Boson-induced *s*-wave pairing in dilute boson-fermion mixtures

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We show that in dilute boson-fermion mixtures with fermions in two internal states, even when the bare fermion-fermion interaction is repulsive, the exchange of density fluctuations of the Bose condensate may lead to an effective fermion-fermion attraction, and thus to a Cooper instability in the *s*-wave channel. We give a simple condition to know when this is going to happen, and an analytic expression for the associated T_c in the most important limit where the phonon branch of the Bogoliubov excitation spectrum of the bosons is important. We find a T_c of the same order as for a pure Fermi gas with bare attraction.

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I. INTRODUCTION

The interest in the effective interaction between fermions in boson-fermion mixtures is not new. Already in the 1960s Pines [1] suggested that the effective interaction between ^{3}He atoms in solution in superfluid ^{4}He could be attractive due to the exchange of density fluctuations of the bosonic background. This attraction was then observed experimentally by Edwards *et al.* [2] and Anderson *et al.* [3]. From the experimental data Bardeen, Baym, and Pines [4] estimated the expected critical temperature for Cooper pairing in both the *s*-wave and *p*-wave channels. The situation is similar to that in ordinary superconductors where the effective attraction between electrons is well established to be caused by the exchange of lattice phonons [5].

The renewed interest in the issue stems from the recent availability of trapped atomic gases at the ultralow temperatures required for quantum phenomena to be relevant. After Bose-Einstein condensation (BEC) was observed, huge steps were also made in the cooling of fermions [6–10]. One way to obtain the cooling [10] has been to mix ^{6}Li atoms (fermions) with ^{7}Li atoms (bosons), and to proceed with the standard technique of evaporative cooling on the latter so as to cool the former "sympathetically," i.e., indirectly, by simple thermal contact. At the end of the process a stable nearly pure BEC of ^{7}Li on top of a cloud of degenerate ^{6}Li was observed. It is therefore timely to calculate the effective interaction between fermions in a dilute boson-fermion mixture when fluctuations of the BEC are taken into account, and to study the consequences on Cooper pairing.

In two recent works Heiselberg *et al.* [11] and Bijlsma *et al.* [12] found the same expression for the boson-induced contribution to the interaction, but proceeded in different ways to analyze the effects on *s*-wave pairing. The equivalence of the two approaches in dealing with the latter problem is not obvious, and the results were given in very different forms, one analytical [11] and the other numerical [12]. Moreover, the calculation in Ref. [11] is restricted to the case of an already attractive bare fermion-fermion interaction, and that in Ref. [12] is limited by the lack of transparency and physical understanding of numerical calculations. In this paper a third way to treat the problem is developed. The idea is to give an approximate, but under usual circumstances accurate, analytical solution to the numerical approach of Ref. [12]. This third method has several advantages. First of all it applies to the case of bare fermion-fermion repulsion, and confirms that, as conjectured in [12], even if the bare fermion-fermion interaction is repulsive, the effective one may be attractive when the boson-induced contribution is taken into account. But much more than that, one can derive a precise and simple criterion to know when this is going to happen, and an elegant expression for the associated BCS critical temperature. Since the method is valid for bare fermion-fermion attraction as well, we show that it gives, within its range of applicability, the same results as were found in Ref. [11], and thus prove the consistency of the three methods. Finally, the result in Ref. [11] was derived under the assumption that the induced interaction is much smaller than the bare one, but we shall prove that, in the appropriate limit in which their result coincides with ours, this assumption is not necessary.

