

Quantum-trajectory approach to stochastically induced quantum-interference effects in coherently driven two-level atoms

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Stochastic perturbation of two-level atoms strongly driven by a coherent light field is analyzed by the quantum-trajectory method. A method is developed for calculating the resonance fluorescence spectra from numerical simulations. It is shown that in the case of dominant incoherent perturbation, the stochastic noise can unexpectedly create phase correlation between the neighboring atomic dressed states. This phase correlation is responsible for quantum interference between the related transitions resulting in anomalous modifications of the resonance fluorescence spectra.

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I. INTRODUCTION

Quantum interference is one of the most intriguing phenomena of quantum mechanics. Over the past decade several effects in atom-light interaction, which have their origin in quantum interference, have been predicted and demonstrated experimentally [1]. Some characteristic examples are reduction and cancellation of absorption [2–6] and spontaneous emission [7–10], and narrow resonances in fluorescence [11,12]. A prerequisite of quantum interference between the transition channels is the existence of some stable time correlation of the atomic system under consideration. A possible way of achieving such correlation is the application of coherent coupling in a multilevel atomic system. Although some interference effects have also been found in two-level systems interacting with two light beams [13], so far quantum interference has been observed exclusively in at least three-level systems.

Generally, various incoherent perturbations destroy the phase correlation between the states involved in the interfering transition pathways and the coherently induced quantum interference disappears. However, under special circumstances, even incoherent perturbation can be responsible for quantum interference. For example, quantum interference can appear in three-level systems due to collisions. Such effects are known as pressure-induced extra resonances and have been studied in four-wave mixing [14,15].

Recently, in an experiment with coherently driven two-level atoms, anomalous resonance fluorescence spectra were found when the collisional relaxation rate exceeded the Rabi frequency [16]. The spectra were of the form of a pressure-broadened line with a narrow, not collisionally broadened dip. These results, unexpected in a collisionally perturbed two-level system, were interpreted as a consequence of quantum interference between different dressed-state transition

channels. In Ref. [16], it was pointed out that these effects can also occur in the case of a nonmonochromatic, e.g., phase-diffusing laser field. Indeed, resonance fluorescence spectra with the phase-diffusing laser field have been calculated by Zhou *et al.* [17], who obtained the same results as Gawlik *et al.* [16]. What in both cases appears essential for observation of quantum interference and anomalous spectra is that the incoherent perturbation (collisions or phase diffusion of the light field) dominates over the Rabi oscillations.

These two examples raise an intriguing question, how can a stochastic noise lead to stable time correlation resulting in quantum interference in two-level systems.

Quantum-trajectory methods are widely used powerful tools for treating the stochastic evolution of open quantum systems [18–23]. They can provide the solution of any master equation that is of Lindblad form [24–27]. Moreover, individual quantum trajectories, as state evolutions conditioned on a particular sequence of observed events, make it possible to reveal phase correlations in the given system.

While resonance fluorescence spectra can be adequately modeled by the master equation, we find that the use of quantum trajectories provides more physical insight. Besides, this method allows one to study the stochastic evolution of the atomic system and, eventually, reveal the phase correlation of its dressed states. For that reason, in this paper we analyze in detail the system of stochastically perturbed two-level atoms applying both the master equation and the quantum-trajectory method to explain quantum-interference effects and the underlying physical processes.

The paper is organized as follows: In Sec. II we introduce our model for the system of a coherently driven and incoherently perturbed two-level atom. In Sec. III the method of quantum trajectories is applied to the system and a method is developed for calculating the spectrum from the simulation results. Section IV presents the numerical results for the spectra and compares them with the analytical solution of the master equation. In Sec. V the phase difference between the dressed states of an atom is analyzed and the phase correlation is revealed. It is shown that the phase correlation emerges as the noise magnitude increases and the related quantum-interference effect is discussed.

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II. THE MODEL

The system of incoherently perturbed and coherently driven two-level atoms can be modeled in several ways. Here we make a rather general assumption that the stochastic perturbation is responsible for fluctuations of the atomic resonance frequency that obeys the Gaussian statistics. In particular, such fluctuations may result from, e.g., elastic, dephasing collisions.

In our model the Hamiltonian of the strongly, coherently driven atom subjected to stochastic perturbation has the form

$$H_{AL} = \hbar[\omega_a(t) - \omega_L]S^z + \frac{1}{2}\hbar\Omega(S^- + S^+) \quad (1)$$

in the interaction picture, where $\omega_a(t) = \omega_a + \delta\omega_a(t)$ is the fluctuating atomic transition frequency, ω_L is the frequency of the laser, Ω is the Rabi frequency, and S^z , S^+ , and S^- are the atomic operators defined in the excited state ($|e\rangle$)–ground state ($|g\rangle$) basis,

$$S^z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad S^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad S^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (2)$$

We assume that the noise in the transition frequency satisfies

$$\langle \delta\omega_a(t) \delta\omega_a(t') \rangle = 2\Gamma \delta(t-t'), \quad (3)$$

where Γ stands for the magnitude of the stochastic noise. If the noise is due to collisions, this quantity is the collision rate between the atoms. This model can also describe the system of two-level atoms driven by a laser field with fluctuating phase, if the phase drift is neglected [17]. In such case, Γ represents the laser linewidth.

