Scattering of magnetized electrons by ions

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Binary collisions between ions and electrons in an external magnetic field are treated in second-order perturbation theory, starting from the unperturbed helical motion of the electrons. For the transfer of relative velocity in a collision, three kinematical regimes are identified, depending on the relative size of the cyclotron radius, the pitch of the helices, and the distance of the closest approach. The magnetic field suppresses the velocity transfer in the transverse direction, but it enhances the longitudinal velocity transfer, provided that the ion velocity itself has a transverse component. In order to relate the velocity transfer to the energy loss of the ions, particular attention must be paid to the nonconservation of the center-of-mass motion in a magnetic field. Hard collisions are accounted for by regularizing the energy transfer at small distances. For ions interacting with monochromatic beams closed expressions for the energy loss can be derived, which are averaged with respect to the velocity distribution of the electrons. The magnetic field reduces the energy loss for ion motion parallel to the magnetic field while it enhances the energy loss for transverse ion motion.

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I. INTRODUCTION

In the presence of an external magnetic field B the problem of two charged particles cannot be solved in a closed form as the relative motion and the motion of the center of mass are coupled to each other. Therefore no theory exists for a solution of this problem that is uniformly valid for any strength of the magnetic field and the Coulomb force between the particles. Numerical calculations have been performed for binary collisions between magnetized electrons [1,2] and for collisions between magnetized electrons and ions [3–5]. As an ion is much heavier than an electron, its uniform motion is only weakly perturbed by collisions with the electrons and the magnetic field. There exists a conserved energy K involving the energy of relative motion and a magnetic term. In this paper we treat the Coulomb interaction with the ion as a perturbation to the helical motion of the magnetized electrons. This has been done previously up to first order in the ion charge Z [6], it is shown here that a second-order treatment is both necessary and sufficient for the conservation of the generalized energy K. Three regimes are identified, depending on the relative size of the parameters R (the cyclotron radius), r_0 (the distance of the closest approach), and δ (the pitch of the helix).

In earlier kinetic approaches [7–11] only two regimes have been distinguished: Fast collisions for $r_0<R$, where the Coulomb interaction is dominant and adiabatic collisions for $r_0>R$, where the magnetic field is important, as the electron performs many gyrations during the collision with the ion. The change ΔE_i of the energy of the ion has been related to the square of the momentum transfer Δp, which has been calculated up to $O(Z)$. This is somewhat unsatisfactory, as the first-order treatment violates energy conservation and there is another $O(Z^2)$ contribution to ΔE_i, in which the second-order momentum transfer enters linearly. Moreover, the friction force on the ion has been written in analogy to electrostatics as a gradient of a pseudopotential in velocity space. This requires that the spatial integration with respect to the impact parameters be performed after averaging with respect to the velocity distribution of the electrons. At low relative velocities this is doubtful. In the present paper we work out the binary collision model up to $O(Z^2)$ and regularize the spatial integration in a manner that leads to the exact result for Rutherford scattering and keeps the resulting modified Coulomb logarithm within the velocity integral.

In applications of the binary collision model to plasma physics, e.g., the stopping of ions by electrons, the polarization is only accounted for by shielding the Coulomb interaction at large distances. In a complementary picture one calculates the energy loss of the ion through its interaction with the polarization cloud it has created in its wake. This dielectric theory of collective excitations requires a cutoff at small distances, where hard collisions cannot be treated any more in linear response. In the absence of a magnetic field both approaches give the same results, if physically reasonable cutoffs are used in the Coulomb logarithms [9,10].

However, the presence of a magnetic field introduces complications, in that case the dielectric theory of the energy loss has, to our best knowledge, not yet been worked out completely. This is desirable, as the effects of collective excitations interfere strongly with the influence of the magnetic field for low ion velocities v_i. Already the underlying expression for the dielectric function ε is quite involved, see, for example, Ref. [12], and for the friction force one has to integrate $\mathrm{Im}(\varepsilon^{-1})$ with respect to the wave numbers. These integrations are facilitated, if one assumes a velocity distribution of the electrons which is completely flat in the direction of the magnetic field (i.e., the temperature parallel to the magnetic field is zero). Then $\mathrm{Im}(\varepsilon^{-1})$ can be approximated by a sum of δ-functions at the plasma frequency and the cyclotron frequency. This facilitates not only the integrations, but also allows a separation into contributions from the plasma mode and from binary collisions [9,10]. However, in

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the electron cooling of ion beams the velocity distribution of the electrons has a small, but finite temperature parallel to the beam. The cooling is most effective for ions with a velocity that corresponds to that temperature, and results for the completely flattened distribution cannot be readily applied in this situation. In fact, it has been argued earlier that plasma-wave excitations are suppressed, if the cyclotron frequency is larger than the plasma frequency [13,14].

In the present paper we attempt to optimize the binary collision model. Keeping the role of collective excitations in mind, we also compare our results with previous numerical solutions of the Vlasov-Poisson equation, in which the electrons are represented by test particles [15]. Such a treatment accounts for the nonlinear response of the electrons to the perturbing ion. The numerical simulations show statistical fluctuations, which tend to obscure the asymptotic behavior at small ion velocities. A linear-response treatment of the energy loss of ions in a magnetized plasma with a finite temperature anisotropy remains, therefore, highly desirable.

The paper is organized as follows: For pedagogical purposes we treat in Sec. II the trivial case where the distance of the closest approach r_0 is smaller than the cyclotron radius R, so that the magnetic field can be neglected. It is shown that the velocity transfer must be calculated to second order $O(Z^2)$ in order to fulfill energy conservation on this level. In Sec. III we consider the scattering of magnetized particles in the framework of the Lagrangian formalism. As the ion mass M is much larger than the electron mass m there exists a conserved generalized energy K, which is the sum of the energy of relative motion and a magnetic term. The equations of motion are solved in an iterative manner up to $O(Z^2)$ starting from the unperturbed helical motion of the electrons in the magnetic field. For strong magnetic fields $R<r_0$ two subregimes can be identified. For stretched helices with a pitch $\delta>r_0$ the guiding center approximation applies. For tight helices with $\delta<r_0$ there is a velocity transfer parallel to the magnetic field; in fact, this contribution is dominant for small relative velocities. The second-order treatment fulfills the generalized conservation law for K. Hard collisions are taken into account by regularizing the integrals leading to Coulomb logarithms at the lower boundary in a manner that leads to the exact result for Rutherford scattering. In Sec. IV the theory is applied to the energy loss of ions in a magnetized electron plasma. This is a theme of large current interest in connection with the electron cooling of heavy-ion beams in storage rings as well as the cooling of (anti-)particles in traps. The influence of the magnetic field on the energy loss is ambiguous: With increasing magnetic field one expects a longitudinal motion of the electrons along the field lines like beads on a wire, with no energy loss at all for $\vec{v}_i \| \vec{B}$ in the limit $B \rightarrow \infty$, except for possible collective effects. On the other hand, as the magnetic field quenches the transversal motion of the electrons, the inverse square law for the dependence of the energy loss on the ion velocity persists down to the longitudinal thermal velocity of the electrons. As this is small in the electron coolers of storage rings the energy loss should be enhanced. We show that, as far as binary collisions are involved, the first effect (reduction of the energy loss) is dominant when the ion moves parallel to the magnetic field, while the enhancement becomes dominant with a transverse motion of the ions. The results of the binary collision model are compared with simulations, in which ensembles of magnetized electrons are scattered from ions [classical trajectory Monte Carlo (CTMC)], and with the dielectric theory both in linear response and by a numerical solution [particle in cell (PIC)] of the Vlasov-Poisson equation. In Sec. V the results are summed up; some formulas and techniques for the second-order treatment are presented in the Appendix.

II. PERTURBATIVE APPROACH TO RUTHERFORD SCATTERING

A. First-order velocity transfer

For pedagogical reasons we consider first the scattering of unmagnetized electrons by a fixed point charge Ze, e.g., an ion, which rests at the origin. The electrons move and in the electric field due to the ion

$$\vec{E}(\vec{r}(t)) = \frac{Ze}{4\pi\varepsilon_0} \frac{\vec{r}(t)}{r^3(t)}, \tag{2.1}$$

where ε_0 is the permittivity of the vacuum. In view of the later inclusion of the magnetic field we do not integrate the equations of motion

$$\frac{d\vec{r}}{dt} = \vec{v}, \tag{2.2}$$

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}\vec{E}(\vec{r}(t)) \tag{2.3}$$

exactly, but seek an approximate solution, in which the Coulomb field (2.1) is treated in a perturbative manner. The first-order velocity transfer is obtained by integrating Eq. (2.3) using the unperturbed electron trajectory for rectilinear motion along the z axis,

$$\vec{r}(t) = \begin{pmatrix} r_0 \sin\theta \\ -r_0 \cos\theta \\ vt \end{pmatrix}. \tag{2.4}$$

Here

$$\vec{r}_0 = r_0 \begin{pmatrix} \sin\theta \\ -\cos\theta \\ 0 \end{pmatrix} \tag{2.5}$$

is the vector of the closest approach, which takes place at $t = 0$. Insertion of Eqs. (2.4) and (2.5) into the equation of motion (2.3) yields the first-order velocity transfer

$$\delta\vec{v}_C(t) = -\frac{Z\not{e}^2}{m}\int_{-\infty}^{t} \frac{dt'}{(r_0^2 + v^2 t'^2)^{3/2}} \begin{pmatrix} r_0 \sin\theta \\ r_0 \cos\theta \\ v_\| t' \end{pmatrix} \tag{2.6}$$

$$= -\frac{2Z\not{e}^2}{mv}\frac{\vec{r}_0}{r_0^2}, \quad t \rightarrow +\infty, \tag{2.7}$$

where $\not{e}^2 = e^2/4\pi\varepsilon_0$.

At this stage it should be noted that the first-order treatment is not yet physically meaningful.

(i) The energy

$$E=\tfrac{1}{2}m\vec{v}^2-\frac{Z\mathbf{e}^2}{r} \tag{2.8}$$

is not conserved, as the velocity transfer (2.7) is parallel to the distance of the closest approach $\vec{r}_0$. Thus

$$\Delta E=m(\vec{v}\cdot\delta\vec{v}_C+\tfrac{1}{2}(\delta\vec{v}_C)^2)=\tfrac{1}{2}m(\delta\vec{v}_C)^2\neq 0. \tag{2.9}$$

(ii) In many applications one has to deal with beams of electrons, which can be considered as homogeneous on the scale set by r_0. Then the first-order velocity transfer vanishes due to symmetry reasons when integrating with respect to $\vec{r}_0$. Indeed, the transport phenomena, etc., are of order $O(Z^2)$ in the ion charge.