II. EFFECTIVE INTERACTION

When the gases of the mixture are dilute all interactions can be described by one parameter each: the appropriate scattering length [13,14]. In a mixture of fermions in two different internal states 1 and 2 (with the same mass), and bosons in one internal state only, the scattering lengths a_{12}, a_{BB}, a_{B1}, and a_{B2} characterize all the relevant interactions, since the Pauli principle allows us to ignore interactions between identical fermions at the temperatures and densities of interest here. In the following we shall suppose, without loss of generality, that $a_{B1}=a_{B2}=a_{BF}$, and rename $a_{12}=a_{FF}$. Often in the text pseudopotentials will be used instead of scattering lengths. They are defined as follows: $U_{FF}=4\pi\hbar^2 a_{FF}/m_F$, $U_{BB}=4\pi\hbar^2 a_{BB}/m_B$, and $U_{BF}=4\pi\hbar^2 a_{BF}/m_{BF}$, where $m_{BF}=2m_Bm_F/(m_B+m_F)$ is twice the reduced mass for a boson with mass m_B and a fermion with mass m_F. Since we are interested in *s*-wave pairing we shall suppose that the densities of 1 and 2 fermions are the same, $n_1=n_2=n_F$, and therefore also their Fermi energies and momenta, ϵ_F and $\hbar k_F$. Moreover, we shall introduce in the theory small parameters of the type k_Fa and $n_B^{1/3}a$, where by a we indicated a generic length of the order of a scattering length and by n_B the boson density.

In a mixture the effective interaction between a type 1 and a type 2 fermion is the sum of the direct one U_{FF}, the one arising from polarization of the bosonic medium U_{FBF}, and that due to polarization of the fermionic medium itself U_{FFF}. We shall show that under suitable conditions the first two interactions can be of the same order, and it is therefore essential to consider both of them to predict whether the system undergoes Cooper pairing or not, and if so what is the critical temperature T_c. The third one is instead of order $a(k_F a)$ and is important for the renormalization of the prefactor in the expression for the BCS critical temperature. The way to deal with U_{FFF} was shown in Ref. [15] (see also [11]). There it was found that it causes a decrease in the prefactor of T_c by a factor $(4e)^{1/3}$, and we take this for granted here.

The boson-induced interaction between fermions introduced in Refs. [11,12] is

$$U_{FBF}(\mathbf{q},\omega)=U_{BF}^2\chi(\mathbf{q},\omega), \tag{1}$$

where the boson density-density response function in the Bogoliubov approximation is given by

$$\chi(\mathbf{q},\omega)=\frac{n_B\hbar^2q^2/m_B}{(\hbar\omega)^2-\epsilon_q^0(\epsilon_q^0+2n_BU_{BB})}, \tag{2}$$

$\epsilon_q^0=\hbar^2q^2/2m_B$ and $q=|\mathbf{q}|$. Since we are considering dilute Bose and Fermi gases, we have neglected the renormalization of $\chi(q,\omega)$ due to the presence of the fermions. This is correct to lowest order in the gas parameter. We used the zero temperature response function because the BCS critical temperature is much smaller than the boson condensation one if $n_B \gtrsim n_F$.

The interaction in Eq. (1) provides an attraction between two particles at the Fermi level, since in that case $\omega=0$ and $U_{FBF}(q,0)<0$. We remind the reader that in uniform systems $U_{BB}>0$ is required for stability of the mixture [16].

At this point one has to analyze two possibilities. The bare fermion-fermion interaction can either be attractive $U_{FF}<0$ or repulsive $U_{FF}>0$. In the former case the gas would undergo pairing even in the absence of bosons at the critical temperature $T_{c,0}$. In the presence of bosons the direct and induced contributions add up constructively to a stronger attractive interaction, and the BCS critical temperature rises. This possibility was studied in detail in Ref. [11]. If $U_{FF}>0$ instead, the Fermi gas is not unstable to pairing without the bosons. When the bosons are added, however, if the attractive boson-induced interaction at the Fermi surface is stronger than the bare repulsion, the total *effective* interaction $U_{FF}+U_{FBF}(q,0)$ is attractive, and the gas becomes unstable to pairing. One can immediately recognize a mechanism at work completely analogous to that of electrons in superconductors. Just as in superconductors the induced interaction depends on the energy exchanged, and is attractive only in a band centered about the Fermi surface [5]. Obtaining a solution for T_c for arbitrary densities is complicated. As is well known, however [4,12], if $v_F \ll s$, where $s=(n_BU_{BB}/m_B)^{1/2}$ is the sound velocity in the Bose gas, retardation effects can be neglected and ω can be set to zero always. The induced interaction is thus attractive in the whole Fermi sphere, and we seek a solution to the problem under this assumption.

