# Multiphoton resonant excitation of atoms in strong laser fields and implementation of coherent superposition states

H. K. Avetissian\* and G. F. Mkrtchian

Department of Quantum Electronics, Plasma Physics Laboratory, Yerevan State University, 1 A. Manukian, 375049 Yerevan, Armenia (Received 29 March 2002; published 23 September 2002)

The multiphoton resonant excitation of three-level atoms in a strong laser field is investigated. The creation of various superposition states is shown. For implementation of these effects either the mean dipole moment in the excited states of an atom or, in three-level systems, a transition dipole moment between excited states (with energies close enough to each other) must be nonzero. The time evolution of the system is found using a nonperturbative resonant approach. Our calculations for the hydrogen atom suggest that by the appropriate optical pulses with moderate strong intensities one can achieve various superposition states by multiphoton resonant excitation.

DOI: 10.1103/PhysRevA.66.033403

PACS number(s): 42.50.Hz, 32.80.Wr, 82.53.Kp

# I. INTRODUCTION

Current laser technology enables production of electromagnetic (e.m.) fields comparable with and larger than internal atomic fields. Under these conditions the bound-bound and bound-free transitions in atomic systems have multiphoton character. The increasing interest in the processes of intense laser and atom interaction is due in large part to the problem of high harmonic generation and short wave coherent radiation implementation via multiphoton bound-free transitions through free continuum spectra. During the last two decades numerous investigations have been carried out to study laser-atom interactions both theoretically and experimentally and many monographs (see, e.g., [1-4]) are devoted to this problem.

On the other hand in strong laser fields one can expect multiphoton resonant excitation of atoms [5-8]. It is well known that when the laser frequency is close to the frequency associated with the level difference of a two-level atom then by the appropriate laser pulses one can obtain various coherent superposition states [9], which can lead to cooperative processes such as superradiation, free-induction decay, photon echo, etc. However, the obtaining of such superposition states is large compared with the optical transitions. In this case multiphoton excitation can be realized, which will allow one to observe cooperative effects in the high frequency domain and especially the implementation of high order harmonic generation [10].

In [8] a nonperturbative analysis of the multiphoton excitation of a two-level atom has been done. However, because of the strong dependence of the resonance on the intensity and because of their narrowness, the atom is excited only during a small interval of the laser pulse. As a result the rate of the concurrent process of multiphoton ionization exceeds the excitation rate by several orders of magnitude, making impossible the efficient excitation of an atom in this case.

Regarding three-level systems, most theoretical studies of

laser-assisted resonant transitions have been done in not very strong fields (see, e.g., [11,12]). In [6,7] there is a theoretical treatment of multiphoton two-mode processes in a threelevel atom based on an exactly solvable Jaynes-Cumming type model with a model of the second quantized Hamiltonian. For the efficient multiphoton excitation of atoms, the laser field should be strong enough to induce multiphoton transitions. On the other hand in this case nonresonant levels may play a role. In [13,14] the dynamics and radiation of highly charged hydrogenlike ions in intense high frequency laser pulses have been investigated numerically. As was shown in the near-resonant multiphoton regime only a few resonant levels are involved in the dynamics of the wave packet. Hence, when the ionization process is not dominant one can reduce the dynamics to a few levels rather than considering the whole wave packet as usual in the strong field regime.

In the present work the multiphoton resonant excitation of three-level atoms subjected to a strong laser field is studied. We consider a three-level atom when the latter has a mean dipole moment in the excited states. Otherwise, the energies of the excited states of a three-level atom should be close enough to each other (the frequency associated with the level difference should be small compared with the Rabi frequency) and the transition dipole moment between these states must be nonzero. The time evolution of such systems is found using a nonperturbative resonant approach. The analysis shows that quasienergy levels close to the ground state arise. As a result the problem is reduced to the usual Rabi problem with a generalized "Rabi frequency," which has a nonlinear dependence on the amplitude of the e.m. field. A hydrogenlike atomic system may serve as a good candidate for the considered model, where due to random degeneration upon an orbital moment the atom has a mean dipole moment in excited stationary states. Our calculations for the hydrogen atom suggest that by using appropriate optical pulses with moderately strong intensities one can achieve various superposition states by multiphoton resonant excitation.