It is therefore necessary to calculate the velocity transfer to second order.

B. Second-order velocity transfer for Rutherford scattering

With the dimensionless time variable

$$\tau=|v|t/r_0, \tag{2.10}$$

one obtains after another time integration the first-order correction to the trajectory,

$$\delta\vec{r}(t)=-\frac{Z\mathbf{e}^2}{m}\frac{1}{v^2}\int_{-\infty}^{\tau}d\tau'\left[\left(\frac{\tau'}{(1+\tau'^2)^{1/2}}+1\right)\begin{pmatrix}\sin\theta\\-\cos\theta\\0\end{pmatrix}-\frac{\mathrm{sgn}(v)}{(1+\tau'^2)^{1/2}}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]. \tag{2.11}$$

This is a small correction if

$$r_{\min}:=\frac{Z\mathbf{e}^2}{mv^2}<r_0. \tag{2.12}$$

Here and in the following we will not worry about apparent divergences as it will turn out that these cancel exactly when physically observable quantities such as the velocity transfer are calculated.

The correction to the electric field is calculated with the help of the expansion

$$\begin{aligned}\delta E_k&=E_k(\vec{r}+\delta\vec{r})-E_k(\vec{r})\\&=\frac{Ze}{4\pi\varepsilon_0}\delta r_i\frac{\partial}{\partial r_i}\frac{r_k}{(r_jr_j)^{3/2}}\\&=\frac{Ze}{4\pi\varepsilon_0(r_jr_j)^{3/2}}\left(\delta r_k-3r_k\frac{r_i\delta r_i}{r_jr_j}\right),\end{aligned} \tag{2.13}$$

where the summation convention has been used. The second-order correction to the velocity is then

$$\begin{aligned}\delta^{(2)}\vec{v}_C&=-\frac{e}{m}\int_{-\infty}^{\infty}dt\,\delta\vec{E}(t)\\&=-\left(\frac{Z\mathbf{e}^2}{m}\right)^2\frac{2\,\mathrm{sgn}(v)}{v^3r_0^2}\begin{pmatrix}0\\0\\1\end{pmatrix}\\&=-\left(\frac{Z\mathbf{e}^2}{m}\right)^2\frac{2}{v^4r_0^2}\vec{v}.\end{aligned} \tag{2.14}$$

We note that the sum

$$\Delta\vec{v}=\delta\vec{v}_C+\delta^{(2)}\vec{v}_C \tag{2.15}$$

guarantees energy conservation up to $O(Z^2)$,

$$\begin{aligned}\Delta E&=\frac{m}{2}[2\vec{v}\cdot\delta\vec{v}_C+2\vec{v}\cdot\delta^{(2)}\vec{v}_C+(\delta\vec{v}_C)^2]\\&=\frac{m}{2}\left[0-4\left(\frac{Z\mathbf{e}^2}{m}\right)^2\frac{1}{v^2r_0^2}+4\left(\frac{Z\mathbf{e}^2}{m}\right)^2\frac{1}{v^2r_0^2}\right]=0.\end{aligned} \tag{2.16}$$

In fact, we could have proceeded backwards from here. By noting $\delta\vec{v}_C\perp\vec{v}$ the second-order contribution $\delta^{(2)}\vec{v}_C\|\vec{v}$ could have been calculated uniquely by enforcing energy conservation.

III. SCATTERING OF MAGNETIZED ELECTRONS BY IONS MOVING TRANSVERSELY TO THE MAGNETIC FIELD

A. Relative motion and conservation law

We assume now slow ions with mass $M\gg m$, which move uniformly with a velocity

$$\vec{v}_i=\begin{pmatrix}v_{i\perp}\\0\\v_{i\|}\end{pmatrix}. \tag{3.1}$$

Because of Galilei invariance, the results obtained previously in Sec. II for the case $\vec{B}=\vec{0}$ remain valid, if the velocities, distances, etc., are interpreted as relative quantities, e.g.,

$$\begin{aligned}\vec{r}(t)&=\vec{r}_e(t)-\vec{r}_i(t)=\vec{r}_e(t)-\vec{v}_it,\\\vec{v}(t)&=\vec{v}_e(t)-\vec{v}_i.\end{aligned} \tag{3.2}$$

However, if a magnetic field $\vec{B}=\vec{\nabla}\times\vec{A}$ is present the Lagrangian of the electron-ion system is

$$\mathcal{L}=\frac{m}{2}\vec{v}_e^2+\frac{M}{2}\vec{v}_i^2-e\vec{A}(\vec{r}_e)\cdot\vec{v}_e+Ze\vec{A}(\vec{r}_i)\cdot\vec{v}_i+\frac{Z\mathbf{e}^2}{r}. \tag{3.3}$$

For a homogeneous field $\vec{B}$ the vector potential may be chosen as $\vec{A}(\vec{x})=\tfrac{1}{2}\vec{B}\times\vec{x}$, this yields

$$\mathcal{L}=\frac{m}{2}\vec{v}_e^2+\frac{M}{2}\vec{v}_i^2-\frac{e}{2}(\vec{B}\times\vec{r}_e)\cdot\vec{v}_e+\frac{Ze}{2}(\vec{B}\times\vec{r}_i)\cdot\vec{v}_i+\frac{Z\not{e}^2}{r}. \tag{3.4}$$

This Lagrangian cannot, in general, be separated into parts describing the relative motion and the motion of the center of mass: With $\vec{r}_{\text{c.m.}}=(m\vec{r}_e+M\vec{r}_i)/(M+m)$, $\vec{v}_{\text{c.m.}}=(m\vec{v}_e+M\vec{v}_i)/(m+M)$ and the reduced mass $\mu^{-1}=m^{-1}+M^{-1}$ the Lagrangian is

$$\begin{aligned}\mathcal{L}=&\frac{m+M}{2}\vec{v}_{\text{c.m.}}^2+\frac{\mu}{2}\vec{v}^2+\frac{Z\not{e}^2}{r}+\frac{(Z-1)e}{2}(\vec{B}\times\vec{r}_{\text{c.m.}})\cdot\vec{v}_{\text{c.m.}}\\&+\frac{\mu^2}{2}\left(\frac{Ze}{M^2}-\frac{e}{m^2}\right)(\vec{B}\times\vec{r})\cdot\vec{v}-\frac{\mu}{2}\left(\frac{Ze}{M}+\frac{e}{m}\right)\\&\times[(\vec{B}\times\vec{r}_{\text{c.m.}})\cdot\vec{v}+(\vec{B}\times\vec{r})\cdot\vec{v}_{\text{c.m.}}].\end{aligned} \tag{3.5}$$

Here, under the assumptions mentioned above, a heavy, uniformly moving ion can be considered as a time-dependent perturbation on the electron motion. Using $\mu\to m$ and dropping unimportant constants the Lagrangian for the relative motion is

$$\begin{aligned}\mathcal{L}=&\frac{m}{2}\vec{v}^2+\frac{Z\not{e}^2}{r}-\frac{e}{2}(\vec{B}\times\vec{r})\cdot\vec{v}-\frac{e}{2}\\&\times[(\vec{B}\times\vec{v}_it)\cdot\vec{v}+(\vec{B}\times\vec{r})\cdot\vec{v}_i],\end{aligned} \tag{3.6}$$

with the equations of motion

$$\begin{aligned}\vec{v}&=\frac{d\vec{r}}{dt},\\ m\frac{d\vec{v}}{dt}&=m\frac{d\vec{v}_e}{dt}\\&=\vec{\nabla}\frac{Z\not{e}^2}{r}-e(\vec{v}\times\vec{B})-e(\vec{v}_i\times\vec{B})\\&=-\vec{\nabla}\frac{Z\not{e}^2}{r}-e(\vec{v}_e\times\vec{B}).\end{aligned} \tag{3.7}$$

As $\mathcal{L}$ depends on the time explicitly it is not the energy E of relative motion which is conserved, but rather the quantity

$$K=E+e(\vec{v}_i\times\vec{B})\cdot\vec{r}=\frac{m}{2}\vec{v}^2-\frac{Z\not{e}^2}{r}+e(\vec{v}_i\times\vec{B})\cdot\vec{r}, \tag{3.8}$$

which can be easily proved with the help of the equations of motion (3.7). According to the ansatz (3.1) the ion moves in the x-z plane, for an evaluation of the last term in K one needs, therefore, the y component of the trajectory of relative motion

$$e(\vec{v}_i\times\vec{B})\cdot\vec{r}=-ev_{i\perp}Br_y. \tag{3.9}$$

As before, we aim at an iterative solution of Eqs. (3.7) starting from the zero-order helical motion

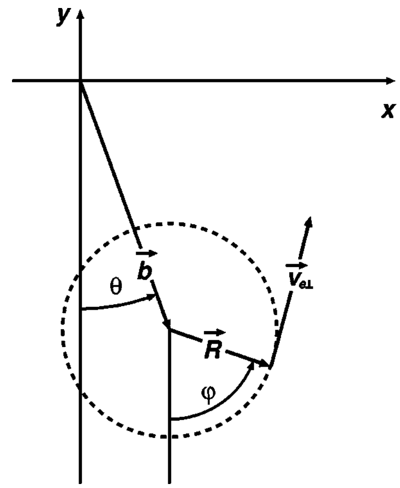

FIG. 1. Relation between the parameters b, R, ρ, θ, and φ describing the trajectories according to Eq. (3.10) in the transverse plane at $t=0$.

$$\vec{r}(t)=\begin{pmatrix}b\sin\theta+R\sin(\Omega t+\varphi)-v_{i\perp}t\\-b\cos\theta-R\cos(\Omega t+\varphi)\\(v_{e\parallel}-v_{i\parallel})t\end{pmatrix}, \tag{3.10}$$

see Fig. 1, and

$$\vec{v}(t)=\begin{pmatrix}v_{e\perp}\cos(\Omega t+\varphi)-v_{i\perp}\\v_{e\perp}\sin(\Omega t+\varphi)\\v_{e\parallel}-v_{i\parallel}\end{pmatrix} \tag{3.11}$$

with the cyclotron frequency

$$\Omega=\frac{eB}{m} \tag{3.12}$$

and the cyclotron radius

$$R=\frac{v_{e\perp}}{\Omega}=\frac{mv_{e\perp}}{eB} \tag{3.13}$$

with $v_{e\perp}\geqslant 0$. We will need the time t_0 and the distance vector $\vec{r}_0$ of the closest approach between the electron and the ion. For that purpose we expand $r^{-3}(t)$, which will be needed in the electric field, with respect to R,

$$\begin{aligned}r^{-3}(t)=&\{b^2+R^2+(v_{e\parallel}-v_{e\parallel})^2t^2+v_{i\perp}^2t^2-2v_{i\perp}bt\sin\theta\\&+2R[b\cos(\Omega t+\varphi-\theta)-v_{i\perp}t\sin(\Omega t+\varphi)]\}^{-3/2}\\\approx&[b^2+(v_{e\parallel}-v_{i\parallel})^2t^2+v_{i\perp}t^2-2v_{i\perp}bt\sin\theta]^{-3/2}\\&\times\left(1-\frac{3R[b\cos(\Omega t+\varphi-\theta)-v_{i\perp}t\sin(\Omega t+\varphi)]}{b^2+(v_{e\parallel}-v_{i\parallel})^2t^2+v_{i\perp}^2t^2-2v_{i\perp}bt\sin\theta}\right).\end{aligned} \tag{3.14}$$

