Causality in quantum teleportation: Information extraction and noise effects in entanglement distribution

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Quantum teleportation is possible because entanglement allows a definition of precise correlations between the noncommuting properties of a local system and corresponding noncommuting properties of a remote system. In this paper, the exact causality achieved by maximal entanglement is analyzed and the results are applied to the transfer of effects acting on the entanglement distribution channels to the teleported output state. In particular, it is shown how measurements performed on the entangled system distributed to the sender provide information on the teleported state while transferring the corresponding backaction to the teleported quantum state.

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I. INTRODUCTION

Quantum teleportation is one of the most fundamental applications of entanglement between well separated physical systems [1–5]. The transmission of a quantum state through local measurements and classical communications illustrates some of the essential features of quantum physics. It is therefore of great interest to analyze the transmission processes in detail, especially in realistic circumstances where entanglement may not be maximal [6–12]. In this paper, the principles of quantum teleportation are reviewed and a representation of the teleportation process emphasizing the precise causality implied by maximal entanglement is proposed. This representation is especially useful to investigate the transfer of effects on the entanglement distribution channels to the teleported output state. The general theory of this transfer is formulated and the results are applied to measurements on the entangled signal sent from the source of entanglement to the sender. Such measurements could be used by a third party to extract information during the teleportation process, e.g., for the purpose of eavesdropping. Moreover, the effects of this measurement illustrate the distribution of information and noise in the teleportation process, providing insights into the dynamics of quantum information processes.

II. PROPERTIES OF ENTANGLEMENT

Entanglement is the quantum-mechanical property that makes teleportation possible. As was already pointed out by Schrödinger [13], the essential feature of entanglement is that all of the relative properties of two systems can be defined with precision if the individual properties are maximally uncertain. In teleportation, the quantum state in the input can therefore be reconstructed in the output by exclusively referring to these precise relations between the systems, while avoiding any direct measurement of individual properties.

In order to express entanglement in terms of the relation between physical properties in the two systems, it is useful to write the maximally entangled state in a common basis $|n\rangle$. Maximal entanglement between two N-level systems, R and B, can then be expressed by the quantum state

$$|E_{\max}\rangle_{R,B}=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}(\hat{U}_0|n\rangle)_R\otimes|n\rangle_B. \qquad (1)$$

The precise properties of this quantum state are defined by the unitary transformation $\hat{U}_0$. Specifically, any observable property $\hat{O}_B$ of system B corresponds to an observable $\hat{O}_R$ in system R, such that the measurement values obtained for $\hat{O}_B$ will always be equal to the measurement values obtained for $\hat{O}_R$. $\hat{U}_0$ defines the relation between $\hat{O}_B$ and $\hat{O}_R$ as

$$\hat{O}_R=\hat{U}_0\hat{O}_B^T\hat{U}_0^{-1}, \qquad (2)$$

where the transpose $\hat{O}_B^T$ is defined with respect to the basis $|n\rangle$. It is then possible to formulate the unusual properties of entanglement in the spirit of the Einstein-Podolsky-Rosen paradox [14,15] as an apparent violation of local uncertainty relations by nonlocal correlations. Given two noncommuting properties of system B, $\hat{X}_B$ and $\hat{Y}_B$, measurement results for both these properties can be predicted by an observer in R from measurements of the corresponding properties $\hat{X}_R$ and $\hat{Y}_R$ since the maximally entangled state is an eigenstate of the two operator properties