The organization of the paper is as follows. In Sec. II we present our model and solution of the set of equations in the resonant approximation. In Sec. III we apply the results ob-

<sup>\*</sup>FAX: (3741) 570-597. Electronic address: avetissian@ysu.am

tained for the hydrogen atom and present some numerical calculations. Finally, conclusions are given in Sec. IV.

#### **II. BASIC MODEL AND RESONANT SOLUTION**

Let us consider a three-level atom in an e.m. wave field. The latter will be treated semiclassically and in the dipole approximation. We assume the e.m. wave is linearly polarized and characterized by a slowly varying amplitude  $E_0(t)$ and carrier frequency  $\omega$ . The pulse duration time  $\tau$  is assumed to be smaller than any relaxation time in our model system. Therefore we will use the Schrödinger equation instead of the density matrix treatment. We denote the atomic states by  $|\eta\rangle$ , where  $\eta=0,1,2$  indicates the set of quantum numbers that characterizes the state. The Schrödinger equation in the energetic representation, that is, the set of equations for the probability amplitudes  $a_{\eta}(t)$ , is the following (throughout this paper we use atomic units:  $e=m=\hbar=1$ , c=137):

$$i\frac{da_{\eta}}{dt} = \varepsilon_{\eta}a_{\eta} + \sum_{\nu=1}^{3} V_{\eta\nu}a_{\nu}, \qquad (2.1)$$

where  $\varepsilon_n$  is the energy of the state and

$$V_{n\nu} = -\Lambda_{n\nu} \cos \omega t \tag{2.2}$$

is the interaction Hamiltonian. Here

$$\Lambda_{\eta\nu} = \langle \eta | \mathbf{r} \cdot \hat{\mathbf{e}} | \nu \rangle E_0(t)$$
(2.3)

are the transition amplitudes,  $\hat{\mathbf{e}}$  is the unit polarization vector, and  $\mathbf{r}$  is the radius vector. For concreteness we assume that  $V_{12}, V_{21}=0$  and  $V_{11}, V_{22}\neq 0$ , which means that dipole transitions between the excited stationary states of an atom are forbidden but the atom has a mean dipole moment in those states. To simplify the set of equations and to have the physically more appropriate form for multiphoton resonant transitions we apply the following unitarian transformation:

$$a_{\eta}(t) = b_{\eta}(t) \exp\left(-i\varepsilon_{\eta}t - i\int_{0}^{t} V_{\eta\eta}dt\right), \qquad (2.4)$$

providing the same initial conditions for the new amplitudes  $b_{\eta}(t)$  at t=0. From Eq. (2.1) and Eq. (2.4) for  $b_{\eta}(t)$  we obtain the following set of equations:

$$i\frac{db_0}{dt} = F(t)b_1 + G(t)b_2,$$
 (2.5a)

$$i\frac{db_1}{dt} = F^{\dagger}(t)b_0,$$
 (2.5b)

$$i\frac{db_2}{dt} = G^{\dagger}(t)b_0, \qquad (2.5c)$$

$$F(t) = V_{01}(t) \exp\left(i(\varepsilon_0 - \varepsilon_1)t - i\int_0^t V_{11}(t)dt\right), \quad (2.6)$$

$$G(t) = V_{02}(t) \exp\left(i(\varepsilon_0 - \varepsilon_2)t - i\int_0^t V_{22}(t)dt\right), \quad (2.7)$$

and  $F^{\dagger}$  denotes the complex conjugation of *F*. In this representation the quasienergy levels  $\varepsilon_{1,2} \pm s \omega$  (s = 1, 2, ...) close to the ground state arise. The probabilities of multiphoton transitions between these quasienergy levels will have maximal values for the resonant transitions

$$\varepsilon_0 - \varepsilon_1 + n\,\omega \simeq 0, \tag{2.8}$$

$$\varepsilon_0 - \varepsilon_2 + n \, \omega \simeq 0 \qquad (n = 1, 2, \dots).$$
 (2.9)