The new denominator describes the relative motion excluding the transverse motion of the electrons, which is quenched by the magnetic field

$$\vec{\bar{r}}^2(t) = b^2 + [(v_{e\parallel} - v_{i\parallel})^2 + v_{i\perp}^2] t^2 - 2 v_{i\perp} b t \sin\theta$$
$$= \vec{\bar{v}}^2 (t - t_0)^2 + r_0^2, \tag{3.15}$$

where

$$\vec{\bar{v}} = \vec{v}_\parallel + \vec{v}_\perp = \begin{pmatrix} 0 \\ 0 \\ v_{e\parallel} - v_{i\parallel} \end{pmatrix} + \begin{pmatrix} -v_{i\perp} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -v_{i\perp} \\ 0 \\ v_{e\parallel} - v_{i\parallel} \end{pmatrix} \tag{3.16}$$

is the relative velocity vector between the guiding center and the ion and its components in an obvious notation. Its magnitude $\bar{v}$ is related to the pitch of the helices

$$\delta = \frac{\bar{v}}{\Omega}. \tag{3.17}$$

The distance $\vec{\bar{r}}(t_0) = \vec{r}_0$ and time t_0 of the closest approach between the guiding center and the ion are

$$t_0 = -\frac{\bar{v}_\perp b \sin\theta}{\bar{v}^2} \tag{3.18}$$

and

$$\vec{\bar{r}}^2(t_0) = r_0^2 = b^2 \left[1 - \left(\frac{\bar{v}_\perp \sin\theta}{\bar{v}} \right) \right]^2 = b^2 - (\bar{v} t_0)^2, \tag{3.19}$$

so that

$$\vec{\bar{r}}(t) = \begin{pmatrix} b \sin\theta + \bar{v}_\perp t_0 \\ -b\cos\theta \\ +\bar{v}_\parallel t_0 \end{pmatrix} + \begin{pmatrix} \bar{v}_\perp \\ 0 \\ \bar{v}_\parallel \end{pmatrix} (t - t_0) = \vec{r}_0 + \vec{\bar{v}}(t - t_0). \tag{3.20}$$

Here we continue to use the notation $\vec{r}_0$ for the distance vector and t_0 for the time of the closest approach, although the relation between these quantities and the parameters describing the trajectories is different from that in the preceding section. This is done because $\vec{r}_0$ is the independent variable with respect to which integrations must be performed when the interaction of ions with an electron beam is considered.

It is useful to introduce dimensionless variables such as

$$\tau = \frac{\bar{v} t}{r_0} \tag{3.21}$$

and

$$\sigma = \tau - \tau_0 = \frac{\bar{v}(t - t_0)}{r_0} \tag{3.22}$$

for the time,

$$\omega = \frac{r_0}{\delta} = \frac{r_0 \Omega}{\bar{v}} \tag{3.23}$$

for the pitch,

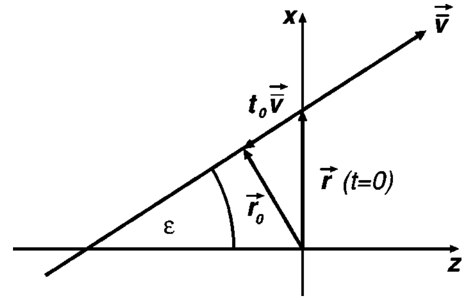

FIG. 2. Illustration of the relation (3.27) between the vector of relative motion $\vec{\bar{v}}$ and the orthogonal vector of closest approach $\vec{r}_0$.

$$\gamma_\parallel = \frac{\bar{v}_\parallel}{\bar{v}} \tag{3.24}$$

and

$$\gamma_\perp = \frac{\bar{v}_\perp}{\bar{v}} \tag{3.25}$$

as components of a reduced relative velocity as well as

$$\beta = \frac{b}{r_0} \tag{3.26}$$

for the trajectory. Furthermore, polar coordinates as shown in Fig. 2 will be useful,

$$\vec{r}_0 = r_0 \begin{pmatrix} \cos\varepsilon \cos\psi \\ \sin\psi \\ -\sin\varepsilon \cos\psi \end{pmatrix}, \tag{3.27}$$

where ε is the angle between the direction of flux $\vec{\bar{v}}$ and the direction of the magnetic field, i.e.,

$$\cos\varepsilon = \frac{v_{e\parallel} - v_{i\parallel}}{\bar{v}} = \gamma_\parallel. \tag{3.28}$$

Comparing the perpendicular components of Eqs. (3.20) and (3.27) one obtains

$$\tau_0 = -\beta \gamma_\perp \sin\theta = -\frac{\gamma_\perp}{\gamma_\parallel} \cos\psi \tag{3.29}$$

and

$$\beta \cos\theta = -\sin\psi. \tag{3.30}$$

Thus β, τ_0, and θ can be expressed by ψ and the reduced velocities

$$\beta \sin\theta + \gamma_\perp \tau_0 = \gamma_\parallel \cos\psi, \tag{3.31}$$

$$\beta^2 = 1 + \tau_0^2. \tag{3.32}$$

B. First-order velocity transfer, trajectory correction, and K conservation

The equations of motion (3.7) are solved iteratively by treating the Coulomb field as a perturbation. The magnetic field is taken into account by transforming to a rotating system

$$\vec{v}_e(t) = T(\Omega t)\vec{V}_e(t) \tag{3.33}$$

with

$$T(\Omega t) = \begin{pmatrix} \cos\Omega t & -\sin\Omega t & 0 \\ \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.34}$$

which yields

$$\frac{d\vec{V}_e}{dt} = -\frac{e}{m} T^{-1}(\Omega t)\vec{E}(\vec{r}(t)). \tag{3.35}$$

The first-order velocity transfer is obtained by integrating this equation. For strong magnetic fields, $R < r_0$, the leading term is

$$\begin{aligned}\delta\vec{V}_B(t) &= -\frac{Ze^2}{m}\int_{-\infty}^{t}\frac{dt'}{\bar{r}^3(t')}T^{-1}(\Omega t')\bar{\vec{r}}(t') \\ &= -\frac{Ze^2}{m}\frac{1}{\bar{v}r_0}\int_{-\infty}^{\sigma}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}} \\ &\quad\times\begin{pmatrix} \cos\omega\tau' & \sin\omega\tau' & 0 \\ -\sin\omega\tau' & \cos\omega\tau' & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\quad\times\begin{pmatrix} \beta\sin\theta + \gamma_\perp\tau' \\ -\beta\cos\theta \\ \gamma_\parallel\tau' \end{pmatrix}.\end{aligned} \tag{3.36}$$

The asymptotic velocity transfer is

$$\delta\vec{V}_B(t\to\infty) = -\frac{Ze^2}{m}\frac{1}{\bar{v}r_0}\begin{pmatrix} 2\omega K_1(\omega)(\gamma_\parallel\cos\psi\cos\omega\tau_0 + \sin\psi\sin\omega\tau_0) - 2\omega K_0(\omega)\gamma_\perp\sin\omega\tau_0 \\ -2\omega K_1(\omega)(\gamma_\parallel\cos\psi\sin\omega\tau_0 - \sin\psi\cos\omega\tau_0) - 2\omega K_0(\omega)\gamma_\perp\cos\omega\tau_0 \\ 2\gamma_\parallel\tau_0 \end{pmatrix}, \tag{3.37}$$

where K_0 and K_1 are the modified Bessel functions [16]. This generalizes the result of Ref. [6] to ions that have a transverse component of motion with respect to the magnetic field. It will turn out to be useful to distinguish two subcases of strong magnetic fields: Stretched helices (case *s*), where the pitch δ is larger than the distance of the closest approach r_0 and tight helices (case *t*) in the opposite case, see Fig. 3. In the limit $\omega \ll 1$ of stretched helices one obtains

$$\delta\vec{V}_B \to \delta\vec{V}_s = -\frac{Ze^2}{m}\frac{2}{\bar{v}r_0^2}\vec{r}_0, \tag{3.38}$$

which should be compared with Eq. (2.7). In the opposite limit $\omega\to\infty$, the first-order velocity transfer is parallel to the magnetic field,

$$\delta\vec{V}_B \to \delta\vec{V}_t = -\frac{Ze^2}{m\bar{v}r_0}2\gamma_\parallel\tau_0\begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{3.39}$$

The first-order trajectory correction is

$$\begin{aligned}\delta\vec{r}(t) &= -\frac{Ze^2}{m\bar{v}^2}\int_{-\infty}^{\sigma}d\sigma' T(\omega\tau') \\ &\quad\times\int_{-\infty}^{\sigma'}\frac{d\sigma''}{(1+\sigma''^2)^{3/2}}T^{-1}(\omega\tau'')\begin{pmatrix}\beta\sin\theta+\gamma_\perp\tau'' \\ -\beta\cos\theta \\ \gamma_\parallel\tau''\end{pmatrix}.\end{aligned} \tag{3.40}$$

The parallel component is not affected by the rotations *T*,

$$\delta r_\parallel(t) = -\frac{Ze^2}{m\bar{v}^2}\gamma_\parallel\int_{-\infty}^{\sigma}d\sigma'\left(\frac{\tau_0\sigma'-1}{(1+\sigma'^2)^{1/2}}+\tau_0\right). \tag{3.41}$$

For the perpendicular components one performs a partial integration in the outer (σ') integral, see the Appendix. This yields

$$\begin{aligned}\delta\vec{r}_\perp(t) &= \frac{r_0}{\bar{v}\omega}\begin{pmatrix}\sin\omega\tau & \cos\omega\tau \\ -\cos\omega\tau & \sin\omega\tau\end{pmatrix}\delta\vec{V}_{B,\perp}(t) \\ &\quad-\frac{Ze^2}{m\bar{v}^2\omega}\int_{-\infty}^{\sigma}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}}\begin{pmatrix}\beta\cos\theta \\ \beta\sin\theta+\gamma_\perp\tau'\end{pmatrix}.\end{aligned} \tag{3.42}$$

