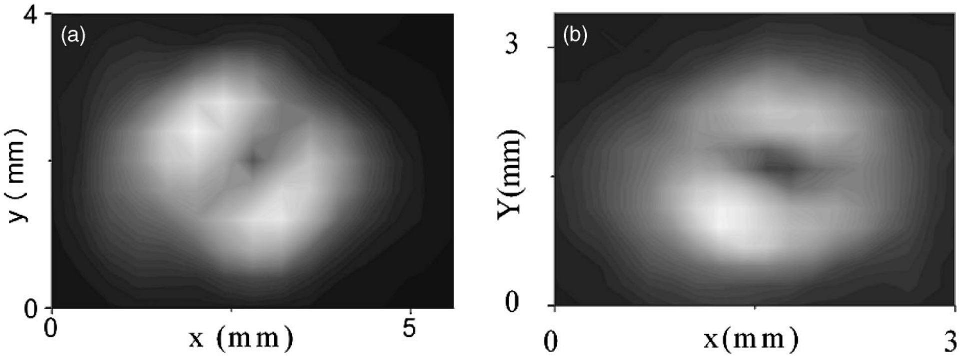

FIG. 3. Gray scale bitmap plotted from a 20×20 matrix with the measured transverse intensity of the idler beam. (a) Pump $LG_{0,1}$ mode $m_p=+1$ and (b) auxiliary $LG_{0,1}$ mode $m_s=+1$.

The pump beam was prepared in an *LG* mode with $m_p=+1$. After crossing the crystal, the beam is directed to a Michelson interferometer, in the same fashion as described above for the idler, in order to be able to compare the interference patterns for pump and idler. All interference patterns are measured by scanning the detector in the transverse plane. The resulting matrix with the intensities at different positions is converted into a gray scale bitmap where the higher intensities are white and the lower ones are black. The interference pattern measured for the pump beam is shown in Fig. 2. In Fig. 2(a) we have presented a charge-coupled device (CCD) camera picture of the interference for a $\mathrm{LG}_{0,0}$ mode and in Fig. 2(b) the interference pattern for the $\mathrm{LG}_{0,1}$ mode detected with photon counting detectors is shown. The two forks would be oriented along the vertical axis if the misalignment were only in the horizontal direction. Due to a