The boson-induced interaction when $v_F \ll s$ is then

$$U_{FBF}(q)=-\frac{U_{BF}^2}{U_{BB}}\frac{1}{1+(\hbar q/2m_Bs)^2}. \tag{3}$$

Notice that if $m_B \gtrsim m_F$, since the typical momentum exchanged in an interaction is $\hbar q \sim m_F v_F \ll 2m_B s$, we expect $U_{FBF}(q)\simeq -U_{BF}^2/U_{BB}$, i.e., a constant independent of q [cf. Eq. (12) below].

III. COOPER PAIRING

We now want to properly take into account both the bare and the induced interactions. According to Emery [17] if the fermions with opposite spins interact via a potential $U(r)$ and if $\tan\delta_0(k_F)$ is greater than zero, where $\delta_0(k_F)$ is the $l=0$ phase shift associated with the potential $U(r)$ and evaluated at the Fermi wave number, the system undergoes s-wave pairing at the critical temperature,

$$k_BT_c=\frac{\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\epsilon_F\, e^{-\pi/2\tan\delta_0(k_F)}. \tag{4}$$

In writing Eq. (4) we have already included the correction to the prefactor due to the polarization of the fermions.

In the case of a pure two-species Fermi gas with bare attraction $U(r)$ is the bare potential. The associated scattering length and pseudopotential are a_{FF} and U_{FF} (both negative), respectively. Since by assumption $k_F|a_{FF}|\ll 1$, then $\tan\delta_0(k_F)\simeq -k_Fa_{FF}$, and Eq. (4) reduces to the well known formula [15]

$$k_BT_c=\frac{\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\epsilon_F\, e^{\pi/2k_Fa_{FF}}. \tag{5}$$

In a mixture $U(r)$ is more complicated but the same principle applies. This approach for finding the effects of the boson-induced interaction has been used by Stoof and coworkers [12]. To use Eq. (4) in our case one has to take the Fourier transform into real space coordinates of Eq. (3):

$$U_{FBF}(r)=-\frac{U_{BF}^2}{4\pi U_{BB}\xi_B^2}\frac{1}{r}e^{-r/\xi_B}, \tag{6}$$

where $\xi_B=\hbar/2m_Bs$ is the boson coherence length. The $1/r$ divergence at $r=0$ is artificial, since the potential must in any case be cut off at a distance r_0 of the order of a scattering length. $U_{FBF}(r)$ is a Yukawa potential with range $\xi_B=a_{BB}(16\pi n_Ba_{BB}^3)^{-1/2}$, which is much greater than a_{BB} if the gas is dilute.

The fermions interact via both $U_{\rm bare}(r)$ and $U_{FBF}(r)$ so that the total interaction potential is given by $U_{\rm tot}(r)=U_{\rm bare}(r)+U_{FBF}(r)$. The aim is then to calculate the s-wave phase shift due to the total potential $U_{\rm tot}(r)$. In principle this is a difficult problem, since one should solve the radial Schrödinger equation

$$\left(-\frac{d^2}{dr^2}+\frac{m_F}{\hbar^2}U_{\rm tot}(r)-k^2\right)u_0^{\rm tot}(r;k)=0, \tag{7}$$

where, however, the bare potential is not known, as only the scattering length a_{FF} is a measured quantity. Fortunately, it is not necessary to know also the detailed variation with r of $U_{\rm bare}$. Equation (7) can be greatly simplified by noticing that $U_{\rm bare}$ and U_{FBF} act on two different length scales, the first one from $r=0$ to $r\sim r_0$ and the second one from $r\sim r_0$ to $r\sim\xi_B\gg r_0$. One can then just solve Eq. (7) for $r>r_0$, with $U_{\rm tot}(r)=U_{FBF}(r)$, and introduce a boundary condition on $u_0^{\rm tot}(r;k)$ at $r=r_0$, which accounts for the phase shift due to the bare potential. This is due to the fact that, by the time the wave function reaches the region of distances where $U_{FBF}(r)$ is relevant, the bare potential has stopped acting and the wave function has recovered its sinusoidal form with phase shift $\delta_0^{\rm bare}(k)$.