It is now easy to show *K* conservation up to first order. From Eqs. (3.8) and (3.9) we have to show

$$\begin{aligned}\delta K &= \frac{m}{2}(2\bar{\vec{v}}\cdot\delta\vec{v}_B - ev_{i\perp}B\,\delta r_y) = \frac{m}{2}(2\bar{\vec{V}}\cdot\delta\vec{V}_B + e\bar{v}_\perp B\,\delta r_y) \\ &= 0.\end{aligned} \tag{3.43}$$

Now

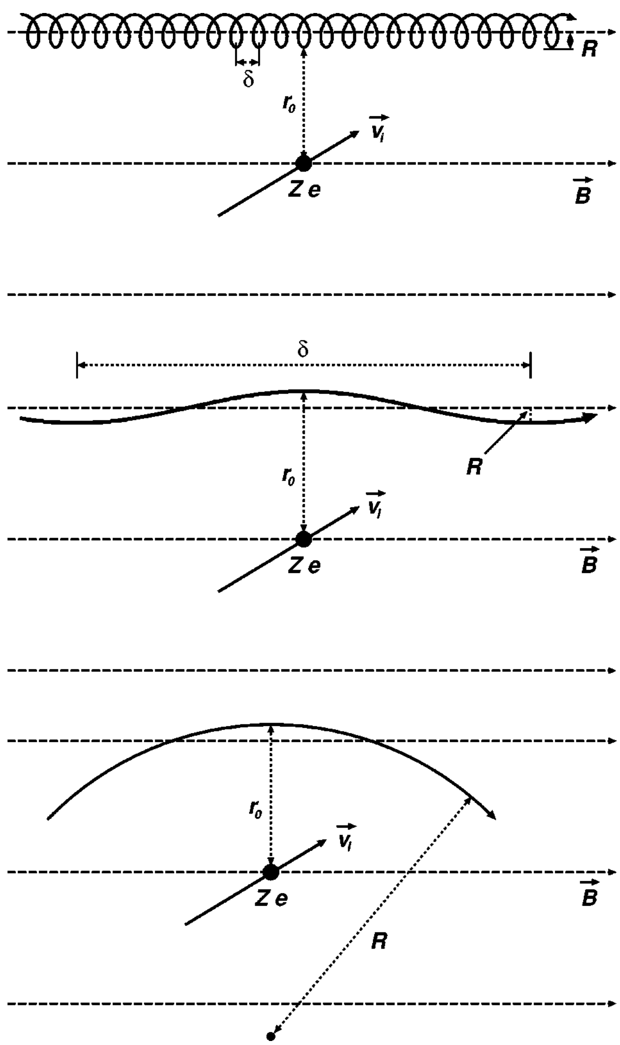

FIG. 3. Trajectories of ions and electrons in a magnetic field. In the upper part the magnetic field is strong in the sense that the cyclotron radius is small and the helix is tight compared to the distance of closest approach, $R<r_0$ and $\delta<r_0$. In the central part the magnetic field is still strong with respect to the transverse motion, but weak with respect to the longitudinal motion, the helix is stretched, $R<r_0$ and $\delta>r_0$. The corresponding expressions for the velocity transfers are similar to the case of a weak magnetic field, $R>r_0$, which is shown in the lower part of the figure.

$$e\overline{v_\perp}B\,\delta r_y = m\overline{v_\perp}(-\delta V_{B,x}\cos\omega\tau + \delta V_{B,y}\sin\omega\tau) - \frac{Z\not{e}^2}{\overline{v}r_0}\overline{v_\perp}\int_{-\infty}^{\infty}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}}(\beta\sin\theta + \gamma_\perp\tau') \tag{3.44}$$

and

$$m\vec{\overline{v}}\cdot\delta\vec{v}_B = m\vec{\overline{V}}\cdot\delta\vec{V}_B = m(T^{-1}\vec{\overline{v}})\cdot\delta\vec{V}_B = m\overline{v_\perp}(\delta V_{B,x}\cos\omega\tau - \delta V_{B,y}\sin\omega\tau) - \frac{Z\not{e}^2}{\overline{v}r_0}\overline{v_\parallel}\int_{-\infty}^{\infty}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}}\gamma_\parallel\tau'. \tag{3.45}$$

As the perpendicular components cancel, there remains

$$\delta K = -\frac{Z\not{e}^2}{\overline{v}r_0}\int_{-\infty}^{\infty}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}}[\overline{v_\perp}(\beta\sin\theta + \gamma_\perp(\sigma'+\tau_0)) + \overline{v_\parallel}\gamma_\parallel(\sigma'+\tau_0)] = \frac{Z\not{e}^2}{r_0}2\gamma_\perp\beta\sin\theta(\gamma_\perp^2+\gamma_\parallel^2-1)=0, \tag{3.46}$$

where Eq. (3.18) has been used to relate τ_0 to $\sin\,\theta$.

C. Electric-field correction, second-order velocity correction, and K conservation

The electric-field correction in the guiding center approximation is

$$\delta E_k = \frac{Ze}{4\pi\varepsilon_0(\overline{r_j}\overline{r_j})^{3/2}}\left(\delta r_k - 3r_k\frac{\overline{r_i}\delta r_i}{\overline{r_j}\overline{r_j}}\right), \tag{3.47}$$

with the guiding center trajectory (3.20) and the second-order velocity correction

$$\delta^{(2)}\vec{V}_B(t) = -\frac{e}{m}\int_{-\infty}^{t}dt'\,T^{-1}(\Omega t')\,\delta\vec{E}(t'). \tag{3.48}$$

We first calculate the scalar product $\overline{r_i}\delta r_i$ with the help of Eqs. (3.20), (3.41), and (3.42):

$$(\overline{r_i}\delta r_i)(t) = -\frac{Z\not{e}^2}{m}\frac{r_0}{\overline{v}^2}\Bigg\{\frac{1}{\omega}\int_{-\infty}^{\sigma}\frac{d\sigma'}{(1+\sigma'^2)^{3/2}} \times\{\beta^2(\sin\omega\tau\cos\omega\tau' - \cos\omega\tau\sin\omega\tau') + \beta\gamma_\perp[\sin\omega\tau(\sin\theta(\tau+\tau')\cos\omega\tau' - \cos\theta(\tau-\tau')\sin\omega\tau')] + \cos\omega\tau[-\sin\theta(\tau+\tau')\sin\omega\tau' - \cos\theta(\tau-\tau')\cos\omega\tau')] + \gamma_\perp^2\tau\tau'(\sin\omega\tau\cos\omega\tau' - \cos\omega\tau\sin\omega\tau') + \beta\gamma_\perp\cos\theta(\tau-\tau')\Bigg\} + \gamma_\parallel^2(\sigma+\tau_0) \times\int_{-\infty}^{\sigma}d\sigma'\left(\frac{\tau_0\sigma'-1}{(1+\sigma'^2)^{1/2}}+\tau_0\right). \tag{3.49}$$

Here and in the following the integrals can not more be evaluated in a closed form for arbitrary ω. In the limit of tight and stretched helices, there remains

$$(\overline{r_i}\delta r_i)(t) \to -\frac{Z\not{e}^2}{m}\frac{r_0}{\overline{v}^2}\gamma_\parallel^2(\sigma+\tau_0) \times\int_{-\infty}^{\sigma}d\sigma'\left(\frac{\tau_0\sigma'-1}{(1+\sigma'^2)^{1/2}}+\tau_0\right),\quad \omega\to\infty \tag{3.50}$$

and

$$(\overline{r_i}\delta r_i)(t)\to -\frac{Z e^2}{m}\frac{r_0}{\overline{v}^2}\left\{\int_{-\infty}^{\sigma}d\sigma'\left(\frac{\tau_0\sigma'-1}{(1+\sigma'^2)^{1/2}}+\tau_0\right)\right.$$
$$\times[1-\gamma_\parallel^2\tau_0^2-\gamma_\parallel^2\tau_0\sigma-\gamma_\parallel^2\tau_0\sigma'+\gamma_\perp^2\sigma\sigma']$$
$$\left.+\gamma_\parallel^2(\sigma+\tau_0)\int_{-\infty}^{\sigma}d\sigma'\left(\frac{\tau_0\sigma'-1}{(1+\sigma'^2)^{1/2}+\tau_0}\right)\right\},$$
$$\omega\to 0, \tag{3.51}$$

respectively. For the parallel component of the second-order velocity transfer in the limit $\omega\to\infty$, Eqs. (3.20), (3.42) and (3.50) must be inserted into the $k=3$ component of Eq. (3.47). As mentioned above we are interested in the interaction of ions with beams of magnetized electrons, which requires, among other things, an integration with respect to the angle ψ defined in Eq. (4.23). Averaging with respect to ψ all terms odd in τ_0 vanish while

$$\langle\tau_0^2\rangle=\frac{1}{2}\frac{\gamma_\perp^2}{\gamma_\parallel^2},$$
$$\langle\tau_0^4\rangle=\frac{3}{8}\frac{\gamma_\perp^4}{\gamma_\parallel^4}. \tag{3.52}$$

The multiple integrations are done with the help of the techniques in the Appendix: The result for the averaged parallel second-order velocity transfer is

$$\langle\delta^{(2)}V_{t,\parallel}\rangle=-\left(\frac{Z e^2}{m r_0}\right)^2\frac{2\gamma_\perp^2\gamma_\parallel}{\overline{v}^3}. \tag{3.53}$$

In the opposite limit $\omega\to 0$ the scalar product (3.51) contains additional terms. Performing the ψ averages (3.52) and carrying out the integrations results in

$$\delta^{(2)}V_{s,\parallel}=-\left(\frac{Z e^2}{m r_0}\right)^2\frac{2\gamma_\parallel}{\overline{v}^3}, \tag{3.54}$$

again this corresponds to the Coulomb result (2.15) if $\overline{\vec{v}}\leftrightarrow\vec{v}$.