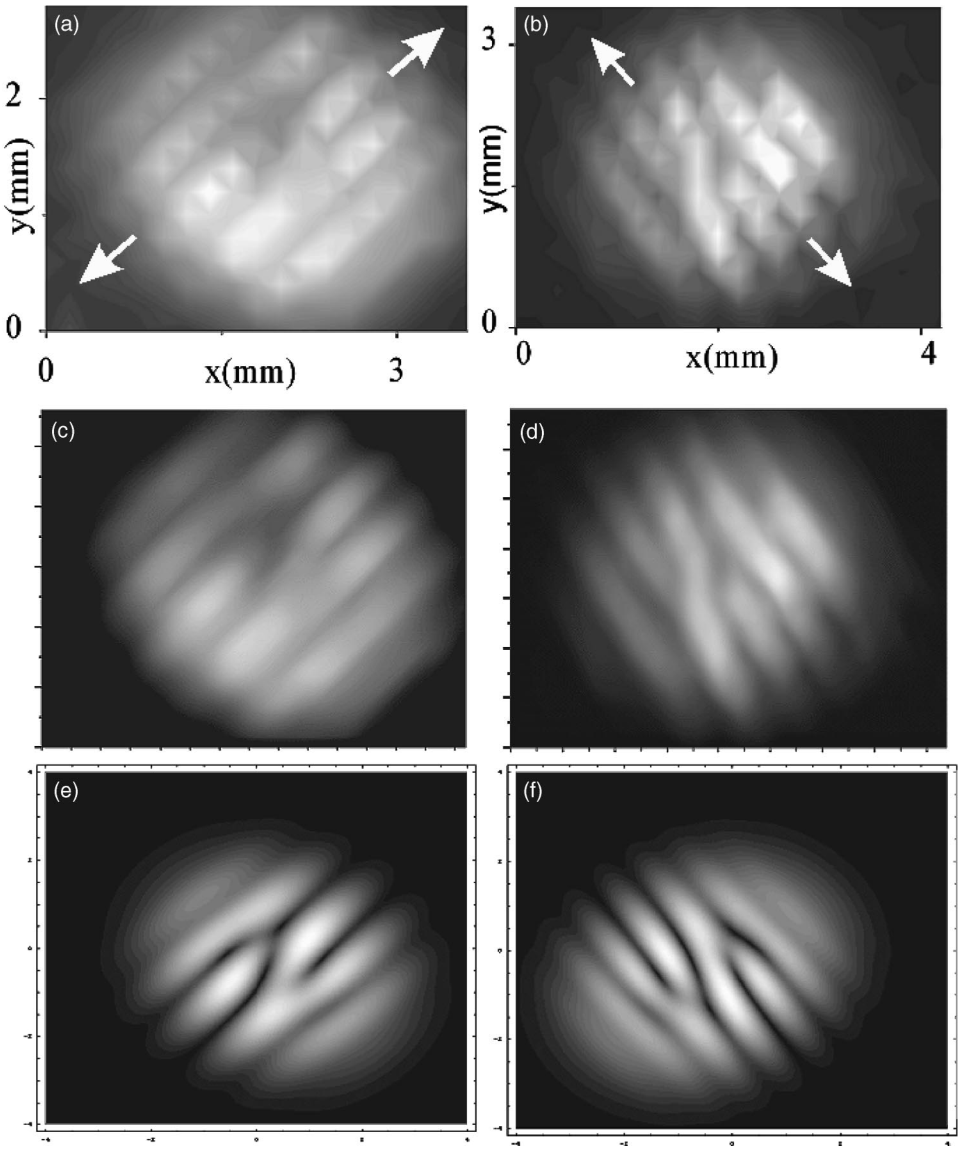

FIG. 4. Gray scale bitmap plotted from a 20×20 matrix with the measured transverse interference pattern of the idler beam. (a) Pump $LG_{0,1}$ mode $m_p=+1$ (raw data). (b) auxiliary $LG_{0,1}$ mode $m_s=+1$ (raw data). The arrows follow the forks orientation. Treated image of the experimental results. (c) Pump $LG_{0,1}$ mode $m_p=+1$, (d) auxiliary $LG_{0,1}$ mode $m_s=+1$. Theoretical simulations, (e) pump $LG_{0,1}$ mode $m_s=+1$, (f) auxiliary $LG_{0,1}$ mode $m_s=+1$.

small vertical misalignment [18], the forks are oriented along an axis making an angle with the vertical direction. From the orientation of the forks it is possible to identify the topological charge, $m_p = +1$. As a consequence of OAM conservation, when the pump is prepared with $m_p = +1$ and the auxiliary laser with $m_s = 0$, the idler must have $m_i = +1$. However, when the auxiliary laser is prepared with $m_s = +1$ and the pump with $m_p = 0$, the idler must have $m_i = -1$.

The idler beam obtained in the stimulated down-conversion is then analyzed with the Michelson interferometer in the same way as described above for the pump. Auxiliary laser power is high enough to ensure that the spontaneous emission is negligible compared to the stimulated one (signal to noise ratio is about 300). As a result, when either the pump or the auxiliary laser is prepared in an $LG_{0,m}(m \neq 0)$ mode, the idler beam also propagates as an $LG_{0,m}(m \neq 0)$ mode and its intensity distribution looks like a doughnut. Idler intensity distributions are shown in Fig. 3, when (a) $m_p = +1$ and $m_s = 0$ and (b) $m_p = 0$ and $m_s = +1$. In Fig. 4 we have shown the interference patterns, again for (a) $m_p = +1$ and $m_s = 0$, and (b) $m_p = 0$ and $m_s = +1$. The experimental results presented in Figs. 4(a) and 4(b) are raw data. We also display the respective images in Figs. 4(c) and 4(d), treated with some smoothing and blurring, in order to improve the visualization of the features of the patterns. The interference patterns for the $\mathrm{LG}_{0,1}$ modes can also be obtained theoretically. In Figs. 4(e) and 4(f) the theoretical simulations are shown. The parameters of the simulation were adjusted in order to provide patterns as similar as possible to the measurements. Note that the orientation of the forks of the idler in Fig. 4(e) $(m_i = +1)$ is inverted (mirror image) when compared to the idler in (f) $(m_i = -1)$.

The interference patterns in Fig. 4 have a rather low visibility. Even though the spontaneous emission is negligible in our experiment, we still have some sources of noise, as the residual light background in the room and the dark counting rate of the APD detector, because the final intensities detected are very low. The data acquisition takes a rather long time (about 1 h) and small drifts of the phase difference may take place in the interferometer. The most important point is, however, the fact that the stimulated emission only gives rise to coherent light. In this case, if the OAM transfer was not complete, other coherent modes would be present and interference between them would be detected in the intensity patterns of Figs. 3(a) and 3(b). Therefore, the OAM transfer and conservation is guaranteed by both the intensity profiles of Fig. 3 and interference fringes of Fig. 4.

In the results presented above, the OAM was actually transferred from the pump and auxiliary lasers to the stimulated idler beam. When the OAM comes from the pump with $m_p = +1$, the idler is changed into a $m_i = +1$ $LG_{0,1}$ mode. When the OAM comes from the auxiliary laser with $m_s = +1$, the idler is changed into a $m_i = -1$ $LG_{0,-1}$ mode. This is compatible with the conservation of the total topological charge $m_p = m_s + m_i$. The OAM is conserved within the light beams in the same way as it is for the second-harmonic generation and spontaneous down-conversion at the single-photon level. The relation, $m_i = -m_s$ when $m_p = 0$, can be understood in terms of the phase conjugation of the idler in comparison with the auxiliary laser [5], as an *LG* beam with $m = +1$ looks like an *LG* beam with $m = -1$ propagating backwards.

In conclusion, we have observed experimentally the transfer of orbital angular momentum from the pump and auxiliary lasers to the stimulated parametric down-conversion idler beam. This transfer implies the conservation of the topological charge.

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