Using these replacements Eq. (7) has been solved numerically by Bijlsma *et al.* [12] for various sample elements and densities, and the results for $\delta_0^{\rm tot}(k)$ are in their publication. Whenever $\tan\delta_0^{\rm tot}(k_F)>0$ the critical temperature is found by replacing the value obtained into Eq. (4). Analyzing their numerical results, the authors of Ref. [12] noticed that, for a ^{87}Rb-^{40}K mixture with the values of the scattering lengths they use, the bare repulsion is overcome by the boson-induced attraction and the system becomes unstable to pairing [18]. However, by their approach one cannot tell when this is going to happen. Moreover, they do not give numerical results for the critical temperature but only for the phase shifts, so that the temperature scales involved in the effect are not predicted.

We now want to show that if $k_F\xi_B\ll 1$ (a condition automatically satisfied if $m_B\gtrsim m_F$, since by assumption $v_F\ll s$) then $\tan\delta_0^{\rm tot}(k_F)$ can in fact be found analytically.

It is well known (see, for instance, [19] that, given any two potentials $U^{(1)}(r)$ and $U^{(2)}(r)$ for the interaction of two particles with reduced mass $m_{\rm red}$, and given the solutions with wave number k to the corresponding radial Schrödinger equations, $u_l^{(1)}(r;k)$ and $u_l^{(2)}(r;k)$, the associated phase shifts $\delta_l^{(1)}(k)$ and $\delta_l^{(2)}(k)$ are related by

$$\tan\delta_l^{(1)}(k)-\tan\delta_l^{(2)}(k)=-k\frac{2m_{\rm red}}{\hbar^2}\int_0^\infty u_l^{(2)}(r;k)\times[U^{(1)}(r)-U^{(2)}(r)]u_l^{(1)}(r;k)\,dr. \tag{8}$$

This formula can be applied to our case by letting $U^{(1)}(r)=U_{\rm tot}(r)$ and $U^{(2)}(r)=U_{\rm bare}(r)$, $m_{\rm red}=m_F/2$, and $k=k_F$. For the $l=0$ channel then

$$\tan\delta_0^{\rm tot}(k_F)=\tan\delta_0^{\rm bare}(k_F)-k_F\frac{m_F}{\hbar^2}\times\int_0^\infty u_0^{\rm bare}(r;k_F)U_{FBF}(r)u_0^{\rm tot}(r;k_F). \tag{9}$$

Again the bare phase shift is

$$\tan\delta_0^{\rm bare}(k_F)\simeq -k_Fa_{FF}.$$

We can now use the special form of our potentials. Because $U_{FBF}(r)$ is zero for $r\lesssim r_0$, the relevant lower limit of the integral in Eq. (9) is really r_0. But for $r>r_0$ we can replace $u_0^{\rm bare}$ by its asymptotic value for $r\to\infty$ since the bare potential has decayed by then. Thus $u_0^{\rm bare}(r;k_F)\simeq k_F^{-1}[\sin(k_Fr)-k_Fa_{FF}\cos(k_Fr)]$. If in addition $k_F\xi_B\ll 1$, we can approximate $u_0^{\rm bare}(r;k_F)\simeq r-a_{FF}$.

Later we shall prove that if the gases are dilute and for typical values of the parameters the potential $U_{FBF}(r)$ is shallow, i.e., not strong enough to form "bound states." Thus we can to a first approximation (Born) also let $u_0^{\rm tot}(r;k)=u_0^{\rm bare}(r;k)$. This yields

$$\tan\delta_0^{\rm tot}(k_F)\simeq -k_Fa_{FF}-A, \tag{10}$$

with

$$A=-\frac{m_Fk_F}{4\pi\hbar^2}\frac{U_{BF}^2}{U_{BB}\xi_B^2}\int_{r_0}^\infty dr(r-a_{FF})^2\frac{1}{r}e^{-r/\xi_B} \tag{11}$$

$$=-\frac{k_Fa_{FF}U_{BF}^2}{U_{BB}U_{FF}}\left\{e^{-\tilde r_0}(1+\tilde r_0)-2\tilde a_{FF}e^{-\tilde r_0}+\tilde a_{FF}^2\int_{\tilde r_0}^\infty dx\frac{1}{x}e^{-x}\right\},$$

$\tilde r_0=r_0/\xi_B$, and $\tilde a_{FF}=a_{FF}/\xi_B$.