$$(\hat{X}_B-\hat{X}_R)|E_{\max}\rangle_{R,B}=0$$

$$\text{and}\quad (\hat{Y}_B-\hat{Y}_R)|E_{\max}\rangle_{R,B}=0$$

$$\text{with}\quad \hat{X}_R=\hat{U}_0\hat{X}_B^T\hat{U}_0^{-1}$$

$$\text{and}\quad \hat{Y}_R=\hat{U}_0\hat{Y}_B^T\hat{U}_0^{-1}. \qquad (3)$$

Any measurement result of $\hat{X}_R$ in R must be equal to the measurement result for $\hat{X}_B$ in B, and any measurement result of $\hat{Y}_R$ in R must be equal to the measurement result for $\hat{Y}_B$ in B. Interestingly, this property has a quite intuitive classical interpretation. Effectively, the reference system R is a mirror image of system B. In classical physics, this property is not unusual, since all properties of both systems can be defined with precision: uncertainty is a specifically nonclassical feature of quantum-mechanics. Entanglement is only difficult to understand because we cannot explain the connection between the local uncertainty expressed by product states and the nonlocal uncertainty expressed by inseparable entangled states. Using the precisely defined relation between physical properties in systems R and B, it is therefore possible to explain quantum teleportation without using any purely quantum-mechanical terminology.

III. PRINCIPLES OF QUANTUM TELEPORTATION

In quantum teleportation, entanglement is applied to establish a well-defined relation between an unknown input A and an output B via the reference R. Initially, the output B and the reference R are maximally entangled. This means that there exists a well-defined relation between the properties of R and of B, while the individual properties of the two systems are completely unknown. The relation between the unknown input A and the reference R is then obtained from the joint measurement (also referred to as a Bell measurement) of A and R at the location of the sender. The sender communicates the information obtained on the previously unknown relation between A and R to the receiver, and the receiver obtains the exact relation of the unknown input in A and the quantum system B by linking the measured relation between A and R with the previously known relation between R and B. Figure 1 illustrates this analysis of teleportation. The letters A, R, and B denote a complete set of operator properties of the respective systems, while f_{in} and f_m represent the well-defined relations established by the entanglement resource and the joint measurement.

The main problem with this intuitive approach to quantum teleportation is that it does not explicitly include the statistical properties of the quantum state. To express these aspects of quantum teleportation, one should start with an input state $|\psi_{\text{in}}\rangle_A$ in A and the entangled state $|E_{\max}\rangle_{R,B}$ in R and B. This product state is then measured in the subspace of A and R. If a perfect Bell measurement is performed, the systems are projected into a maximally entangled quantum state

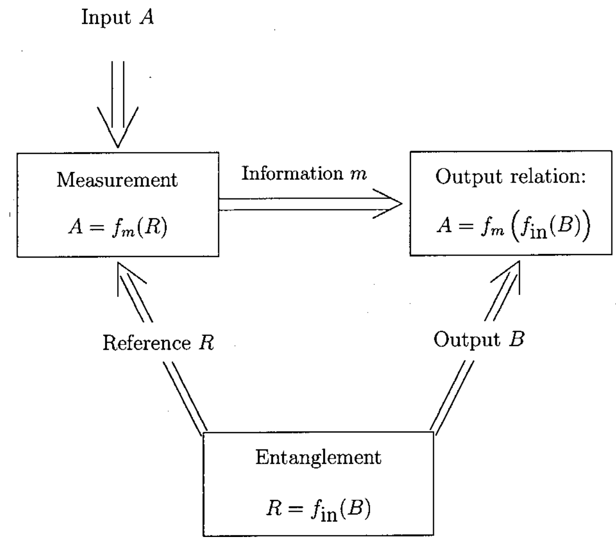

FIG. 1. Illustration of causality in the quantum teleportation process. The letters A, R, B denote the physical properties of the systems. The initial entanglement is described by the relation f_{in} and the result of the Bell measurement is given by m, corresponding to a relation f_m between the input A and the reference R.