The time evolution of the system defined by the Hamiltonian in Eq. (1) is described by the following master equation, taking into account also the spontaneous emission processes:

$$\dot{\rho} = \frac{1}{i\hbar} [\langle H_{AL} \rangle, \rho] + L\rho, \quad (4)$$

where

$$L\rho = (L\rho)_{\text{sp}} + (L\rho)_{\text{st}}, \quad (5a)$$

$$(L\rho)_{\text{sp}} = \gamma \left[-\frac{1}{2}(S^+ S^- \rho + \rho S^+ S^-) + S^- \rho S^+ \right], \quad (5b)$$

$$(L\rho)_{\text{st}} = 4\Gamma \left[-\frac{1}{2}(S^z S^z \rho + \rho S^z S^z) + S^z \rho S^z \right], \quad (5c)$$

and $\langle H_{AL} \rangle$ is the mean atomic Hamiltonian obtained by averaging over the stochastic noise of Eq. (3), and γ is the natural linewidth of the atom.

It is natural to introduce the dressed-state basis in which the atomic Hamiltonian H_{AL} is diagonal,

$$|1\rangle = \cos \Theta |g\rangle + \sin \Theta |e\rangle, \quad (6a)$$

$$|2\rangle = -\sin \Theta |g\rangle + \cos \Theta |e\rangle, \quad (6b)$$

where

$$\Theta = -\frac{1}{2} \arctan \left(\frac{\Omega}{\Delta} \right),$$

and $\Delta = \omega_a - \omega_L$ is the laser detuning.

The mean atomic Hamiltonian $\langle H_{AL} \rangle$ in the dressed-state basis $|1\rangle$, $|2\rangle$ can be written as

$$\langle H_{AL} \rangle = E_1 |1\rangle \langle 1| + E_2 |2\rangle \langle 2|, \quad (7)$$

where

$$E_{1,2} = \mp \frac{1}{2} \hbar \sqrt{\Omega^2 + \Delta^2}.$$

The effect of incoherent perturbation on the pure states can be determined from the Lindblad form (5c) of the master equation in Eq. (4). The action of the operator $2\sqrt{\Gamma}S^z$ corresponds to an event generated by the stochastic noise. Without detuning, this operator generates transitions between the dressed states $|1\rangle$ and $|2\rangle$,

$$2\sqrt{\Gamma}S^z|1\rangle = \sqrt{\Gamma}|2\rangle, \quad (8a)$$

$$2\sqrt{\Gamma}S^z|2\rangle = \sqrt{\Gamma}|1\rangle. \quad (8b)$$

Another way of observing the effect of the stochastic noise is to transform the time-dependent Hamiltonian in the Langevin equation into the dressed-state basis. This must be done carefully since parameter Θ in the definition of the dressed states becomes time dependent in this case,

$$|1,t\rangle \approx |1\rangle - \frac{1}{2} \frac{\Omega}{\Delta^2 + \Omega^2} \delta\omega_a(t) |2\rangle, \quad (9a)$$

$$|2,t\rangle \approx |2\rangle + \frac{1}{2} \frac{\Omega}{\Delta^2 + \Omega^2} \delta\omega_a(t) |1\rangle \quad (9b)$$

to the first order in $\delta\omega_a(t)$. Hamiltonian H_{AL} is diagonal in the time-dependent dressed-state basis, thus

$$\begin{aligned} H_{AL} &= E_1 |1,t\rangle \langle 1,t| + E_2 |2,t\rangle \langle 2,t| \approx E_1 |1\rangle \langle 1| \\ &\quad + E_2 |2\rangle \langle 2| + \delta\omega_a(t) \frac{1}{2} \frac{E_1 \Omega}{\Delta^2 + \Omega^2} (-|1\rangle \langle 2| - |2\rangle \langle 1|) \\ &\quad + \delta\omega_a(t) \frac{1}{2} \frac{E_2 \Omega}{\Delta^2 + \Omega^2} (|1\rangle \langle 2| + |2\rangle \langle 1|) \\ &= \langle H_{AL} \rangle - \hbar \delta\omega_a(t) \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}} (|1\rangle \langle 2| + |2\rangle \langle 1|). \end{aligned} \quad (10)$$

This also shows that the stochastic noise generates transitions between the dressed states $|1\rangle$ and $|2\rangle$.

The master equation has the following form in the dressed-state basis:

$$\frac{d}{dt}\rho_z = -\left(2\frac{\Gamma'\Omega^2}{\Omega^2+\Delta^2} + \gamma\right)\rho_z + 2\frac{\Gamma'\Delta\Omega}{\Omega^2+\Delta^2} \times (\rho_{21} + \rho_{12}) + \gamma\frac{\Delta}{\sqrt{\Omega^2+\Delta^2}}, \quad (11a)$$

$$\begin{aligned} \frac{d}{dt}\rho_{12} = & \frac{\Gamma'\Omega\Delta}{\Omega^2+\Delta^2}\rho_z + \frac{1}{2}\gamma\Omega \\ & - \left(2\Gamma'\frac{\Delta^2}{\Omega^2+\Delta^2} + i\sqrt{\Omega^2+\Delta^2} + \gamma\right)\rho_{12} \\ & - \Gamma'\frac{\Omega^2}{\Omega^2+\Delta^2}(\rho_{12} - \rho_{21}), \end{aligned} \quad (11b)$$

where $\rho_z = \rho_{11} - \rho_{22}$, $\Gamma' = \Gamma - \gamma/4$, and ρ_{11} , ρ_{12} , ρ_{21} , ρ_{22} are the matrix elements of the density operator in the dressed-state basis. The matrix element ρ_{21} is the complex conjugate of ρ_{12} , as ρ is Hermitian. For resonant excitation ($\Delta = 0$) the stochastic noise couples the $|1\rangle$ and $|2\rangle$ states and increases the relaxation rate of the spin- z component. In this case the dressed states become independent of the Rabi frequency, and Eq. (11a) is uncoupled from Eq. (11b). This is however not the case in the general, nonresonant case.