The boson coherence length is in general much larger than the cutoff r_0 which in turn is of order $|a_{FF}|$. Therefore $\tilde r_0$ and $\tilde a_{FF}$ can be set equal to zero in the first two terms in the curly brackets. The last integral is dominated by the logarithmic divergence and it varies as $\sim\tilde a_{FF}^2\ln\tilde r_0$, but since $\tilde a_{FF}\sim\tilde r_0\ll 1$ also the last term can be set to zero. We finally obtain

$$\tan\delta_0^{\rm tot}(k_F)\simeq -k_Fa_{FF}\left(1-\frac{U_{BF}^2}{U_{BB}U_{FF}}\right), \tag{12}$$

which is the main result of the present work. In order for the system to condense we need to have $\tan\delta_0^{\rm tot}(k_F)>0$. This is always the case if $a_{FF}<0$, and we recover the result of Ref. [11], in the limit $k_F\xi_B\ll 1$. Notice that when $k_F\xi_B\sim 1$ we can no longer expand $u_0^{\rm bare}$ as we did, and we expect corrections to our result. However, the boson-induced attraction is maximized in the limit $k_F\xi_B\ll 1$ since it is the phonon branch of the Bogoliubov spectrum that provides most of the attraction, and that is why it is satisfactory for the time being to consider only this limit. It is important to underline that the assumption $U_{BF}^2/U_{BB}\ll U_{FF}$ did not enter the present derivation anywhere, while it did in Ref. [11]. We have thus proved that, as long as $k_F\xi_B \ll 1$, the assumption is in fact unnecessary and the result is valid for arbitrary interactions.

On the other hand, the use of the Born approximation on the approach of Ref. [12] to obtain analytical values of the phase shifts allows a deeper physical understanding and shows a simplicity which was hidden by the numerics. As we anticipated, we now have a precise condition to determine when a bare repulsion is overcome by the boson-induced interaction, making the system unstable. This happens when $U_{BF}^2/U_{BB}U_{FF}>1$. Recall that the condition was obtained assuming $k_F\xi_B \ll 1$, but again that is when the attraction is expected to be strongest.

A comment is in order at this point. With the values of the scattering lengths used in their ^{87}Rb-^{40}K calculation ($a_{BB}=109a_0$, $a_{FF}=160a_0$, and $a_{BF}=100a_0$), the authors of Ref. [12] find that there is an inversion in the sign of the effective interaction due to the bosons. However, using these values we have $U_{BF}^2/U_{BB}U_{FF}=0.66<1$. Therefore we do not expect the sign inversion to take place. Explicit numerical calculations reveal that indeed it does not occur. We believe that the authors must have used a Bose-Fermi interaction a factor of $\sqrt{2}$ larger, or inserted by mistake a factor of 2 in the prefactor of $U_{FBF}(r)$. With this replacement $U_{BF}^2/U_{BB}U_{FF}=1.33$, we then expect the inversion, and our numerical calculations appear to give identical results to the ones published by them. Moreover, in that case one numerically finds that the ratio $\tan\delta_0(k_F)/k_F$ tends to $\approx -50a_0$ for $k_F \ll \xi_B^{-1}$, as we predict analytically.