$|P(m)\rangle_{A,R}$, where m denotes the measurement result obtained. This maximally entangled state can be conveniently expressed in terms of the $\hat{U}_0|n\rangle$ basis in R. It then reads

$$|P(m)\rangle_{A,R}=\sqrt{\frac{\chi(m)}{N}}\sum_{n=0}^{N-1}[\hat{U}(m)|n\rangle]_A\otimes(\hat{U}_0|n\rangle)_R. \quad (4)$$

The normalization factor $\chi(m)$ is necessary to provide the correct probability for measuring m if a complete set of nonorthogonal measurements is considered. The relation between properties of R and properties of A is expressed by the combination of the unitary transformations $\hat{U}_0$ and $\hat{U}(m)$. Since $\hat{U}_0$ represents the relation between R and B, the unitary transformation $\hat{U}(m)$ effectively describes the relation between A and B necessary for the reconstruction of the quantum state. This can be verified by applying the measurement projection to the input state of the quantum teleportation.

$$\begin{array}{llllll}
\text{Initial state} & \frac{1}{\sqrt{N}}\sum_n & |\psi_{\text{in}}\rangle_A & \otimes & \hat{U}_0|n\rangle_R & \otimes\ |n\rangle_B, \\
\text{measurement} & \sqrt{\frac{\chi(m)}{N}}\sum_{n'} & (\langle n'|\hat{U}^{-1}(m))_A & \otimes & (\langle n'|\hat{U}_0^{-1})_R, & \\
\text{conditional output} & \frac{\sqrt{\chi(m)}}{N}\sum_n & \langle n|\hat{U}^{-1}(m)|\psi_{\text{in}}\rangle & & & |n\rangle_B
\end{array}$$

$$=\hat{U}^{-1}(m)\underbrace{\frac{\sqrt{\chi(m)}}{N}|\psi_{\text{in}}\rangle_B}_{\text{Teleported state}} \quad (5)$$

The unitary transformation $\hat{U}(m)$ therefore describes the relation between the unknown input in A and the fluctuating output B, even though there has been no interaction or other connection between A and B. To understand this effect, it is useful to compare the calculation with the explanation of teleportation illustrated in Fig. 1. This analysis indicates that the measurement performed on A and R merely obtains information about the previously unknown state in B. This information can be represented by a decomposition of the density matrix in B into subensembles corresponding to the different measurement results m. The quantum-mechanical features then arise from the nonclassical properties of density-matrix decompositions. In the present case, it is possible to consider a complete Bell measurement with

$$\sum_m |P(m)\rangle\langle P(m)|_{A,R} = \hat{1}_{A,R}. \tag{6}$$

If this condition is fulfilled, the initial density matrix at B can be decomposed into a mixture of unitary transformations of the input state $|\psi_{\text{in}}\rangle$ with

$$\hat{\rho}_B = \sum_m \frac{\chi(m)}{N^2} [\hat{U}^{-1}(m)|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|\hat{U}(m)]_B = \frac{1}{N}\hat{1}_B. \tag{7}$$

Because of the properties of $\hat{U}(m)$ associated with the completeness relation (6), this decomposition is valid for any input state $|\psi_{\text{in}}\rangle$.

By obtaining the measurement information m, the receiver identifies the subensemble of the density matrix according to its relation with the teleported state. This selection process can be understood entirely in analogy with classical physics. In quantum mechanics, however, there is no fundamental decomposition of the density matrix simultaneously valid for all possible input states. The selection of a subensemble density matrix therefore appears to be a choice between mutually exclusive possibilities. Interpretational problems arise if one tries to reconcile different decompositions with each other. Quantum teleportation provides an input state dependent decomposition of the maximally mixed density matrix in B. It is this dependence of the decomposition of the density matrix in B on the quantum state in A which appears to introduce a nonlocal effect beyond the classical nonlocality of statistical correlations between remote objects. Nevertheless, each subensemble is always potentially contained in the initial mixed state of B, and an observer in B will not be able to identify a specific decomposition without information about the measurement performed on R.