III. NUMERICAL SIMULATION

In the system of coherently driven stochastically perturbed two-level atoms, quantum-interference effects can be seen in the resonance fluorescence spectra [16,17]. The resonance fluorescence can be described by transitions between appropriate dressed states of the atom. If spectral modifications are due to quantum interference, some time correlation should exist between the dressed states of the atom involved in the interfering transition channels.

For analyzing time correlations in a quantum system, quantum-trajectory methods are particularly appropriate. These methods are based on the simulation of quantum trajectories, which are individual realizations of the evolution of the system conditioned on a particular sequence of observed events. By tracking the time evolution of a single quantum trajectory, the time correlations can be revealed.

We apply the quantum-trajectory method of Ref. [18] for simulating the time evolution of the coherently driven, stochastically perturbed two-level atom. In this system, a single quantum-trajectory evolves coherently according to the Hamiltonian of Eq. (7), interrupted by incoherent gedanken measurements due to noise events and spontaneous emission. The evolution of the density operator of the system is obtained by averaging the density operators of the individual quantum trajectories. The resulting density operator is the solution of the master equation of Eq. (4).

The accuracy of the simulation is limited by two factors: the length Δt of the time step and the number N of the simulated quantum trajectories. Δt should be much less than the characteristic time of any process in the system. N should be large enough to obtain the right ensemble averages for the

density operator at the given stochastic noise magnitude. In our simulations N was $\approx 5 \times 10^5$.

Within dipole approximation the resonance fluorescence spectrum $S(\omega)$ can be calculated as the real part of the two-time correlation function

$$\Gamma_1^N(\omega) = \lim_{t \rightarrow \infty} \int_0^\infty \exp(-i\omega\tau) \langle S^+(t+\tau)S^-(t) \rangle d\tau, \quad (12)$$

for an arbitrary initial condition

$$S(\omega) = \text{Re} \Gamma_1^N(\omega), \quad (13)$$

where ω is the detuning of the emitted light from ω_L . There are different methods in the literature for obtaining the spectrum using a numerical simulation [18,25,26]. One kind of them simulates not only the atom, but also the quantized electromagnetic field [25]. Such methods seem to be excessive when the field can be treated classically.

The method presented by Dalibard *et al.* [18] simulates only the atom, and obtains the spectrum by calculating two-time averages and taking their Fourier transform. The computation time of this method increases as $1/\Delta t^2$, where Δt is the time step of the simulation, because, for each time step, an additional simulation is started to calculate the two-time averages. That can be time consuming in the case when a large number of quantum trajectories are simulated and small time steps are used. This is the situation in our problem when we simulate the system in the high-noise magnitude regime.

The question arises whether it is possible to develop a method that simulates only the atom, without the need of starting extra simulations for calculating the two-time averages. Below, we briefly outline the essentials of our proposed method for spectrum calculation. More details will be published elsewhere.

Let us consider a general two-level atom-field system. Let $\rho(t)$ be the density operator of the whole system, A an operator in the Schrödinger picture acting only on the atom, and $U(t)$ the unitary time-evolution operator. Then

$$\begin{aligned} \text{Tr}[A\rho(t)] &= \text{Tr}[AU(t)\rho(0)U^\dagger(t)] \\ &= \text{Tr}[U^\dagger(t)AU(t)\rho(0)] \\ &= \text{Tr}[A(t)\rho(0)], \end{aligned}$$

where $A(t)$ is the operator A in the Heisenberg picture. One can define a time-dependent $A'(t)$ operator for which

$$\begin{aligned} \text{Tr}[A\rho(t)] &= \text{Tr}_A[A\rho_A(t)] \\ &= \text{Tr}_A[A'(t)\rho_A(0)], \end{aligned} \quad (14)$$

where $\rho_A(t) = \text{Tr}_L\rho(t)$ is the reduced density operator of the atom. $A'(t)$ depends also on the laser field. Let $R_i(0)$ be a set of reduced density operators of the atom which form a \mathbb{C} -linear basis in the set of the operators acting on the atom. In the case of a two-level atom the basis consists of four elements. These basis elements may evolve also, their value at time t is denoted by $R_i(t)$. Using this basis, any operator X that acts on the atom can be written in the form

$$X = \sum_i x_i R_i(0). \quad (15)$$

The coefficients can be expressed as

$$x_i = \sum_j (T^{-1})_{ij} \text{Tr}[X R_j(0)], \quad (16)$$

where $T_{ij} = \text{Tr}[R_i(0)R_j(0)]$. Matrix T is invertable since the operators $R_i(0)$ form a basis and they are linearly independent. Using Eqs. (14)–(16), the following form can be derived for the $A'(t)$ operator:

$$A'(t) = \sum_{ik} (T^{-1})_{jk} \text{Tr}[R_k(t)R_i(0)] \lambda_i(0) R_j(0), \quad (17)$$

where $\lambda_i(0)$ is expressed by the atomic operator A using Eq. (16) as

$$\lambda_i(0) = (T^{-1})_{ij} \text{Tr}[A R_j(0)].$$

Equation (14) holds for all density operators $\rho(0)$ with $A'(t)$ of Eq. (17),