The transversal components of the second-order velocity transfer are

$$\delta^{(2)}\vec{V}_{B,\perp}(t\to\infty)=-\frac{e}{m}\int_{-\infty}^{\infty}d\tau'\begin{pmatrix}\cos\omega\tau\sin\omega\tau'\\ -\sin\omega\tau'\cos\omega\tau'\end{pmatrix}\delta\vec{E}_\perp\,. \tag{3.55}$$

The lengthy calculations yield a plausible result: For tight helices the transversal velocity transfer $\delta^{(2)}\vec{V}_{t,\perp}$ vanishes because of the transformation to the rotating system. For stretched helices one obtains the Coulomb expression (2.15) with $\vec{v}\to\overline{\vec{v}}$ also for $\delta^{(2)}\vec{V}_{s,\perp}$. Thus

$$\langle\delta^{(2)}\vec{V}_B(t\to\infty)\rangle\to\begin{cases}\langle\delta^{(2)}\vec{V}_t\rangle, & \omega\to\infty\\ \langle\delta^{(2)}\vec{V}_s\rangle, & \omega\to 0,\end{cases} \tag{3.56}$$

with

$$\langle\delta^{(2)}\vec{V}_t\rangle=\langle\delta^{(2)}V_{t,\parallel}\rangle\begin{pmatrix}0\\0\\1\end{pmatrix}=-\left(\frac{Z e^2}{m r_0}\right)^2\frac{2\gamma_\perp^2\gamma_\parallel}{\overline{v}^3}\begin{pmatrix}0\\0\\1\end{pmatrix} \tag{3.57}$$

from Eq. (4.46) and

$$\langle\delta^{(2)}\vec{V}_s\rangle=-\left(\frac{Z e^2}{m r_0}\right)^2\frac{2\overline{\vec{v}}}{\overline{v}^4}. \tag{3.58}$$

With $\gamma_\perp\neq 0$ there is a second-order velocity transfer parallel to the magnetic field even for tight helices with $\omega\to\infty$.

There remains to test the conservation of K, Eq. (3.8), up to second order. In Eq. (3.46) we have shown that the first-order corrections to K cancel, there remains

$$\Delta K=\frac{m}{2}[2\overline{\vec{v}}\cdot\delta^{(2)}\vec{v}_B+(\delta\vec{v}_B)^2]+e\overline{v}_\perp B\,\delta^{(2)}r_y=0. \tag{3.59}$$

For that purpose the second-order trajectory correction $\delta^{(2)}r_y$ must be calculated. This is the y component of

$$\delta^{(2)}\vec{r}_\perp(t)=\int_{-\infty}^{t}dt'\,T(\Omega t')\,\delta^{(2)}\vec{V}_{B,\perp}(t')$$
$$=\delta^{(2)}\vec{r}_\perp^{(a)}(t)+\delta^{(2)}\vec{r}_\perp^{(b)}(t)$$
$$=\frac{1}{2}\begin{pmatrix}\sin\Omega t\cos\Omega t\\ -\cos\Omega t\sin\Omega t\end{pmatrix}\delta^{(2)}\vec{V}_{B,\perp}$$
$$+\frac{e}{m\Omega}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\int_{-\infty}^{t}dt'\,\delta\vec{E}_\perp(t'), \tag{3.60}$$

where a partial integration has been performed, see the Appendix. The first, integrated term gives

$$e\overline{v}_\perp B\,\delta^{(2)}r_y^{(a)}(t)=m\overline{v}_\perp[-\delta^{(2)}V_{B,x}(t)\cos\Omega t$$
$$+\delta^{(2)}V_{B,y}(t)\sin\Omega t] \tag{3.61}$$

and this cancels for any value of Ω with the transversal part of the second-order kinetic energy correction

$$m\overline{\vec{v}}_\perp\,\delta^{(2)}\vec{v}_{B,\perp}(t)=m\vec{V}_\perp\,\delta^{(2)}\vec{V}_{B,\perp}$$
$$=M(T^{-1}\overline{v})_\perp\,\delta^{(2)}\vec{V}_{B,\perp}$$
$$=m\overline{v}_\perp(\delta^{(2)}\vec{V}_{B,x}\cos\Omega t-\delta^{(2)}\vec{V}_{B,y}\sin\Omega t). \tag{3.62}$$

The second term in Eq. (3.60) gives

$$e\overline{v}_\perp B\,\delta^{(2)}r_y^{(b)}(t)=-e\overline{v}_\perp\int_{-\infty}^{t}dt'\,\delta E_x(t'), \tag{3.63}$$

which involves the scalar product (3.49). In the limit $\omega\to 0$ (stretched helices), one obtains immediately from Eq. (3.43)

$$e\overline{v_\perp}B\,\delta^{(2)}r_y^{(b)}(t)=-e\overline{v_\perp}\left(-\frac{m}{e}\delta^{(2)}V_{s,x}\right), \qquad (3.64)$$

and after ψ averaging from Eq. (3.58),

$$\langle e\overline{v_\perp}B\,\delta^{(2)}r_y^{(b)}\rangle=-\left(\frac{Ze^2}{r_0}\right)^2\frac{2\overline{v_\perp^2}}{m\overline{v^4}}. \qquad (3.65)$$

Collecting terms from Eqs. (3.38), (3.58), and (3.65) one has then for the ψ-averaged K correction,

$$(\Delta K)=\left\langle m\overline{v_\parallel}\delta^{(2)}v_{s,\parallel}+\frac{m}{2}(\delta\vec{V}_s)^2+e\overline{v_\perp}B\,\delta^{(2)}r_y^{(b)}\right\rangle$$
$$=\left(\frac{Ze^2}{r_0}\right)^2\frac{1}{m}\left(-\frac{2\overline{v_\parallel^2}}{\overline{v^4}}+\frac{2}{\overline{v^2}}-\frac{2\overline{v_\perp^2}}{\overline{v^4}}\right)=0, \qquad (3.66)$$

which proves K conservation for stretched helices $\omega\to0$. In the opposite limit $\omega\to\infty$, the integral in Eq. (3.63) is a special case of Eq. (3.55), in which δE_x is evaluated for $\omega\to\infty$ and the final rotation is dropped. This yields, after ψ averaging,

$$\langle e\overline{v_\perp}B\,\delta^{(2)}r_y^{(b)}\rangle=-e\overline{v_\perp}\int_{-\infty}^{\infty}dt'\langle\delta E_x(t')\rangle$$
$$=\left(\frac{Ze^2}{r_0}\right)^2\frac{\overline{v_\perp^2}\,\overline{v_\parallel^2}}{m\overline{v^6}}\left(1-\frac{\overline{v_\perp^2}}{\overline{v_\parallel^2}}\right). \qquad (3.67)$$

The ψ-averaged ΔK correction is

$$\langle\Delta K\rangle=\left\langle m\overline{v_\parallel}\delta^{(2)}v_{t,\parallel}+\frac{m}{2}(\delta\vec{V}_t)^2+e\overline{v_\perp}B\,\delta^{(2)}r_y^{(b)}\right\rangle$$
$$=\left(\frac{Ze^2}{r_0}\right)^2\frac{1}{m}\left(-\frac{2\overline{v_\perp^2}\,\overline{v_\parallel^2}}{\overline{v^6}}+\frac{\overline{v_\perp^2}}{\overline{v^4}}+\frac{\overline{v_\perp^2}\,\overline{v_\parallel^2}}{\overline{v^6}}-\frac{\overline{v_\perp^4}}{\overline{v^6}}\right)=0, \qquad (3.68)$$

which establishes K conservation up to second order also for tight helices.

IV. APPLICATION TO THE ENERGY LOSS OF IONS IN COLLISIONS WITH MAGNETIZED ELECTRONS

A. From velocity transfer to energy loss

The energy change of the ion in leading order is

$$\Delta E_i=\frac{M}{2}(\vec{v}_i'^2-\vec{v}_i^2)=-M\vec{v}_i\cdot\Delta\vec{v}_i \qquad (4.1)$$

connected to the velocity transfer

$$\Delta\vec{v}_i=\vec{v}_i'-\vec{v}_i=\Delta\vec{v}_{\text{c.m.}}-\frac{m}{M}\Delta\vec{v} \qquad (4.2)$$

in a collision. The change in the center-of-mass velocity is obtained from the Lagrangian (3.5),

$$Md(\Delta\vec{v}_{\text{c.m.}})/dt+(Z-1)e(\vec{B}\times\Delta\vec{v}_{\text{c.m.}})=e\vec{B}\times\Delta\vec{v}. \qquad (4.3)$$

We have calculated $\Delta\vec{v}(t)$ in the preceding section. The integration of this c.m. equation could be done in a coordinate system that rotates with the cyclotron frequency $(Z-1)eB/M$, which is much smaller than the cyclotron frequency of the electron. So this rotation can be neglected. Integration of the remaining equation relates the change $\Delta\vec{v}_{\text{c.m.}}$ to the magnetic term occurring in the conserved quantity K of Eq. (3.8),

$$\Delta\vec{v}_{\text{c.m.}}=\frac{e}{M}(\vec{B}\times\Delta\vec{r}). \qquad (4.4)$$

Substitution into Eq. (4.2) shows that the change in the ion velocity

$$\Delta\vec{v}_i=\frac{e}{M}(\vec{B}\times\Delta\vec{r})-\frac{m}{M}\Delta\vec{v} \qquad (4.5)$$

yields

$$\Delta E_i=-m\vec{v}_i\cdot\Delta\vec{v}+e\vec{v}_i\cdot(\vec{B}\times\Delta\vec{r})$$
$$=-m\vec{v}_i\cdot\Delta\vec{v}-\frac{m}{2}(\vec{v}'^2-\vec{v}^2), \qquad (4.6)$$

where K conservation has been used in the last step.

In the case C of a weak magnetic field, $r_0<R$, the energy of relative motion is conserved. Inserting the first- and second-velocity transfers from Eqs. (2.7) and (2.14), respectively,

$$\Delta\vec{v}_C=\delta\vec{v}_C+\delta^{(2)}\vec{v}_C=-\frac{Ze^2}{mv}\frac{2\vec{r}_0}{r_0^2}-\left(\frac{Ze^2}{mr_0}\right)^2\frac{2\vec{v}}{v^4}. \qquad (4.7)$$

As discussed in Sec. III, ΔE_i will be averaged with respect to the azimuthal angle ψ of $\vec{r}_0$, see Eq. (3.27). The $O(Z)$ term $\delta\vec{v}_C$ gives then no contribution due to symmetry, and the averaged energy change is

$$\langle\Delta E_i\rangle_C=\left(\frac{Ze^2}{r_0}\right)^2\frac{2\vec{v}_i\cdot\vec{v}}{mv^4}. \qquad (4.8)$$

For the case s of stretched helices, $R<r_0$ and $\delta>r_0$, the relative velocity $\vec{v}$ must be replaced by the relative velocity $\overline{\vec{v}}$ (3.16) of the guiding center,

$$\langle\Delta E_i\rangle_s=\left(\frac{Ze^2}{r_0}\right)^2\frac{2\vec{v}_i\cdot\overline{\vec{v}}}{m\overline{v^4}}. \qquad (4.9)$$

More interesting is the case t of tight helices, $R<r_0$ and $\delta<r_0$, where only K, but not E, is conserved and the magnetic term in Eq. (4.6) is essential.