In the case of ideal quantum teleportation described in this section, no information whatsoever is obtained about the

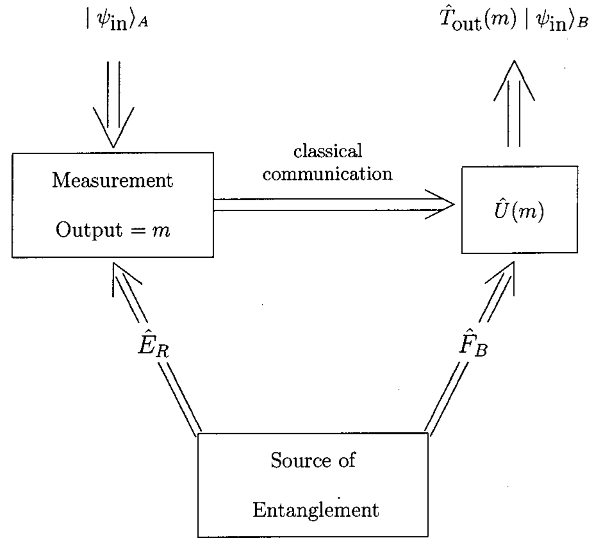

FIG. 2. Illustration of the transfer of effects from entanglement distribution channels to the output state. The output effect $\hat{T}_{\text{out}}$ is a function of the input effects $\hat{E}_R$, $\hat{F}_B$, and of the measurement result m.

properties of the teleported state. Both the information initially available and the information obtained in the Bell measurement are about relations between the systems; neither is about individual systems. However, this condition is only fulfilled if maximal entanglement is available. Any modification to the entangled state of R and B changes the dynamics of teleportation by modifying the information about the individual systems R and B, as well as the implications of the Bell measurement for system A. In the following, the effects of such modifications will be investigated.

IV. TRANSFER OF EFFECTS FROM ENTANGLEMENT DISTRIBUTION TO THE OUTPUT STATE

The entanglement distribution channels may be subject to a variety of effects, such as measurements, decoherence, or unitary transformations [8,16,17]. Such effects can be described by operators $\hat{E}_R$ and $\hat{F}_B$ acting on the quantum states of these channels. $\hat{E}_R$ and $\hat{F}_B$ can represent any combination of unitary operations and measurement projections. Decoherence effects can be represented by random mixtures of such operators. The general transfer properties derived in the following, therefore apply to all kinds of interactions with the entanglement distribution channels.

As shown in Fig. 2, the effects on the entanglement distribution channels will be transferred to a single effect on the output state, described by the output operator $\hat{T}_{\text{out}}$. This output operator can be obtained by an analysis similar to the one applied to the ideal teleportation case in Eq. (5),

$$
\begin{array}{llllll}
\text{Initial state} & \frac{1}{\sqrt{N}}\sum_n & |\psi_{\text{in}}\rangle_A & \otimes & \hat{E}_R\hat{U}_0|n\rangle_R \quad \otimes & \hat{F}_B|n\rangle_B, \\
\text{measurement} & \sqrt{\frac{\chi(m)}{N}}\sum_{n'} & (\langle n'|\hat{U}^{-1}(m))_A & \otimes & (\langle n'|\hat{U}_0^{-1})_R, & \\
\text{output} & \frac{\sqrt{\chi(m)}}{N}\sum_{n,n'} & \langle n'|\hat{U}^{-1}(m)|\psi_{\text{in}}\rangle & & \langle n'|\hat{U}_0^{-1}\hat{E}_R\hat{U}_0|n\rangle & \hat{F}_B|n\rangle_B
\end{array}
$$

$$
= \hat{U}^{-1}(m)\underbrace{\frac{\sqrt{\chi(m)}}{N}\hat{U}(m)\hat{F}_B(\hat{U}_0^{-1}\hat{E}_R\hat{U}_0)^T\hat{U}^{-1}(m)}_{\hat{T}_{\text{out}}}|\psi_{\text{in}B}. \tag{8}
$$