$$\text{Tr}[A(t)\rho(0)] = \text{Tr}_A[A'(t)\rho_A(0)]. \quad (18)$$

Having an operator $B(t)$ in the Heisenberg picture such that $B(0)$ acts only on the atom, $B(0)\rho(0)$ can be expressed as a C-linear combination of density operators. The linearity of the trace in Eq. (18) yields

$$\text{Tr}[A(t)B(0)\rho(0)] = \text{Tr}[A'(t)B(0)\rho_A(0)]. \quad (19)$$

For calculating two-time correlation functions of the form $\text{Tr}A(t)B(t')\rho(0)$, the above equation can be modified by using the cyclic property of the trace,

$$\begin{aligned} \text{Tr}A(t)B(t')\rho(0) &= \text{Tr}A(t)U(-t')B(0)U(t')U(-t') \\ &\quad \times \rho(t')U(t') \\ &= \text{Tr}U(t')A(t)U(-t')B(0)\rho(t') \\ &= \text{Tr}A(t-t')B(0)\rho(t') \\ &= \text{Tr}_A A'(t-t')B(0)\rho_A(t'), \end{aligned} \quad (20)$$

where $\rho_A(t')$ is the reduced density operator in the Schrödinger picture at time t' .

Let us apply the general expressions presented above to the atomic operators S^+ and S^- . Calculating the correlation function, the quantity $\text{Tr}S^+(t+\tau)S^-(t)\rho(0)$ should be determined from the simulation. Using Eq. (20),

$$\begin{aligned} \lim_{t \rightarrow \infty} \text{Tr}S^+(t+\tau)S^-(t)\rho(0) &= \lim_{t \rightarrow \infty} \text{Tr}S^+(\tau)S^-(0)\rho(t) \\ &= \text{Tr}_A S^{+'}(\tau)S^-(0)\rho_A(\infty). \end{aligned} \quad (21)$$

In order to obtain operator $S^{+'}(\tau)$ from the simulation, one needs to choose a basis consisting of density operators, according to Eq. (15) and start independent simulations using the elements of this basis as initial states. In our simulation, we choose the density operators that in the dressed-state basis $|1\rangle, |2\rangle$ are

$$R_1(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_3(0) = \frac{1}{2} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix},$$

$$R_2(0) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad R_4(0) = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \quad (22)$$

For all time steps of the simulation we calculate the $S^{+'}(\tau)$ operator using Eq. (17) and record it for later use. After the simulation has been completed, i.e., the time t has reached its final value T , the correlation function defined by Eq. (12) is calculated numerically by evaluating the expression

$$\Gamma_1^N(\omega) = \sum_{\tau=0}^T \exp(-i\omega\tau) \text{Tr}S^{+'}(\tau)S^-(0)\overline{\rho(T)}\Delta t, \quad (23)$$

where the summation is done over all time steps between 0 and T and $\overline{\rho(T)}$ is the average of all four $R_i(T)$ density operators.

The advantage of this method is that there is no need to start a new simulation in each time step, and it is sufficient to simulate only the atomic system for obtaining the spectra.

IV. THE FLUORESCENCE SPECTRUM

In order to check our numerical results, we compare them with the spectra calculated analytically. After determining the time evolution of the averages of the Bloch vector components $\langle S^z(t) \rangle$, $\langle S^+(t) \rangle$, $\langle S^-(t) \rangle$, the quantum regression theorem is used for expressing the two-time average $\langle S^+(t+\tau)S^-(t) \rangle$ in Eq. (12) as a function of one-time averages [28]. The Bloch equations are

$$\langle S^z(\dot{t}) \rangle = -\gamma \langle S^z(t) \rangle + \frac{1}{2} i \Omega (\langle S^-(t) \rangle - \langle S^+(t) \rangle) - \frac{1}{2} \gamma,$$

$$\langle S^+(\dot{t}) \rangle = -i \Omega \langle S^z(t) \rangle + (i \Delta - 2\Gamma - \frac{1}{2} \gamma) \langle S^+(t) \rangle,$$

$$\langle S^-(\dot{t}) \rangle = i \Omega \langle S^z(t) \rangle + (-i \Delta - 2\Gamma - \frac{1}{2} \gamma) \langle S^-(t) \rangle.$$

After some calculation we obtain

$$\Gamma_1^N(\omega) = \frac{i\Omega \left(i\omega - i\Delta + 2\Gamma + \frac{1}{2}\gamma \right) \left(-\frac{1}{2}K_3 - K_1K_3 \right)}{(i\omega + \gamma) \left[\left(i\omega + 2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right] + \Omega^2 \left(i\omega + 2\Gamma + \frac{1}{2}\gamma \right)} + \frac{\left[\frac{1}{2}\Omega^2 + (i\omega + \gamma) \left(i\omega - i\Delta + 2\Gamma + \frac{1}{2}\gamma \right) \right] \left(\frac{1}{2} + K_1 - K_2K_3 \right) + \frac{1}{2}\Omega^2(-K_3K_3)}{(i\omega + \gamma) \left[\left(i\omega + 2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right] + \Omega^2 \left(i\omega + 2\Gamma + \frac{1}{2}\gamma \right)}, \quad (24)$$

where

$$K_1 = \frac{\frac{1}{2}\gamma \left[\left(2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right]}{\gamma \left[\left(2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right] + \Omega^2 \left(2\Gamma + \frac{1}{2}\gamma \right)}, \quad (25a)$$

$$K_2 = \frac{\frac{1}{2}\gamma i\Omega \left(i\Delta - 2\Gamma - \frac{1}{2}\gamma \right)}{\gamma \left[\left(2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right] + \Omega^2 \left(2\Gamma + \frac{1}{2}\gamma \right)}, \quad (25b)$$

$$K_3 = \frac{\frac{1}{2}\gamma i\Omega \left(i\Delta + 2\Gamma + \frac{1}{2}\gamma \right)}{\gamma \left[\left(2\Gamma + \frac{1}{2}\gamma \right)^2 + \Delta^2 \right] + \Omega^2 \left(2\Gamma + \frac{1}{2}\gamma \right)}. \quad (25c)$$