In all cases in which λ is negative we have, according to Eq. (4), that the critical temperature is given by

$$k_B T_c = \frac{\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\epsilon_F\, e^{1/\lambda}, \tag{13}$$

where

$$\lambda = N(0)U_{FF}\left\{1-\frac{U_{BF}^2}{U_{BB}U_{FF}}\right\}, \tag{14}$$

and $N(0)=m_F k_F/(2\pi^2\hbar^2)$. This shows, as we already anticipated, that the critical temperature in the case of boson-induced pairing is of the same order as that of pairing with a bare fermion-fermion attraction, and can be very large if $U_{BF}^2/U_{BB}U_{FF} \gg 1$. The condition can be achieved by a suitable choice of elements. At this time it is difficult to suggest an appropriate choice since the scattering lengths for collisions between bosonic and fermionic atoms are mostly being studied at the time of writing, and precisely in view of the present developments to which this work is a contribution. We also point out that an interesting consequence of Eqs. (13) and (14) is that, so long as $k_F\xi_B \ll 1$, the new critical temperature is independent of the boson density. The strong density dependence in the plots of Ref. [12] is explained by the fact that the regime $k_F\xi_B \gtrsim 1$ is also probed. We remark that from our conclusion it follows that, when the Born approximation is valid, as for all cases in Figs. 1 and 2 in Ref. [12], the curves for the phase shifts tend to a common tangent not only for large k_F's, as pointed out by the authors, but also for small k_F's. This is because, as we have proved, the phase shifts are independent of n_B when $k_F \ll \xi_B^{-1}$. This important aspect was not clear from the plots, since the scale chosen did not reveal it. The anomaly of curve 3 in Fig. 2 of Ref. [12] cannot be explained and we believe something must have gone wrong. We again carried out numerical calculations and confirmed this hypothesis.

IV. NOTE ON VALIDITY AND CONCLUSIONS

In the derivation above we have used the Born approximation. This is valid only if the potential is sufficiently shallow. But, as we see from Eq. (6), the depth of the induced potential depends linearly on n_B through the square of the coherence length in the denominator. At low boson densities the potential is very weak and all the considerations above are certainly valid. But when n_B is large enough for the induced potential to be able to host a "bound state" they fail, and the new scattering length and critical temperature may depend dramatically on n_B as the calculations in Ref. [12] indicate (see their Figs. 3 and 4). This regime is interesting and worth studying in greater detail, but is beyond the scope of this work.

By a simple argument we may estimate the highest boson density allowed for our model to apply. The typical potential energy of a particle confined in the potential (6) is, apart from the sign, $E_p \sim U_{BF}^2/4\pi U_{BB}\xi_B^3$, and the kinetic one $E_k \sim \hbar^2/m_F\xi_B^2$. To be safe then we need to require $E_p/E_k \ll 1$, which implies

$$n_B^{1/3}a_{BB} \ll \frac{1}{(16\,\pi)^{1/3}}\left(\frac{U_{BB}}{U_{BF}}\right)^{4/3}\left(\frac{m_B}{m_F}\right)^{2/3}. \tag{15}$$

Thus if the scattering lengths and the masses are approximately the same, as in typical conditions, it is enough to require that the boson gas is dilute for the Born approximation to apply, but specific checks may be necessary, especially for large U_{BF}. Recall that the boson density cannot be too low though if the condition $k_F\xi_B \ll 1$ also has to be fulfilled.

In conclusion we have shown that the boson-induced interaction in a boson-fermion mixture can cause *s*-wave Cooper pairing in a Fermi gas with bare repulsion, and we have given a simple criterion to predict when this is going to happen. We have calculated the associated critical temperature in the limit $k_F\xi_B \ll 1$ where the highly efficient attraction due to the phonon branch of the Bogoliubov spectrum is important, and found that T_c is of the same order as for *s*-wave pairing in a gas with bare attraction [see Eq. (13)].

Note added in proof: Recently Roati *et al.* [20] have experimentally realized a degenerate Fermi gas of ^{40}K in the hyperfine state $|F=9/2,\ M_F=9/2\rangle$, on top of a Bose-Einstein condensate of ^{89}Rb in the $|F=2,\ M_F=2\rangle$ state. They predict a $a_{BF}=-300(100)a_0$. Such a large value of a_{BF} opens very interesting possibilities for the experimental study of the phenomenon presented here.

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