Inserting the first- and second-order velocity transfer $\Delta\vec{v}_t=\delta\vec{v}_t+\delta^{(2)}\vec{v}_t$ into Eq. (4.6) one obtains

$$\langle \Delta E_i \rangle_t = -m\vec{v}_i \cdot (\langle \delta \vec{v}_t \rangle + \langle \delta^{(2)} \vec{v}_t \rangle) - \frac{m}{2}(2\bar{\vec{v}} \cdot \langle \delta \vec{v}_t \rangle + 2\bar{\vec{v}} \cdot \langle \delta^{(2)} \vec{v}_t \rangle + \langle \delta \vec{v}_t^2 \rangle). \tag{4.10}$$

As $\delta\vec{v}_t$ is linear in τ_0, see Eq. (3.39), the ψ-average $\langle \delta\vec{v}_t \rangle$ vanishes, while Eq. (3.52) yields

$$\langle \delta \vec{v}_t^2 \rangle = \left(\frac{Z e^2}{m r_0} \right)^2 \frac{2\bar{v}_\perp^2}{\bar{v}^4}. \tag{4.11}$$

Inserting $\langle \delta^{(2)} \vec{v}_t \rangle$ from Eq. (3.57) the energy change is

$$\langle \Delta E_i \rangle_t = \left(\frac{Z e^2}{r_0} \right)^2 \frac{v_{i\perp}^2}{m \bar{v}^6} (v_{e\parallel}^2 - v_i^2). \tag{4.12}$$

This result has already been given in Eq. (53) of Ref. [9] for the special case $v_{e\parallel} = 0$. In the general case this term leads to an energy gain for $v_i^2 < v_{e\parallel}^2$. The energy loss of the ion in a homogeneous monochromatic electron beam is obtained by integrating over an area element $\hat{\bar{v}} d\psi r_0 dr_0$ parallel to the relative current density $n_e \bar{\vec{v}}$,

$$\frac{dE_i}{ds} = \frac{1}{v_i} \frac{dE_i}{dt} = \frac{2\pi}{v_i} n_e \bar{\vec{v}} \cdot \hat{\bar{v}} \int_{r_{\min}}^{r_{\max}} dr_0 r_0 \langle \Delta E_i \rangle, \tag{4.13}$$

where $r_{\max}$ is an upper cutoff that accounts for shielding, while $r_{\min}$ is the cutoff (2.12), below which the perturbative treatment of the Coulomb interaction fails. It is well known, however, that for Rutherford scattering, hard collisions are taken into account by regularizing the r_0 integral according to

$$L = \int_{r_{\min}}^{r_{\max}} \frac{dr_0}{r_0} = \ln \frac{r_{\max}}{r_{\min}} \rightarrow \Lambda(r_{\max}) = \int_0^{r_{\max}} \frac{r_0 dr_0}{r_0^2 + r_{\min}^2} = \frac{1}{2} \ln\left(1 + \frac{r_{\max}^2}{r_{\min}^2}\right), \tag{4.14}$$

a procedure that yields the exact result [17]. As the hard collisions are dominated by the Coulomb interaction we adopt this regularization also for the magnetic cases. In these integrals, with respect to r_0, the appropriate expressions for $\langle \Delta E_i \rangle$ must be inserted, i.e., Eq. (4.8) for $r_0 < R$, Eq. (4.9) for $r_0 > R$, $\delta > r_0$, and Eq. (4.12) for $r_0 > R$, $\delta < r_0$. Adding these contributions yields

$$\frac{dE_i}{ds} = \frac{4\pi (Z e^2)^2 n_e}{m} \Bigg\{ \frac{\hat{\bar{v}}_i \cdot \bar{\vec{v}}}{\bar{v}^3} \Lambda(\min(R, r_{\max})) + \Theta(R-\delta) \times \frac{1}{2} \frac{v_{i\perp}^2 (v_{e\parallel}^2 - v_i^2)}{\bar{v}^5 v_i} [\Lambda(r_{\max}) - \Lambda(\min(R, r_{\max}))] + \Theta(\delta - R) \Bigg[\frac{1}{2} \frac{v_{i\perp}^2 (v_{e\parallel}^2 - v_i^2)}{\bar{v}^5 v_i} [\Lambda(r_{\max}) - \Lambda(\min(\delta, r_{\max}))] + \frac{\hat{\bar{v}}_i \bar{\vec{v}}}{\bar{v}^3} [\Lambda(\min(\delta, r_{\max})) - \Lambda(\min(R, r_{\max}))] \Bigg] \Bigg\}. \tag{4.15}$$

In electron cooling of ion beams, the velocity distribution of the electrons is not isotropic. It is usually modeled by an anisotropic Maxwellian

$$f(\vec{v}_e) = \frac{m}{k_B T_\perp} \left(\frac{m}{2\pi k_B T_\parallel} \right)^{1/2} \frac{1}{2\pi} \exp\left(-\frac{m v_{e\perp}^2}{2 k_B T_\perp} - \frac{m v_{e\parallel}^2}{2 k_B T_\parallel} \right), \tag{4.16}$$

where k_B is the Boltzmann constant. For the upper cutoffs in the Coulomb logarithms of Eq. (4.15), which account for the shielding of the electron-ion interaction, we assume dynamic screening, see, for example, Refs. [18,19]:

$$r_{\max} = \lambda_D (1 + v_i^2 / \bar{v}_{\text{th}}^2)^{1/2}, \tag{4.17}$$

where λ_D is the Debye length,

$$\lambda_D = \left(\frac{\varepsilon_0 k_B \bar{T}}{n_e e^2} \right)^{1/2} = \left(\frac{k_B \bar{T}}{4\pi n_e e^2} \right)^{1/2}, \tag{4.18}$$

with $\bar{T} = \frac{1}{3} T_\parallel + \frac{2}{3} T_\perp$ and $\bar{v}_{\text{th}} = (k_B \bar{T}/m)^{1/2}$. The velocity-averaged energy loss is then calculated in cylindrical coordinates

$$\left\langle \frac{dE_i}{ds} \right\rangle = \frac{4\pi Z^2 e^4 n_e}{m} \frac{m}{k_B T_\perp} \left(\frac{m}{2\pi k_B T_\parallel} \right)^{1/2} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \times \int_0^\infty dv_{e\perp} v_{e\perp} \int_{-\infty}^\infty dv_{e\parallel} \exp\left(-\frac{m v_{e\perp}^2}{2 k_B T_\perp} - \frac{m v_{e\parallel}^2}{2 k_B T_\parallel} \right) \times \Bigg\{ \frac{\hat{\bar{v}} \cdot \vec{v}}{v^3} \Lambda(\min(R, r_{\max})) + \Theta(R-\delta) \frac{1}{2} \times \frac{v_{i\perp}^2 (v_{e\parallel}^2 - v_i^2)}{\bar{v}^5 v_i} [\Lambda(r_{\max}) - \Lambda(\min(R, r_{\max}))] + \Theta(\delta - R) \Bigg[\frac{1}{2} \frac{v_{i\perp}^2 (v_{e\parallel}^2 - v_i^2)}{\bar{v}^5 v_i} [\Lambda(r_{\max}) - \Lambda(\min(\delta, r_{\max}))] + \frac{\hat{\bar{v}}_i \cdot \bar{\vec{v}}}{\bar{v}^3} [\Lambda(\min(\delta, r_{\max})) - \Lambda(\min(R, r_{\max}))] \Bigg] \Bigg\}. \tag{4.19}$$

Here $\langle \cdots \rangle$ indicates the average with respect to the distribution (4.6). We note that the two magnetic terms do not involve the angle φ at all, so this integral is trivial for these terms. Moreover, the magnetic terms involve only the relative velocity $\bar{\vec{v}}$, in which $v_{e\perp}$ is quenched. In previous kinematical approaches average Coulomb logarithms L have been taken out of the velocity integral, which could then be

TABLE I. Parameters of ion and electron cooling beams in the Heidelberg test storage ring TSR [20] for the calculations leading to the results shown in Figs. 4–11.

Ion	C^{6+}
B (T)	0.005–0.05
Ω (s^{-1})	$0.88\times10^{9}-0.88\times10^{10}$
$\frac{(k_B T_\perp/m)^{1/2}}{\Omega}$ (m)	$5.1\times10^{-6}-5.1\times10^{-5}$
n_e (cm^{-3})	8×10^{6}
$k_B T_\perp$ (meV)	11.5
$k_B T_\parallel$ (meV)	0.1
λ_D (m)	0.73×10^{-4}

done in analogy to electrostatics by calculating a quasipotential due to an anisotropic Gaussian charge distribution. For the contributions where the transverse motion of the electrons is quenched by the magnetic field, this would correspond to the calculation of the potential of a wire carrying a Gaussian distribution. However, the resulting divergent behavior of the energy loss at low ion velocities is artificial. Then hard collisions dominate, we therefore keep the appropriate Coulomb logarithms Λ (4.14) under the velocity integrals in Eq. (4.19). Because of Eq. (2.12) this guarantees an analytical behavior of the energy loss at $v_i \to 0$. However, the velocity integrals cannot be evaluated in a closed form anymore.

B. Results

Actual calculations of the energy loss have been performed under conditions prevailing at the test storage ring TSR in Heidelberg [20]; the parameters are given in Table I. As mentioned in the Introduction, a comparison of our results with other treatments should address the following issues.

(i) The role of the cutoff $r_{\min}$ (2.12) below which the perturbation treatment fails. We have accounted for hard collisions by modifying the Coulomb logarithm according to Eq. (4.14). In a complementary manner the dielectric linear-response treatment fails for wave numbers larger than $r_{\min}^{-1}$ [4,9,10,19].

(ii) The binary collision model presumes a single-particle excitation spectrum of the target electrons, their polarization is only accounted for by dynamic screening according to Eq. (4.17). If exactly worked out the dielectric theory includes collectivity in the linear response of the electrons. This raises the question, how far does this redistribution of strength in the excitation spectrum of the electrons affect the energy loss and its dependence on the magnetic field.

(iii) However, for technical reasons, the complete dielectric linear-response theory has only been worked out for a flattered velocity distribution of the electrons ($T_\parallel = 0$) [9,10]. But the energy loss at small ion velocities depends also very sensitively on the details of the velocity distribution.