For the special case of no detuning ($\Delta=0$), the correlation function has the form

$$\Gamma_1^N(\omega) = \frac{1}{\left[\omega - 2i \left(\Gamma + \frac{1}{4}\gamma \right) \right] \left[i\alpha^2(2\omega^2 - 4i\omega\Gamma'' - \alpha^2) \right]} \times \{ (4\Gamma + \gamma) [(\alpha^2 + \gamma^2)\Omega^2 - 2\Omega^4 - \alpha^4] - 2\Omega^6 + \omega^2 [2\alpha^4 - 2\Omega^2(\alpha^2 + \gamma^2)] + i\omega [(3\gamma + 4\Gamma'')\Omega^2\alpha^2 - 4\Gamma''\alpha^4 - 2\gamma(2\Omega^2 - \gamma^2)\Omega^2] \}, \quad (26)$$

where

$$\alpha^2 = \gamma^2 + 4\Gamma\gamma + \Omega^2, \quad \Gamma'' = \Gamma' + \gamma, \quad \Gamma' = \Gamma - \frac{1}{4}\gamma. \quad (27)$$

The correlation function of Eq. (26) can be split into the sum of three functions:

$$\Gamma_1^N(\omega) = \frac{A_+}{\omega - s_+} + \frac{A_-}{\omega - s_-} + \frac{A_0}{\omega - s_0}, \quad (28)$$

where

$$s_{\pm} = i\Gamma'' \pm i\sqrt{\Gamma'^2 - \Omega^2}, \quad (29a)$$

$$s_0 = 2i\Gamma' + i\gamma, \quad (29b)$$

$$A_{\pm} = i\Omega^2 \frac{\frac{1}{2}\alpha^2 + 3\Gamma'\gamma}{2i\alpha^2(s_+ - s_-)(is_- + \gamma)} - i\Omega^2 \frac{-4\gamma^2\Gamma'' \left(\Gamma + \frac{1}{4}\gamma \right) \pm i\gamma\alpha^2(s_+ - s_-)}{2i(s_+ - s_-)\alpha^4(is_- + \gamma)},$$

$$A_0 = -i \frac{\Omega^2 + (4\Gamma + \gamma)\gamma}{2\alpha^2}.$$

If $\Gamma' < \Omega$, the spectrum has the form

$$S(\omega) = \frac{A_0 s_0}{\omega^2 + |s_0|^2} + \frac{\text{Re } A_+ \omega - \text{Re}(A_+ s_+^*)}{(\omega + \sqrt{\Omega^2 - \Gamma'^2}) + \Gamma''^2} + \frac{\text{Re } A_- \omega - \text{Re}(A_- s_-^*)}{(\omega - \sqrt{\Omega^2 - \Gamma'^2}) + \Gamma''^2}, \quad (30)$$

showing that the centers of the two latter Lorentzians are displaced by $\pm\sqrt{\Omega^2 - \Gamma'^2}$ relative to the laser frequency. Together with the first Lorentzian at the laser frequency, they form the Mollow triplet [29].

In the other case when $\Gamma' > \Omega$, all the Lorentzians are centered at zero frequency, corresponding to ω_L in the

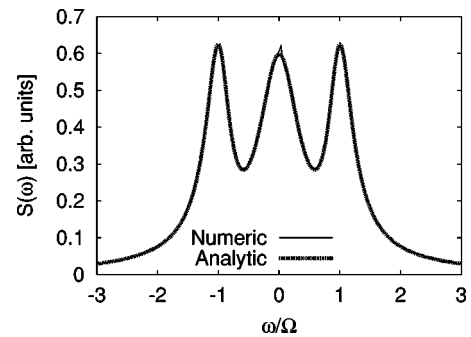


FIG. 1. The resonance fluorescence spectrum for low noise magnitude and strong laser field ($\Gamma/\Omega=0.2$ and $\gamma/\Omega=0.05$) in the case of no detuning ($\Delta=0$).

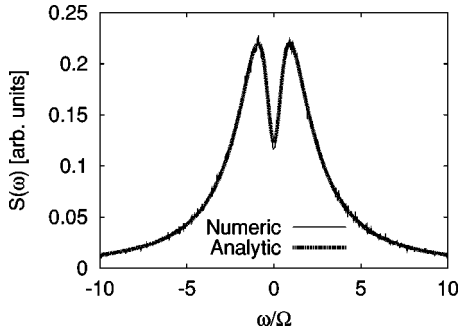


FIG. 2. The resonance fluorescence spectrum for noise magnitude comparable to Rabi frequency and strong laser field ($\Gamma/\Omega = 1.1$ and $\gamma/\Omega = 0.05$) in the case of no detuning ($\Delta = 0$).