The total effect on the teleported state can be separated into contributions from $\hat{F}_B$ and from $\hat{E}_R$. The effect of $\hat{F}_B$ is only modified by the unitary transformation $\hat{U}(m)$, since this transformation represents the only physical change of the output B after the application of $\hat{F}_B$. The effect $\hat{E}_R$ on the reference channel R is transferred to B by the properties of entanglement since there is no direct physical interaction between R and B. The contribution of $\hat{E}_R$ to $\hat{T}_{\text{out}}$ is equal to the operator $\hat{E}_B$ in B corresponding to $\hat{E}_R$ according to Eq. (2). The effects on R are thus transferred to B by the precise correlations between the two systems. Effectively, any modification of the reference R can be understood as a change in the relation between R and B. Since nothing is known about the individual systems, it does not matter whether the effect really acts on R or on B. The effect on the teleported state can then be written as a sequence of effects on B transformed by $\hat{U}(m)$,

$$
\hat{T}_{\text{out}} = \frac{\sqrt{\chi(m)}}{N}\hat{U}(m)\hat{F}_B\underbrace{(\hat{U}_0^{-1}\hat{E}_R\hat{U}_0)^T}_{\hat{E}_B}\hat{U}(m) \tag{9}
$$

It may be interesting to note that $\hat{E}_B$ acts before $\hat{F}_B$, indicating that entanglement always connects the past of system B to system R, regardless of the actual sequence of $\hat{E}_R$ and $\hat{F}_B$ in time.

The mathematical properties of the formalism clearly show that actions on R and B have equivalent effects. However, the entanglement distribution channels are usually well separated in space, and sometimes even in time. It is therefore interesting to trace the causality connecting actions on R to the output in B in more detail. Since the most important aspect of quantum teleportation is the distribution of information, it is convenient to focus this discussion on the possibility of extracting information in the reference channel R by minimal back-action measurements.

V. MEASUREMENTS ON AN ENTANGLEMENT DISTRIBUTION CHANNEL

In ideal quantum teleportation, no information about the teleported quantum state is obtained in the process. Since any quantum measurement has a corresponding backaction, changing the original state unless it happens to be an eigenstate of the measurement operator, this lack of information about the teleported state is a necessary requirement for the precise transfer of any unknown quantum state. As discussed in Sec. III, this requirement is fulfilled because neither the original entanglement nor the Bell measurement reveals any information on the individual systems. Instead, there is a perfect connection of precise relations between the systems defined by entanglement properties. However, this perfect connection can be broken at any point. For example, an eavesdropper might decide to "listen in" on the teleportation by tapping the entanglement distribution line for the reference R. Since the density matrix of R is completely random, the information initially obtained is pure noise. However, the measurement result provides information on the reference used in the Bell measurement. By combining the measurement information m with the noisy result from the entanglement distribution line, information on the teleported quantum state is obtained. At the same time, the measurement backaction has changed the relation between R and B, making it impossible to reconstruct the exact input state. The output state is therefore modified by a measurement backaction equal to the effects of a direct measurement performed on the input state.

Figure 3 illustrates this eavesdropping scheme. The measurement performed on the entanglement distribution channel is represented by the self-adjoint operators $\hat{E}_R(l)$ corresponding to the measurement results l of a minimal backaction measurement [18]. According to Eq. (9), the effect of this measurement on the output state of the teleportation is described by the output operator

$$
\hat{P}(l,m) = \frac{\sqrt{\chi(m)}}{N}\hat{U}(m)[\hat{U}_0^{-1}\hat{E}_R(l)\hat{U}_0]^T\hat{U}^{-1}(m). \tag{10}
$$

This operator is also self-adjoint, indicating that $\hat{P}(l,m)$ also represents a minimal back-action measurement. Effectively, the operator acting on the teleported state is a unitary transformation of the original measurement operator $\hat{E}_R(l)$. The measurement performed on the entanglement distribution channel R, therefore, converts the teleportation process into a measurement of the unknown input state. The information obtained in this measurement is described by the dependence of the probability of the measurement results l and m on the input state