In this case the functions F(t), G(t) can be represented in the following form:

$$F(t) = F_n + f(t),$$
 (2.10)

$$G(t) = G_n + g(t), \qquad (2.11)$$

where

$$F_n = -\omega \frac{\Lambda_{01}}{\Lambda_{11}} n J_n \left( \frac{\Lambda_{11}}{\omega} \right) \exp(i \,\delta_{1n} t), \qquad (2.12)$$

$$G_n = -\omega \frac{\Lambda_{02}}{\Lambda_{22}} n J_n \left( \frac{\Lambda_{22}}{\omega} \right) \exp(i \,\delta_{2n} t)$$
(2.13)

are slowly varying functions on the scale of the e.m. wave period (at the exact resonance constants) and

$$f(t) = -\omega \frac{\Lambda_{01}}{\Lambda_{11}} \exp(i \,\delta_{1n} t)$$

$$\times \sum_{N \neq n, N = -\infty}^{\infty} N J_N \left( \frac{\Lambda_{11}}{\omega} \right) \exp[i(N - n) \omega t],$$
(2.14)

$$g(t) = -\omega \frac{\Lambda_{02}}{\Lambda_{22}} \exp(i \,\delta_{2n} t)$$
$$\times \sum_{N \neq n, N = -\infty}^{\infty} N J_N \left( \frac{\Lambda_{22}}{\omega} \right) \exp[i(N - n) \omega t]$$
(2.15)

are rapidly oscillating functions. To derive Eqs. (2.14), (2.15), (2.12), and (2.13) we have applied the following expansion by Bessel functions:

$$\exp(i\alpha\sin\omega t) = \sum_{N=-\infty}^{\infty} J_N(\alpha)\exp(iN\omega t), \quad (2.16)$$

and introduced the resonance detunings

$$\delta_{1n} = \varepsilon_0 - \varepsilon_1 + n\omega, \quad \delta_{2n} = \varepsilon_0 - \varepsilon_2 + n\omega. \quad (2.17)$$

In consequence of this separation the probability amplitudes can be represented in the form

$$b_{\eta}(t) = \overline{b}_{\eta}(t) + \beta_{\eta}(t) \quad (\eta = 0, 1, 2),$$
 (2.18)

where  $\bar{b}_{\eta}(t)$  are the time averages of  $b_{\eta}(t)$  and  $\beta_{\eta}(t)$  are rapidly oscillating functions. Substituting Eqs. (2.18) into Eqs. (2.5) and separating slow and rapid oscillations taking into account Eqs. (2.10) and (2.11), we obtain the following set of equations for the time average amplitudes  $b_{\eta}(t)$ :

$$i\frac{d\overline{b}_0}{dt} = F_n\overline{b}_1 + G_n\overline{b}_2 + \overline{f(t)\beta_1(t)} + \overline{g(t)\beta_2(t)},$$
(2.19a)

$$i\frac{d\overline{b}_1}{dt} = F_n^{\dagger}\overline{b}_0 + \overline{f^{\dagger}(t)\beta_0(t)}, \qquad (2.19b)$$

$$i\frac{d\overline{b}_2}{dt} = G_n^{\dagger}\overline{b}_0 + \overline{g^{\dagger}(t)\beta_0(t)}, \qquad (2.19c)$$

and consequently

$$\frac{i\frac{d\beta_0}{dt}}{dt} = f(t)\bar{b}_1(t) + g(t)\bar{b}_2(t), \qquad (2.20a)$$

$$i\frac{d\beta_1}{dt} = f^{\dagger}(t)\overline{b}_0, \qquad (2.20b)$$

$$i\frac{d\beta_2}{dt} = g^{\dagger}(t)\overline{b}_0. \qquad (2.20c)$$

In Eqs. (2.19) the overbar denotes averaging over a time much larger than the e.m. wave period. In the set (2.20) we have neglected the terms  $\sim f(t)\beta_{\eta}(t)$  as due to rapid oscillations

$$\left|f(t)\boldsymbol{\beta}_{\eta}(t)\right| \ll \left|\frac{d\boldsymbol{\beta}_{1}}{dt}\right|. \tag{2.21}$$