In Figs. 4–7 results from the present second-order treatment of binary collisions are compared with the energy loss

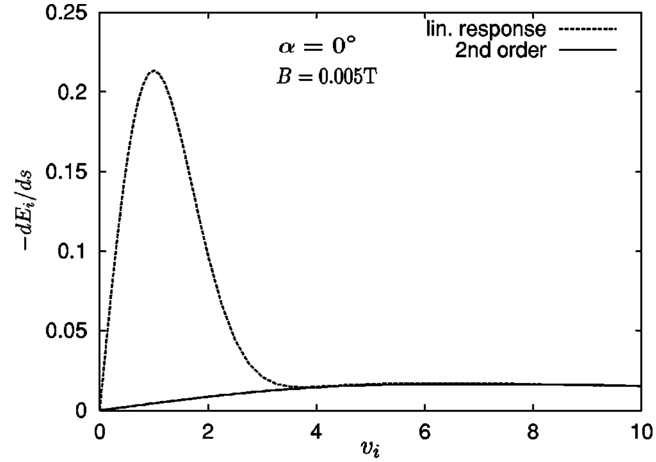

FIG. 4. Energy loss in units of $4\pi(Ze^2)^2 n_e/(m v_{\mathrm{th}\parallel}^2)$ as a function of the ion velocity v_i in units of the longitudinal thermal velocity $v_{\mathrm{th}\parallel}$ for $B = 5$ mT and $\alpha = 0°$. Other parameters are from Table I. Solid curve: present second-order treatment; dashed curve: linear-response theory [22].

obtained in the framework of a linear-response treatment. As mentioned in the Introduction this requires the calculation of the imaginary part of the inverse of the dynamical dielectric function $\mathrm{Im}[1/\varepsilon(\vec{k},\omega)]$. The very intricate linear-response expression for the dielectric function $\varepsilon(\vec{k},\omega)$ [12,21] with the anisotropic distribution (4.16) with $T_\parallel \neq 0$ has recently been evaluated numerically. The corresponding energy loss has been calculated assuming

$$\mathrm{Im}\frac{1}{\varepsilon(\vec{k},\omega)} \to \frac{-\mathrm{Im}\,\varepsilon(\vec{k},\omega)}{|\varepsilon(\vec{k},0)|^2}, \tag{4.20}$$

and introducing a cutoff $O(r_{\min}^{-1})$ in the integration with respect to wave numbers. The agreement between this simplified version of the dielectric linear-response treatment [22] and the binary collision model breaks down if the ion velocity approaches the longitudinal thermal velocity $v_{\mathrm{th}\parallel} = (k_B T_\parallel/m)^{1/2}$ from above. The disagreement is larger for an ion motion along the magnetic field ($\alpha = 0°$, Figs. 4 and 5)

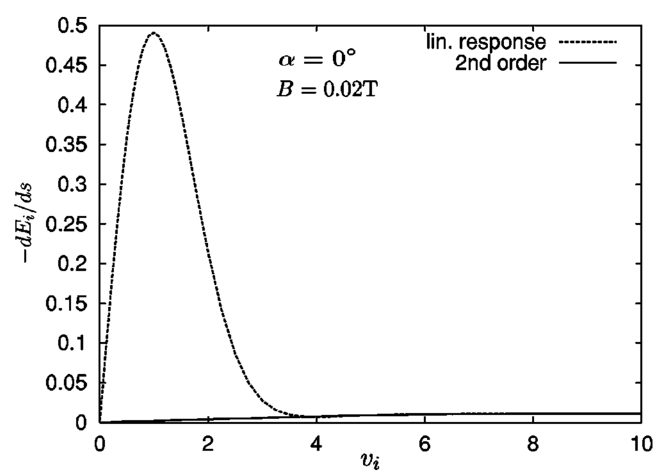

FIG. 5. Same as Fig. 4 for $B = 20$ mT.

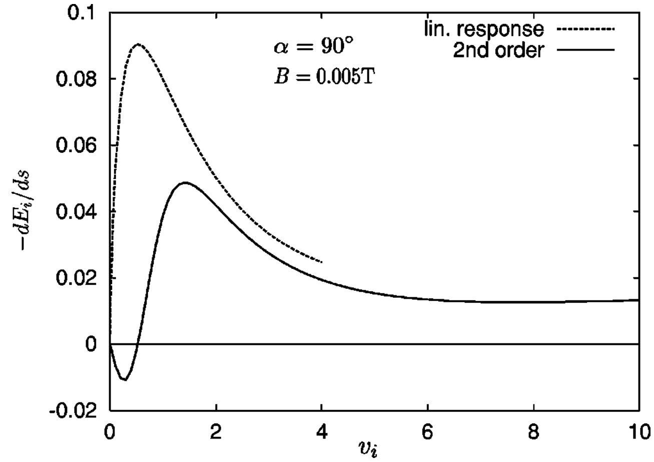

FIG. 6. Same as Fig. 4 for $\alpha = 90°$.

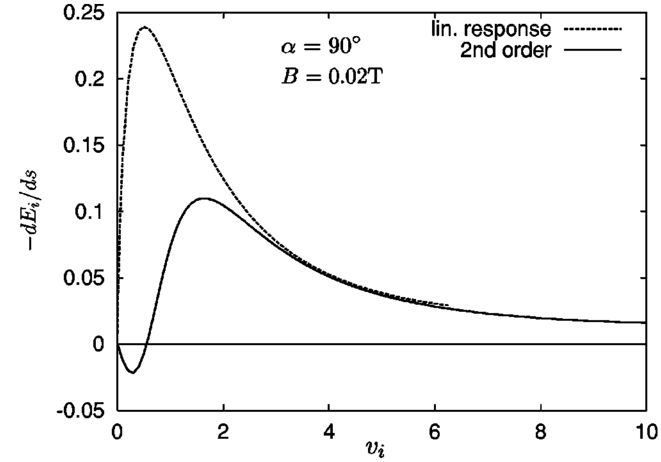

FIG. 7. Same as Fig. 5 for $\alpha = 90°$.

than transversally ($\alpha = 90°$, Figs. 6 and 7). In the latter case the present second-order treatment yields an energy gain for very low ion velocities, which is due to the contribution (4.12) from tight helices for $v_i < v_{\mathrm{th}\parallel}$. As there is no collectivity except screening in either the binary collision model or this version of dielectric linear response, their different behavior at low ion velocities is due to the different treatment of hard collisions: In the present binary collision model the modified Coulomb logarithm Λ (4.14) is employed under the integral with respect to the velocity distribution of the electrons. This approach is self-cutting as $\Lambda \to 0$ for small relative velocities v (or $\overline{v}$). In the dielectric model an average Coulomb logarithm L, evaluated with the thermal velocities of the electrons, is taken out of the velocity integral. This leads to a very large energy loss at low ion velocities, which behaves in a nonanalytical manner $dE_i/ds \propto (v_{i\perp}^2/v_i)\ln v_{i\perp}$ for low ion velocities [21,22].

In order to check the validity of treating hard collisions according to the replacement (4.14) we compare our binary collision model with classical trajectory Monte Carlo (CTMC) calculations, in which, for fixed triples of the parameters $v_{i\perp}$, $v_{e\perp}$ and $(v_{e\parallel} - v_{i\parallel})$, an ensemble of $(1\text{–}4) \times 10^5$ magnetized electrons are scattered in the field of the moving ion [3–5]. The electron-ion interaction has a Yukawa-type shielding with a range $r_{\max} = 1.03\lambda_D$. The CTMC results are shown on the right panels of Figs. 8–11, the agreement with the second-order treatment (left panels) is very satisfactory for all angles between the velocity of the ion and the magnetic field, the energy gains for small ion velocities should be noted. For a purely longitudinal motion (Fig. 8) the magnetic field hinders the loss of energy, in fact, for $B \to \infty$ one expects that the electrons move like beads on a wire with no energy transfer at all. This changes considerably if the ion has a transversal velocity component. Then

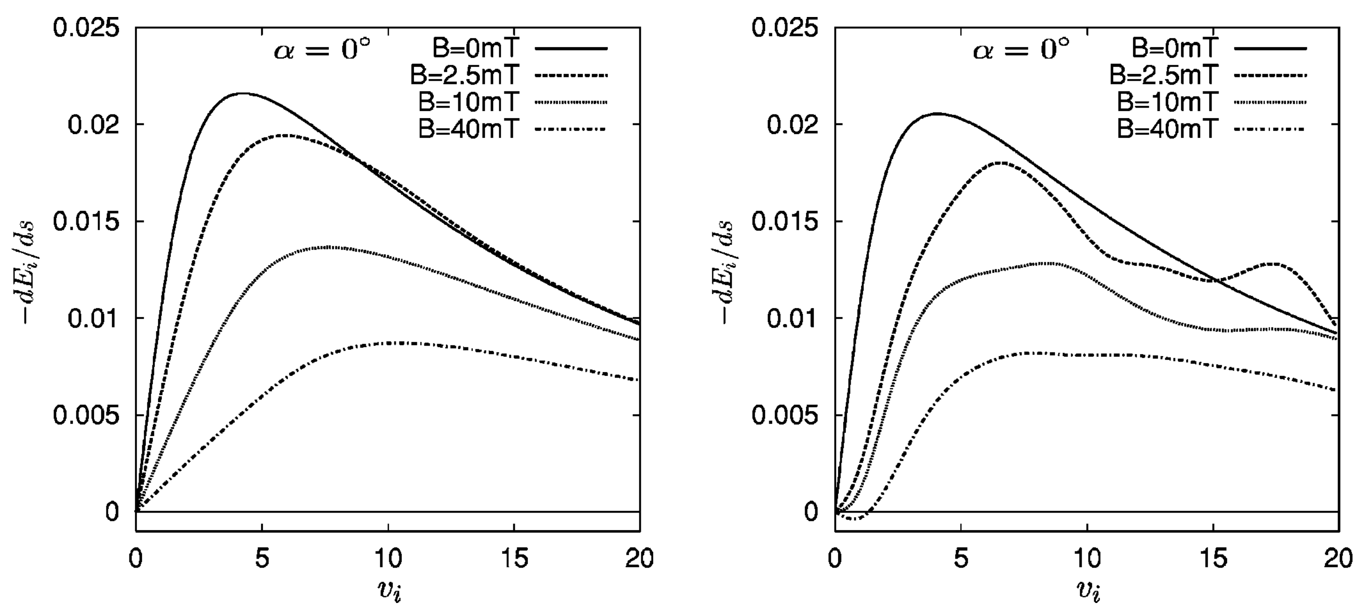

FIG. 8. Energy loss in units of $4\pi(Ze^2)^2 n_e/(m v_{\mathrm{th}\parallel}^2)$ as a function of the ion velocity v_i in units of the longitudinal thermal velocity $v_{\mathrm{th}\parallel}$ for $\alpha = 0°$ and various magnetic-field strengths. Other parameters from Table I. Left panel: present second-order theory; right panel: CTMC results [3–5].