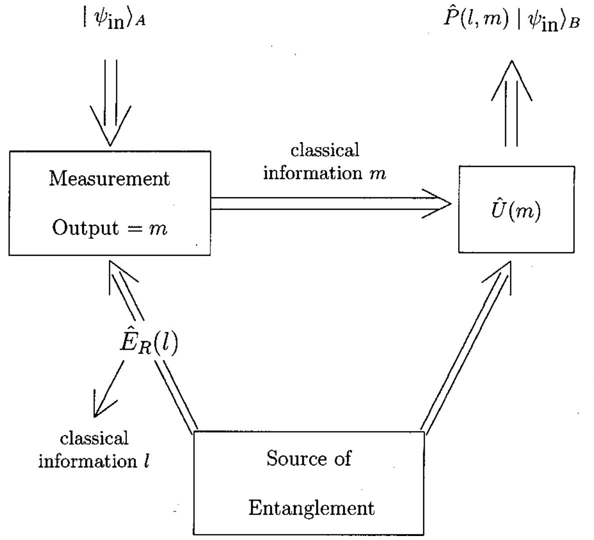

FIG. 3. Measurement on the reference channel of entanglement distribution. $\hat{P}(l,m)$ is the effective measurement operator representing both the information about the quantum state and the measurement backaction associated with the combined measurement results l and m.

$$p(l,m)=\langle\psi_{in}|\hat{P}^2(l,m)|\psi_{in}\rangle$$
$$=\frac{\chi(m)}{N^2}\langle\psi_{in}|\hat{U}(m)[\hat{U}_0^{-1}\hat{E}_R^2(l)\hat{U}_0]^T\hat{U}^{-1}(m)|\psi_{in}\rangle. \tag{11}$$

Equation (11) describes the information extraction achieved by the measurement on the entanglement distribution channel R by combining the information l obtained from R with the information m obtained in the Bell measurement. Specifically, $\hat{P}^2(l,m)$ is the positive operator valued measure of the eavesdropping process, and different input states may be distinguished by the eavesdropper according to the relative statistical weight assigned to them by this measure [19].

Equation (10) shows that the backaction on the teleported state is minimal. It is therefore possible to realize an optimal eavesdropping scheme by performing minimal back-action measurements on the entanglement distribution channels. The precise effect of eavesdropping on the teleported state is given by

$$|\psi_{out}\rangle=\frac{1}{\sqrt{p(l,m)}}\hat{P}(l,m)|\psi_{in}\rangle. \tag{12}$$

The eavesdropping attempt thus modifies the output state, reducing the fidelity of quantum teleportation. A quantitative expression for this loss of fidelity is given by the overlap between the input state and the output state,

$$F(l,m)=|\langle\psi_{out}|\psi_{in}\rangle|^2=\frac{|\langle\psi_{in}|\hat{P}(l,m)|\psi_{in}\rangle|^2}{p(l,m)}. \tag{13}$$

In general, this overlap may strongly depend on the measurement outcome. Equation (13) is therefore a conditional fidelity [10]. If the measurement results are not known, the total fidelity is given by the average over all possible outcomes,

$$F_{total}=\sum_{l,m}|\langle\psi_{in}|\hat{P}(l,m)|\psi_{in}\rangle|^2. \tag{14}$$

Thus the measurement operators $\hat{P}(l,m)$ given in equation (10) fully characterize the loss of fidelity due to the eavesdropping attempt.

Two particularly interesting features of this scheme are the distribution of information about the teleported state between the two measurement results m and l, and the origin of the measurement backaction on the teleported quantum state from a lack of information concerning the the actual relation between the input state in A and the output B. In the following section, these features will be analyzed in greater detail.