Solving the set of equations (2.20), taking into account that  $\bar{b}_{n}(t)$  are slowly varying functions, we obtain

$$\beta_0 = -i \left( \overline{b}_1(t) \int_0^t f(t) dt + \overline{b}_2(t) \int_0^t g(t) dt \right), \quad (2.22)$$

$$\beta_1 = -i\bar{b}_0(t) \int_0^t f^{\dagger}(t)dt,$$
 (2.23)

$$\beta_2 = -i\overline{b}_0(t) \int_0^t g^{\dagger}(t)dt, \qquad (2.24)$$

and substituting  $\beta_{\eta}(t)$  into Eqs. (2.19), we obtain

$$i\frac{d\bar{b}_{0}}{dt} = F_{n}\bar{b}_{1} + G_{n}\bar{b}_{2} + (\Delta_{f} + \Delta_{g})\bar{b}_{0}, \qquad (2.25a)$$

$$i\frac{db_1}{dt} = F_n^{\dagger}\overline{b}_0 - \Delta_f^{\dagger}\overline{b}_1 + \widetilde{\Delta}\overline{b}_2, \qquad (2.25b)$$

$$i\frac{d\bar{b}_2}{dt} = G_n^{\dagger}\bar{b}_0 - \Delta_g^{\dagger}\bar{b}_2 + \tilde{\Delta}^{\dagger}\bar{b}_1, \qquad (2.25c)$$

where

$$\Delta_{f} = -i\overline{f(t)} \int_{0}^{t} f^{\dagger}(t) dt$$
$$= \omega \left(\frac{\Lambda_{01}}{\Lambda_{11}}\right)^{2} \sum_{N \neq n, N = -\infty}^{\infty} \frac{N^{2}}{N - n} J_{N}^{2} \left(\frac{\Lambda_{11}}{\omega}\right), \quad (2.26)$$

$$\Delta_{g} = -i\overline{g(t)} \int_{0}^{t} g^{\dagger}(t) dt$$
$$= \omega \left( \frac{\Lambda_{02}}{\Lambda_{22}} \right)^{2} \sum_{N \neq n, N = -\infty}^{\infty} \frac{N^{2}}{N - n} J_{N}^{2} \left( \frac{\Lambda_{22}}{\omega} \right), \quad (2.27)$$

$$\widetilde{\Delta} = -i\overline{f(t)} \int_{0}^{t} g^{\dagger}(t) dt$$

$$= -\omega \frac{\Lambda_{01} \Lambda_{02}}{\Lambda_{11} \Lambda_{22}} \exp[i(\delta_{1n} - \delta_{2n})t]$$

$$\times \sum_{N \neq n, N = -\infty}^{\infty} \frac{N^{2}}{N - n} J_{N} \left(\frac{\Lambda_{22}}{\omega}\right) J_{N} \left(\frac{\Lambda_{11}}{\omega}\right). \quad (2.28)$$

The physical meaning of these quantities is the dynamic Stark shift. The set of Eqs. (2.25) can be solved in the general case of arbitrary envelope  $E_0(t)$  only numerically. But it admits an exact solution for a monochromatic wave:  $E_0(t) \equiv E_0$  describing Rabi oscillations in the three-level atomic system. In this case the set (2.25) for the phase transformed amplitudes  $\overline{b}_{\eta} \exp(i\delta_{\eta\eta} t)$  is a set of linear ordinary differential equations with fixed coefficients, so its general solution is given by a superposition of three linearly independent solutions

$$b_{\eta} = \sum_{\nu=0}^{2} C_{\nu} M_{\eta \nu} \exp[i(\lambda_{\nu} - \delta_{\eta n})t], \qquad (2.29)$$