Schrödinger picture, but one of them has a negative coefficient, resulting in a dip in the spectrum.

For the $\Gamma' > \Omega$ case, one obtains the following expression for the spectrum:

$$S(\omega) = \frac{A_+ s_+}{\omega^2 + |s_+|^2} + \frac{A_- s_-}{\omega^2 + |s_-|^2} + \frac{A_0 s_0}{\omega^2 + |s_0|^2}, \quad (31)$$

where $A_+ s_+$ and $A_0 s_0$ are always positive and $A_- s_-$ is negative.

In the following we show our numerical results together with the analytical spectra. Figure 1 presents the resonance fluorescence spectrum of the atom irradiated by a resonant ($\Delta = 0$), strong ($\Omega \gg \gamma$) laser field with low noise ($\Omega > \Gamma$). The spectrum exhibits a three-peak structure, but with a suppressed and broadened central peak compared to the standard Mollow triplet. As the noise increases, the central peak disappears and for Γ nearly equal to Ω , we get a two-peak structure with a relatively broad dip, as depicted in Fig. 2. When the noise magnitude is much larger than the Rabi frequency, the dip becomes very narrow, as shown in Fig. 3. The width of the dip is proportional to the value of the parameter $|s_-|$ of Eq. (29a), which approaches the natural linewidth γ when $\Gamma' \gg \Omega$, as shown in Fig. 4. For large detuning ($\Delta \gg \Omega$) and low noise ($\Gamma \ll \Omega$), a two-peak spectrum is obtained with an asymmetric Fano-like structure at the center, as depicted in Fig. 5. Increasing the noise magnitude the Fano-like peak transforms to an asymmetric Fano profile, a

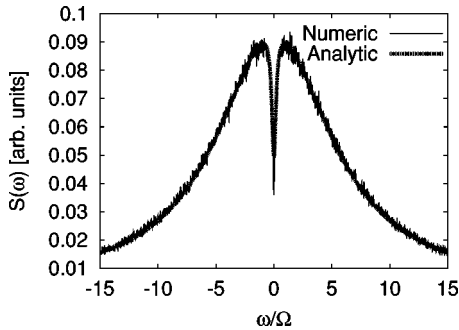


FIG. 3. The resonance fluorescence spectrum for high noise magnitude and strong laser field ($\Gamma/\Omega = 6$ and $\gamma/\Omega = 0.05$) in the case of no detuning ($\Delta = 0$), showing a narrow dip at the center of the spectrum.

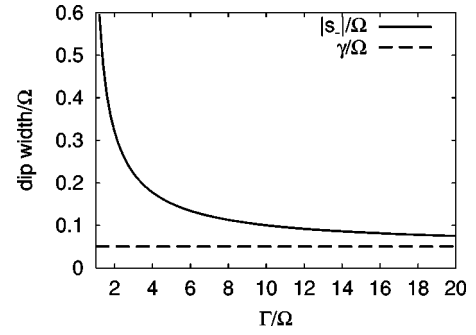


FIG. 4. The width of the dip in the case of no detuning, i.e., the value of $|s_-|$ shown for $\gamma/\Omega = 0.05$. When Γ increases, the width of the dip approaches the natural linewidth γ .

narrow dip on the side of the broadened part of the spectrum due to stochastic noise, and a narrow peak on the other side next to the dip (Fig. 6).

The analytical and numerical results are in very good agreement as one can see in the presented figures, which positively verifies correctness of our numerical simulations.

V. PHASE CORRELATION AND QUANTUM INTERFERENCE

The narrow dip in the spectrum (in the case of a high noise magnitude, resonant excitation) and the asymmetric Fano profile (in the case of large detuning and a high noise magnitude) are signatures of quantum interference in the stochastically perturbed system. The quantum interference emerges if long-time phase correlation exists between quantum states connected by different transition channels. As seen in Sec. II, the stochastic noise generates transitions between the dressed states. It would be interesting to check whether this coupling has any effect on phase correlation between them.

The phase difference $\Delta\phi$ between the dressed states is defined as follows:

$$|\Phi\rangle = a_1 e^{i\phi_1} |1\rangle + a_2 e^{i\phi_2} |2\rangle, \quad \Delta\phi = \phi_2 - \phi_1, \quad (32)$$

where $|\Phi\rangle$ is a pure state of the atom, while $|1\rangle$ and $|2\rangle$ are the dressed states defined in Eqs. (6a) and (6b). The phase

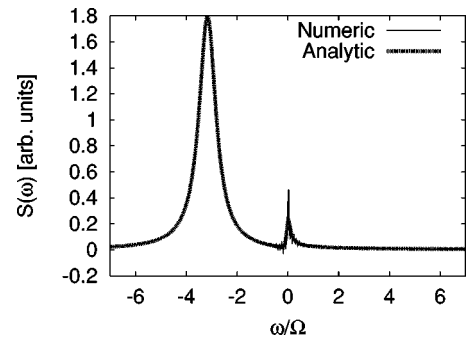


FIG. 5. The resonance fluorescence spectrum for low noise magnitude, strong laser field, and large detuning ($\Gamma/\Omega = 0.2$, $\gamma/\Omega = 0.05$, and $\Delta/\Omega = 3$), showing a Fano-like structure at the driving frequency.