VI. DISTRIBUTION OF INFORMATION AND NOISE

Equation (11) describes the information extracted from the teleported state in terms of the joint probability $p(l,m)$ of obtaining a measurement result of l in R, followed by a Bell measurement result of m. The individual probabilities for measuring l and m can be determined from this joint probability by

$$p(l)=\sum_m p(l,m)$$
$$\text{and}\quad p(m)=\sum_l p(l,m). \tag{15}$$

It is obvious that $p(l)$ should not depend on the input state, since there is no relation between the entangled state in R,B, and the input state in A before the Bell measurement is performed. This independence of $p(l)$ from input state properties can be verified by applying the completeness relation in Eq. (7) to the sum over m,

$$p(l)=\sum_m\frac{\chi(m)}{N^2}\langle\psi_{in}|\hat{U}(m)[\hat{U}_0^{-1}\hat{E}_R^2(l)\hat{U}_0]^T\hat{U}^{-1}(m)|\psi_{in}\rangle$$
$$=\mathrm{Tr}\Bigg\{(\hat{U}_0^{-1}\hat{E}_R^2(l)\hat{U}_0)^T$$
$$\times\left(\sum_m\frac{\chi(m)}{N^2}\hat{U}^{-1}(m)|\psi_{in}\rangle\langle\psi_{in}|\hat{U}(m)\right)\Bigg\}$$
$$=\frac{1}{N}\mathrm{Tr}\{\hat{E}_R^2(l)\}. \tag{16}$$

Note that this probability may also be derived directly from the local density matrix in R before the measurements. This density matrix may also be expressed in terms of the mixture given for $\hat{\rho}_B$ in Eq. (7). Therefore, the sum over m restores the situation before the Bell measurement in R, providing the input independent probability $p(l)$.

The probability $p(m)$ can be derived by making use of the completeness relation of the minimal back-action measurement of l,

$$\sum_l \hat{E}_R^2(l)=\hat{1}_R. \tag{17}$$

In terms of the measurement backaction on the quantum state in R, this completeness relation implies that the density matrix in R after the measurement of l is still equal to $\hat{1}_R/N$ if the measurement result is unknown. Therefore, the statistics of the Bell measurement is unchanged and the probability $p(m)$ does not depend on the input state either. In accordance with this observation, the result of the summation over l reads

$$p(m)=\frac{\chi(m)}{N^2}\langle\psi_{\rm in}|\hat{U}(m)\left[\hat{U}_0^{-1}\left(\sum_l \hat{E}_R^2(l)\right)\hat{U}_0\right]^T \times\hat{U}^{-1}(m)|\psi_{\rm in}\rangle =\frac{\chi(m)}{N^2}. \tag{18}$$

Since the measurement in R does not change the overall statistics of the Bell measurement, an eavesdropping attempt using a minimal back-action measurement on R cannot be detected by the sender, even if some statistical properties of the input states are known. By itself, the measurement result m does not provide any information on the input state. Such information is only obtained by combining the measurement results m with the measurement results l from the entanglement distribution channel R.

As shown in Eq. (7), the main effect of the Bell measurement is the decomposition of the maximally mixed density matrix in B in terms of unitary transformations of the unknown input state. For a maximally entangled state, this decomposition is equally possible for any input state. However, the measurement of l in R provides information about the decomposition of $\hat{\rho}_B$ which is independent of the input state. It is therefore interesting to analyze the decomposition of $\hat{\rho}_B$ in the presence of the measurement on R. The total decomposition can be obtained from $\hat{P}(l,m)$ if the conditional unitary transformation $\hat{U}(m)$ of the output is reversed. It then reads

$$\hat{\rho}_B=\sum_{l,m}\hat{U}^{-1}(m)\hat{P}(l,m)|\psi_{\rm in}\rangle\langle\psi_{\rm in}|\hat{P}(l,m)\hat{U}(m) =\frac{1}{N}\hat{1}_B. \tag{19}$$