where the  $C_v$  are constants and defined by the initial conditions. The factors  $\lambda_v$  are the solutions of the cubic equation

$$\det(A_{\mu\nu}) = 0, \tag{2.30}$$

where

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$$A_{\mu\nu} = \begin{pmatrix} \Delta_f + \Delta_g + \lambda & F_n \exp(-i\,\delta_{1n}t) & G_n \exp(-i\,\delta_{2n}t) \\ F_n^{\dagger} \exp(i\,\delta_{1n}t) & -\Delta_f^{\dagger} - \delta_{1n} + \lambda & \tilde{\Delta} \exp[i(\,\delta_{2n} - \delta_{1n})t] \\ G_n^{\dagger} \exp(i\,\delta_{21n}t) & \tilde{\Delta}^{\dagger} \exp[-i(\,\delta_{2n} - \delta_{1n})t] & -\Delta_g^{\dagger} - \delta_{2n} + \lambda \end{pmatrix}$$
(2.31)

is the time independent characteristic matrix and  $M_{\eta\nu}$  are its minors.

At the end of this section it is relevant to comment on the region of applicability of the theory presented in this paper. The set of equations (2.25) has been derived using the assumption that the amplitudes  $\bar{b}_{\eta}$  are slowly varying functions on the scale of the e.m. wave period, i.e.,

$$\left|\frac{db_{\eta}(t)}{dt}\right| \ll \left|b_{\eta}(t)\right|\omega, \quad \eta = 0, 1, 2.$$
(2.32)

The latter conditions with Eq. (2.21) put the restrictions

$$(|F_n|, |G_n|, |\Delta_f|, |\Delta_g|, |\tilde{\Delta}|, |\delta_{1,2n}|) \leq \omega$$
(2.33)

on the characteristic parameters of the system considered.

Although the main attention has been devoted to the case when the dipole transitions between the excited stationary states of an atom are forbidden but the atom has a mean dipole moment, the current treatment includes a larger class of systems, when  $V_{11}, V_{22}=0$  and  $V_{12}, V_{21}\neq 0$ . If the energies of the excited states are close enough to each other (the frequency associated with the level difference should be small compared with the Rabi frequency) then by the unitarian transformation

$$a'_0 = a_0, \ a'_1 = \frac{a_1 - a_2}{\sqrt{2}}, \ a'_2 = \frac{a_1 + a_2}{\sqrt{2}},$$
 (2.34)

the problem can be reduced to the one considered. This is obvious for the hydrogen atom in parabolic [15] and the usual spherical coordinates. In the first case the atom has a mean dipole moment in the excited states, while in the second case the mean dipole moment is zero for a stationary state but due to the random degeneracy of the orbital moments there is a transition dipole moment between degenerate states. After the transformation (2.34) we will have the same formulas where the quantities  $2^{-1/2}(V_{01}-V_{02})$ ,  $2^{-1/2}(V_{01}+V_{02})$ ,  $-V_{12}$ ,  $V_{12}$  stand for  $V_{01}$ ,  $V_{02}$ ,  $V_{11}$ ,  $V_{22}$ respectively (it is assumed that  $V_{12}=V_{21}$ ).

# **III. APPLICATION FOR HYDROGEN ATOM**

In this section we apply the results obtained for the hydrogen atom and present some numerical simulations. For the hydrogen atom all transitions starting from the ground state fall in the vacuum ultraviolet (VUV) range (10.2 eV for the 1S-2P transition). This is a spectral domain where coherent radiation is difficult to generate and two- or higher-photon resonant excitation is of great interest. Taking into account the problem symmetry it is more appropriate to con-

sider the hydrogen atom in parabolic coordinates. In these coordinates the atomic states are characterized by the set of quantum numbers  $\eta = \{n, m, n_1\}$  where *n* is the principal quantum number, *m* is the magnetic quantum number, and for  $n_1$  we have the following relationship:

$$n = n_1 + n_2 + |m| + 1. \tag{3.1}$$

In this case  $n_1$  and  $n_2$  commute for radial and orbital quantum numbers in the usual representation (we will not consider the fine and hyperfine structure of the levels). Without loss of generality, we can take the polarization vector  $\hat{\mathbf{e}}$  aligned with the Z axis of the parabolic coordinates. Then as long as for dipole transitions there is a selection rule by the magnetic quantum number m=m', then from the ground state  $\{1,0,0\}$  ( $\eta=0$ ) transitions to the  $\{2,0,0\}$  ( $\eta=1$ ) and  $\{2,0,1\}$  ( $\eta=2$ ) states are possible. For the transition amplitudes Eq. (2.3) we have

$$\Lambda_{22} = -\Lambda_{11} = 3E_0, \qquad (3.2)$$

$$\Lambda_{01} = -\Lambda_{02} = \frac{2^7}{3^5} E_0, \qquad (3.3)$$

and for the main characteristic parameters we have

$$\delta_{1n} = \delta_{2n} \equiv \delta, \tag{3.4}$$

$$F_n = (-1)^n G_n = (-1)^n \frac{2^4}{3^5} J_n(8nE_0) \exp(i\,\delta t), \quad (3.5)$$

$$\Delta_f = \Delta_g \equiv \Delta$$
  
=  $\frac{1}{n} \left(\frac{2}{3}\right)^{11} \sum_{N \neq n, N = -\infty}^{\infty} \frac{N^2}{N - n} J_N^2(8nE_0),$  (3.6)

$$\tilde{\Delta} = -\frac{1}{n} \left(\frac{2}{3}\right)^{11} \sum_{N \neq n, N = -\infty}^{\infty} \frac{(-1)^N N^2}{N - n} J_N^2(8nE_0). \quad (3.7)$$

The roots of Eq. (2.30) are

$$\lambda_1 = \Delta + (-1)^n \widetilde{\Delta} + \delta, \qquad (3.8)$$

$$\lambda_{2,3} = -\frac{\Delta + (-1)^n \tilde{\Delta} - \delta}{2} \pm \Omega_n, \qquad (3.9)$$

where

$$\Omega_n = \left(\frac{(3\Delta + (-1)^{n+1}\tilde{\Delta} + \delta)^2}{4} + 2\left|G_n\right|^2\right)^{1/2}.$$
 (3.10)



FIG. 1. Two-photon resonance (n=2). The electric field strength is  $E_0=0.03$  a.u.; the detuning is taken to be  $\delta=0$ . (a) Temporal evolution of the ground state population for a continuous wave. The dotted curve corresponds to numerical calculations; the solid curve corresponds to our approximate solution. (b) Temporal evolution of the state populations for a Gaussian envelope with  $\omega \tau/2\pi=27$ .

We will set the following initial conditions:

$$b_0|_{t=0} = 1, \quad b_1|_{t=0} = 0, \quad b_2|_{t=0} = 0,$$
 (3.11)

assuming that the atom initially is in the ground state.

The average probabilities calculated via the initial conditions Eq. (3.11) and Eq. (2.29) are

$$|\bar{b}_0|^2 = 1 - 2 \frac{|G_n|^2}{\Omega_n^2} \sin^2(\Omega_n t),$$
 (3.12)

$$|\bar{b}_1|^2 = |\bar{b}_2|^2 = \frac{|G_n|^2}{\Omega_n^2} \sin^2(\Omega_n t).$$
 (3.13)

The latter express the Rabi oscillations, but the amplitudes are diminished due to the dynamic Stark shift. When

$$|G_n| \gg \max\{|\Delta|, |\tilde{\Delta}|, |\delta|\}$$
(3.14)

we obtain Rabi oscillations with frequency



FIG. 2. Same as Fig. 1, but for three-photon resonance (n = 3). The electric field strength is  $E_0 = 0.04$  a.u.; to compensate the Stark shift the detuning is taken to be  $\delta/\omega \approx 0.02$ . The pulse duration in (b) is  $\omega \tau/2\pi = 15$ .

$$\Omega_n \simeq 2^{1/2} |G_n|, \tag{3.15}$$

which has a nonlinear dependence on the amplitude of the electromagnetic field. It is easy to see that for one-photon resonance in the case of a two-level atomic system the latter is reduced to the known Rabi frequency [9]. For one- or two-photon resonances the influence of the dynamic Stark shift is not essential, but it becomes significant for large photon numbers, and one can choose an appropriate detuning to compensate the dynamic Stark shift.