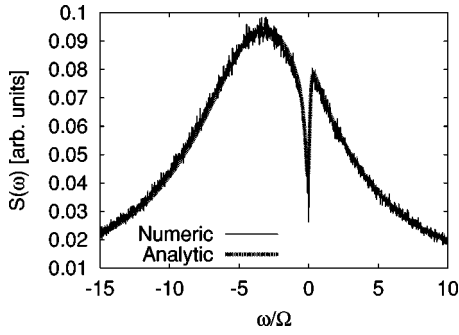


FIG. 6. The resonance fluorescence spectrum for a high noise magnitude, strong laser field, and large detuning ($\Gamma/\Omega=3$, $\gamma/\Omega=0.05$, and $\Delta/\Omega=3$), showing an asymmetric Fano profile at the driving frequency.

difference can be calculated straightforwardly from a single quantum trajectory. It is found that the phase difference behaves differently in the low and high noise magnitude regimes. In Fig. 7(a) the noise magnitude is much less than the

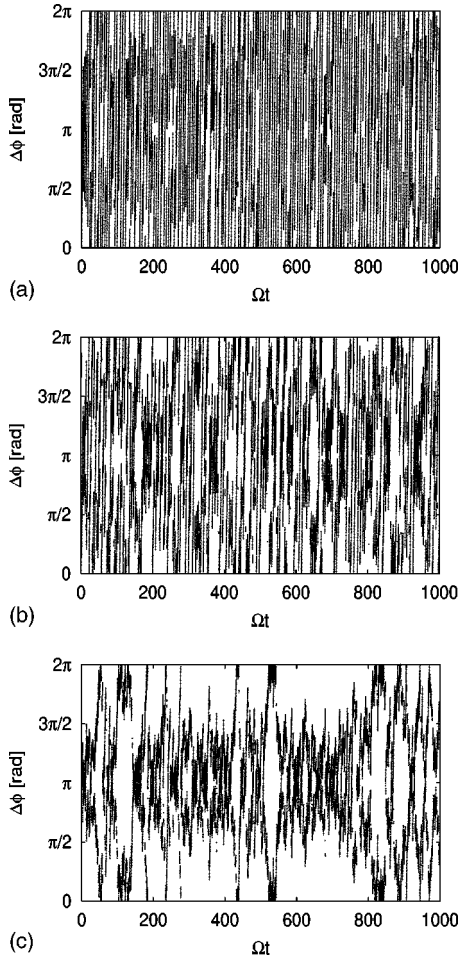


FIG. 7. The phase difference between the dressed states in the case of resonant excitation ($\Delta=0$): (a) for a low noise magnitude ($\Gamma/\Omega=0.2$ and $\gamma/\Omega=0.05$); (b) for a noise magnitude comparable to the Rabi frequency ($\Gamma/\Omega=1.1$ and $\gamma/\Omega=0.05$); and (c) for a high noise magnitude ($\Gamma/\Omega=5$ and $\gamma/\Omega=0.05$). The initial state was the excited state $|e\rangle$ in these simulations.

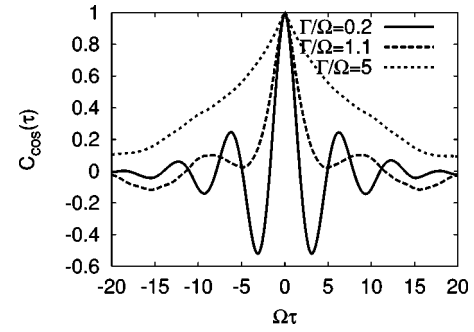


FIG. 8. Correlation function of $\cos \Delta\phi(t)$ for low, medium, and high noise magnitudes, in the case of no detuning and strong laser field ($\Gamma/\Omega \in \{0.2, 1.1, 5\}$, $\Delta=0$, and $\gamma/\Omega=0.05$).

Rabi frequency. Rabi oscillations are rarely disrupted by noise events, hence the phase difference is essentially linearly dependent on time: $\Delta\phi(t)=2\Omega t$. Consequently, the shape of the phase difference as a function of time shows no structure. When the noise magnitude increases, as depicted in Fig. 7(b), the uniform shape changes to a picture showing some structures of gaps appearing from time to time between 0 and π values of the phase difference. For a high noise magnitude [Fig. 7(c)], the phase difference tends to stabilize around values 0 and π for some time intervals.

In order to characterize the observed phenomena quantitatively, we introduce the correlation function of $\cos \Delta\phi$ by the definition

$$C_{\cos}(\tau) = c \int_{t=0}^T [\cos \Delta\phi(t+\tau) - \overline{\cos \Delta\phi}] \times [\cos \Delta\phi(t) - \overline{\cos \Delta\phi}] dt, \quad (33)$$

where $\overline{\cos \Delta\phi}$ is the mean value of the cosine of the phase difference for the simulated time interval and c is a normalization constant fixed by the condition $C_{\cos}(0)=1$. The correlation function of $\sin \Delta\phi(t)$ is defined similarly.