The decomposition into contributions with different l represents the information obtained through the measurement of R, while the teleportation effects are represented by the decomposition into different m. This sequence of the decomposition can be expressed by writing $\hat{\rho}_B$ as

$$\hat{\rho}_B=\sum_l [\hat{U}_0^{-1}\hat{E}_R(l)\hat{U}_0]^T\underbrace{\left(\sum_m\frac{\chi(m)}{N^2}\hat{U}^{-1}(m)|\psi_{\rm in}\rangle\langle\psi_{\rm in}|\hat{U}(m)\right)_B}_{=\hat{1}_B/N}[\hat{U}_0^{-1}\hat{E}_R(l)\hat{U}_0]^T \tag{20}$$

Thus the measurement of l first decomposes the density matrix in B according to the information obtained from R only. The measurement of m then decomposes the components of each result l according to the same statistical weights previously obtained for ideal teleportation in Eq. (7). However, this decomposition is now modified according to the measurement information provided by l, resulting in a distortion of the transformed input state components $\hat{U}^{-1}(m)|\psi_{\rm in}\rangle$. This distortion reflects the reduction of entanglement caused by obtaining information on the local systems R and B. The local uncertainty introduced into R by the measurement backaction is thus transferred to the output state.

It is possible to vary the measurement operators $\hat{E}_R(l)$ continuously between the unit operator $\hat{1}_R$ representing no interaction and precise projections onto a complete orthonormal set of eigenstates $|l\rangle$ of a self-adjoint operator $\hat{L}_R$. In the latter case, entanglement is completely removed by the measurement. It is therefore a particularly simple example and may help to illustrate some of the general features of the information distribution caused by the measurement in R. The projective measurement in R decomposes the density matrix in B into eigenstates $|\phi_l\rangle$ of the variable $\hat{L}_B$ corresponding to $\hat{L}_R$ according to Eq. (2). The measurement in m cannot subdivide this decomposition any more, so it merely provides measurement probabilities,

$$\hat{\rho}_B=\sum_l |\phi_l\rangle\langle\phi_l|\sum_m p(l,m),$$

$$\text{with}\quad p(l,m)=\frac{\chi(m)}{N^2}|\langle\phi_l|\hat{U}^{-1}(m)|\psi_{\rm in}\rangle|^2. \tag{21}$$

As the measurement probability $p(l,m)$ shows, the Bell measurement now projects the input states onto unitary transforms $\hat{U}(m)|\phi_l\rangle$ of the eigenstates of $\hat{L}_B$. This corresponds to a precise measurement of the property $\hat{U}(m)\hat{L}_B\hat{U}^{-1}(m)$ in A. Only this property can then be reproduced in the output state. The measurement backaction randomizes the relation between properties that do not commute with $\hat{L}_R$ in R and their corresponding properties in B. Therefore, the Bell measurement does not provide any information on such variables. This simple example also illustrates the role of l and m in defining the effective measurement performed on the teleported state. Knowledge of m determines the actual variable $\hat{U}(m)\hat{L}_B\hat{U}^{-1}(m)$ defined by the measurement of m, while l provides the measurement outcome for that variable. The measurement result l initially provides only information about a physical property of the reference system R, and the measurement result m is necessary to establish the relation between this property in R and a corresponding property of the input A. Thus, the Bell measurement randomly selects the physical property $\hat{U}(m)\hat{L}_B\hat{U}^{-1}(m)$ measured in A after the measurement result l has been obtained in R.

VII. CONCLUSIONS

Quantum teleportation is an application of the extremely precise correlations possible between two entangled systems. The discussion presented in this paper shows how these correlations can be identified in the quantum-mechanical formalism using unitary operations and density-matrix decompositions. The resulting formulation is especially convenient for tracing the transfer of effects from entanglement distribution channels to the output state.

In the case of a measurement on an entanglement distribution channel, the measurement backaction introduces noise into the teleportation by reducing the precision in the correlation between the entangled systems. The information obtained in the measurement distribution channel may then be combined with the result of the Bell measurement to provide information about the teleported quantum state. This example thus illustrates how the quantum information in the original input state is distributed between the two measurement results and the teleported quantum state in the output.

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