We have also performed numerical simulations solving Eq. (2.1) numerically. Figure 1(a) displays the temporal evolution of the ground state population for two-photon resonance, when  $E_0 = 0.03$  a.u., which corresponds to laser intensity  $I \approx 3 \times 10^{13}$  W/cm<sup>2</sup>; the detuning has been chosen to be  $\delta = 0$ . The dotted curve corresponds to numerical calculations, while the solid curve corresponds to our approximate solution Eq. (3.12) describing the mean value of the probability. The calculations have also been made for the finite wave pulse describing the envelope by the Gaussian function

$$E_0(t) = E_0 \exp\left(-\frac{(t-\sqrt{2}\tau)^2}{2\tau^2}\right).$$
 (3.16)



FIG. 3. Temporal evolution of the state populations for fivephoton resonance (n=5). The electric field strength is  $E_0=0.05$ a.u.; the pulse duration is  $\omega \tau/2\pi = 20$ . To compensate the Stark shift the detuning is taken to be  $\delta/\omega \approx 0.045$ . For visual convenience we have not plotted the  $|2\rangle$  state population, which coincides with  $|1\rangle$ . The inset displays the temporal evolution for the same parameters but  $\delta=0$ .

The state populations are shown in Fig. 1(b) for  $\omega \tau/2\pi$ = 27. The final state is a superposition of three states. Figure 2 displays three-photon resonance. In this case  $E_0 = 0.04$  a.u.  $(I \approx 5.5 \times 10^{13} \text{ W/cm}^2)$  and  $\omega \tau/2\pi = 15$ . For the threephoton resonance the Stark shift is essential and for compensation we have put

$$\delta = -3\Delta - \tilde{\Delta}$$

which gives  $\delta/\omega \approx 0.02$ . The rapid oscillations in Fig. 2(a) in our approximate solutions are described by the  $\beta_n$  functions

which are not included in the figures. In the final state we have ~100% overpopulation. As we see, the numerical results coincide to high accuracy with our approximate solution. Finally, in Fig. 3 the state populations are plotted for five-photon resonance when  $E_0=0.05$  a.u. ( $I=8.7 \times 10^{13}$  W/cm<sup>2</sup>) and  $\omega \tau/2\pi=20$ . The detuning in this case is  $\delta/\omega=0.045$ . As we see the final state is a superposition of three states with the same weights (the curve for  $|2\rangle$  coincides with  $|1\rangle$ ). The inset displays the case  $\delta=0$ . As we see in this case the excitation is very small, which is in accordance with Eq. (3.12)  $(2|G_n|^2/\Omega^2=0.13)$ .

### **IV. CONCLUSION**

We have presented a theoretical treatment of the multiphoton resonant excitation of atoms in a strong laser field. A nonperturbative resonant approach was developed and the time evolution of the three-level system was found. The creation of various superposition states was shown. Particular attention was paid to the hydrogen atom. Numerical simulations are in agreement with the theoretical results. Our calculations for the hydrogen atom suggest that, by using the appropriate optical pulses with moderately strong intensities  $\sim 5 \times 10^{13}$  W/cm<sup>2</sup>, when the rate of the concurrent process of multiphoton ionization is small, one can achieve various superposition states by multiphoton resonant excitation. Although the main attention has been devoted to the hydrogen atom the current treatment includes a larger class of systems.

## ACKNOWLEDGMENT

This work was supported by International Science and Technology Center (ISTC) Project No. A-353.

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