The $C_{\cos}(\tau)$ function is shown in Fig. 8 for the same parameter values as those used in Figs. 7(a)–7(c). The qualitative picture of emerging correlations as the noise magnitude increases is now backed up by the widening of the correlation functions. On the other hand, the correlation of the sine

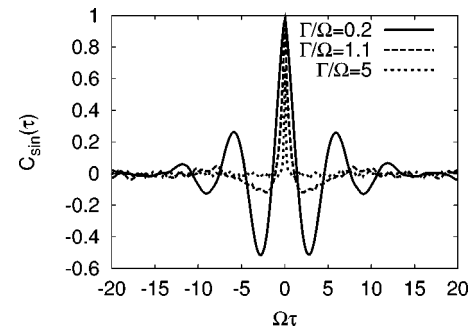


FIG. 9. Correlation function of $\sin \Delta\phi(t)$ for low, medium, and high noise magnitudes, in the case of no detuning and strong laser field ($\Gamma/\Omega \in \{0.2, 1.1, 5\}$, $\Delta=0$, and $\gamma/\Omega=0.05$). The figure shows that $\sin \Delta\phi(t)$ becomes uncorrelated for higher noise magnitude.

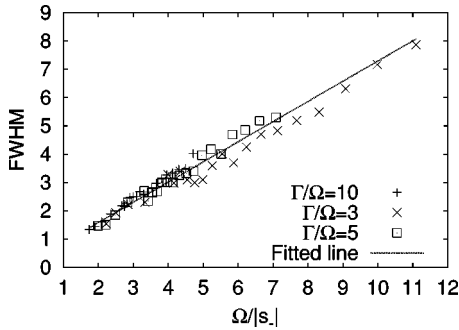


FIG. 10. The FWHM of the correlation function $C_{\cos}(\tau)$ is plotted against the reciprocal of the analytically calculated dip width $|s_-|$.

of the phase difference, $C_{\sin}(\tau)$ [defined similarly as $C_{\cos}(\tau)$] tends towards a δ -like shape when the noise increases (Fig. 9), so when Γ strongly exceeds Ω , $\sin \Delta\phi(t)$ remains uncorrelated. This means that the phase difference is locked to values 0 and π for some time intervals, though it spans a phase interval no less than $\pi/2$ around these phase values.

It is interesting that the widths of the correlation functions are related with the observed narrow dips in the spectra. The full width at half of the maximum value (FWHM) is a good measure of the widths of $C_{\cos}(\tau)$, and $|s_-|$ as defined in Eq. (29a) describes well the spectral dip width. These two quantities are presented in Fig. 10. The FWHM of $C_{\cos}(\tau)$ is roughly proportional to the reciprocal of the width of the dip in the spectrum across a wide range of parameter sets, so the observed phase correlation is indeed responsible for the narrowness of the dip in the spectrum.

The stabilization of the phase difference between the dressed states of the stochastically perturbed and coherently driven two-level atom is the underlying physical process that makes the quantum interference possible. This stabilization supports the following interpretation first suggested for collisional and phase noise-induced quantum-interference effects in resonance fluorescence spectrum in Ref. [16]. Resonance fluorescence of a strongly driven two-level atom is emitted in cascade transitions downward the ladder of the dressed-state doublets. Figure 11 shows two adjacent doublets and all possible spontaneous and noise-induced transitions between the dressed-atom states. According to Eqs. (8) and (10), noise events generate transitions between the dressed states $|1\rangle$ and $|2\rangle$ and couple them as indicated by double arrows in Fig. 11. As we have seen in the preceding section, in the noise-dominated regime, i.e., when $\Gamma > \Omega$, the phase difference between dressed doublets tends to stabilize for some time intervals due to frequent noise events. Moreover, according to Eq. (31), the resonance frequencies of all fluorescence contributions are the same in this regime. Among several possible emission channels there are two

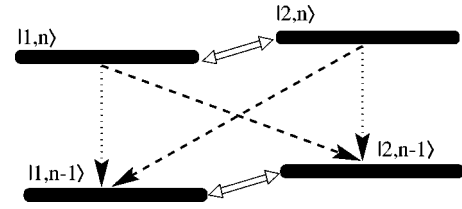


FIG. 11. Different dressed-state transition channels for a coherently driven and stochastically perturbed two-level atom.

pairs: $|1,n\rangle \rightarrow |2,n-1\rangle \rightarrow |1,n-1\rangle$ and $|1,n\rangle \rightarrow |2,n\rangle \rightarrow |1,n-1\rangle$ (or $|2,n\rangle \rightarrow |1,n\rangle \rightarrow |2,n-1\rangle$ and $|2,n\rangle \rightarrow |1,n-1\rangle \rightarrow |2,n-1\rangle$) that differ exclusively by time ordering between collisional mixing and photon emissions. Photons emitted along these channels are indistinguishable, so their interference is possible. Due to opposite signs of the relevant matrix elements this interference is destructive and creates a dip in the line center. On the other hand, other emission channels are not that equivalent, hence the corresponding photons cannot interfere and contribute to nonzero intensity at $\omega=0$. This interference is similar to that seen by Schrama *et al.* [30] in photon correlations of the well-resolved Mollow triplet components in the opposite limit when $\Gamma < \Omega$.

VI. CONCLUSION

We have applied the quantum-trajectory method to the system of two-level atoms strongly driven by a coherent light field and perturbed by stochastic noise. We have developed a method for obtaining the resonance fluorescence spectra from numerical simulations. This method is especially advantageous for physical systems, where the noise dominates the dynamics, and one needs to simulate many quantum trajectories using small time steps. The simulation of a single quantum trajectory revealed that for a high noise magnitude the phase difference between the dressed states tends to stabilize around fixed values. When calculating the resonance fluorescence spectra, narrow resonances as central dip and dispersive Fano-like profile occurred in the regime where the noise dominated the Rabi oscillations. These modifications of the resonance fluorescence spectra are associated with the stabilization of the dressed-state phases and stochastically induced quantum interference between various emission channels.

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