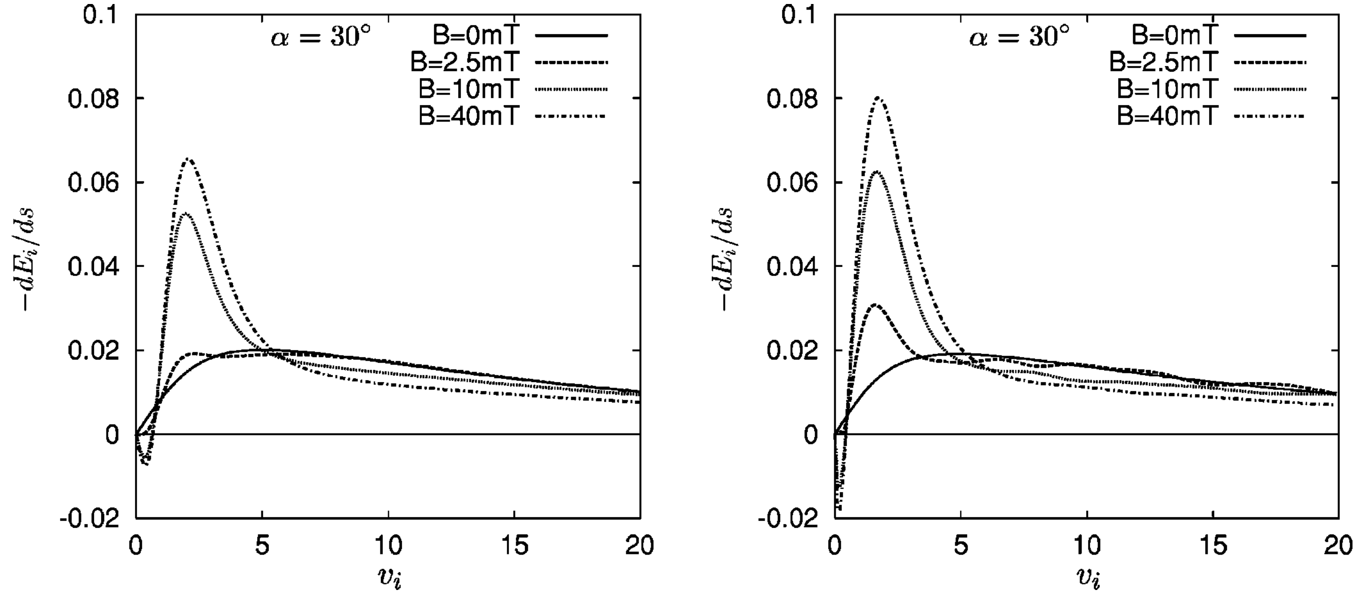

FIG. 9. Same as Fig. 8 for $\alpha = 30°$.

the tight helix contribution (4.12) yields a contribution to the energy loss, which increases both with α and B in the peak region of a few longitudinal thermal velocities. Thus the magnetic field does enhance the energy loss for transversely moving ions, but not as much as in the linear-response treatment [22], as discussed above. We mention that this enhancement is also overestimated in a kinetic model, where the transverse motion of the magnetized electrons is completely quenched and the asymptotic v_i^{-2} behavior of the energy loss prevails down to $v_i \rightarrow v_{\mathrm{th}\parallel}$ [11].

Of course, the results shown in Figs. 3–7 leave the role of the collectivity of the dielectric response open. In this connection the region of low ion velocities is of interest, but there the energy loss depends also very sensitively on the details of the velocity distribution of the electrons. As an example we show in Fig. 12, at fixed $T_\perp$, how the energy loss of protons in the present model increases for $T_\parallel \rightarrow 0$. This should be compared to the energy loss in the dielectric linear-response theory including collective excitations of electrons with a flat velocity distribution (curve from Fig. 8 of Ref. [9]). The effects resulting from the collectivity and the finite anisotropy of the velocity distribution obviously mask each other. The linear response with the more realistic distribution (4.16) is not known in the closed form, but the response of magnetized electrons due to the ion has been calculated by solving the Vlasov-Poisson equation numerically with a particle-in-cell (PIC) code by representing the electrons by test particles [15]. This involves neither a linearization nor a cutoff at small distances (or large wave numbers). As shown in Fig. 13 the PIC results, which include

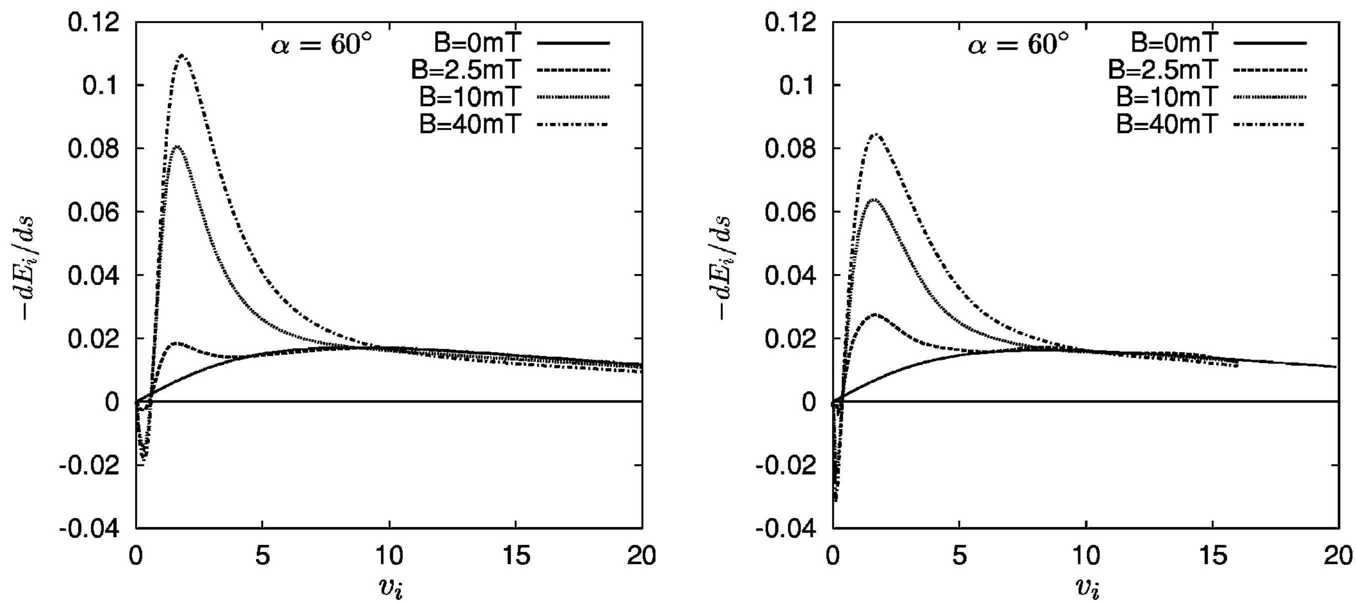

FIG. 10. Same as Fig. 8 for $\alpha = 60°$.

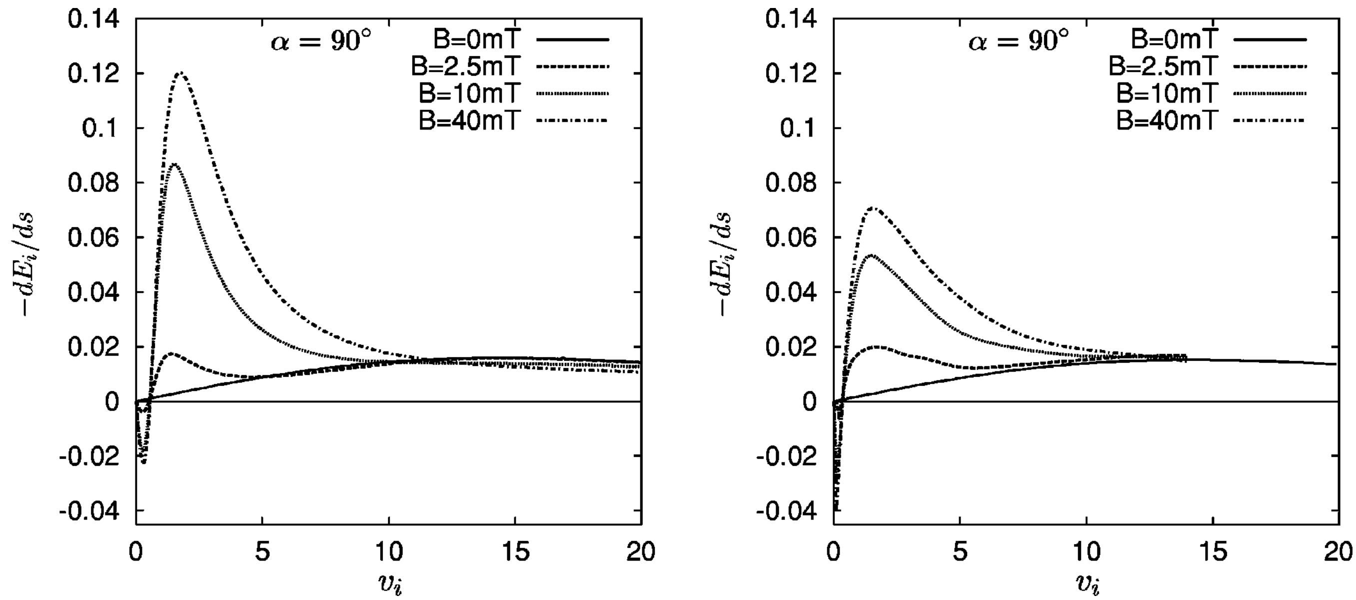

FIG. 11. Same as Fig. 8 for $\alpha = 90°$.

collectivity (points), compare better with the present binary collision model (solid curve) than with the linear-response result in the approximation (4.20) (dashed curve). This indicates that, at least, in the cases considered here the proper treatment of hard collisions, e.g., by Eq. (4.14), seems to be more important than to account for the collectivity in the response of the electrons.

V. SUMMARY AND CONCLUSIONS

We consider collisions between heavy charged particles (ions) and light charged particles (electrons) in a homogeneous external magnetic field. As this problem is not integrable, we seek an approximate solution in which the ion-electron interaction is treated as a perturbation. Three kinematical regimes have been identified: For a weak magnetic field the cyclotron radius R is larger than the distance of the closest approach r_0 and the unperturbed motion is rectilinear. For a strong magnetic field $R < r_0$, the unperturbed motion of the electron is helical. The guiding center approximation applies if the pitch δ of the helix is larger than r_0 (stretched helices) while tight helices with $\delta < r_0$ become important if the ion velocity has a component transverse to the magnetic field.

In the general two-body problem in a magnetic field only the total energy is conserved, but not the energies of the relative and c.m. motions separately. The uniform motion of the ion is only weakly perturbed by the collision with an electron, and there exists a conserved quantity K that involves the energy of relative motion and a magnetic term

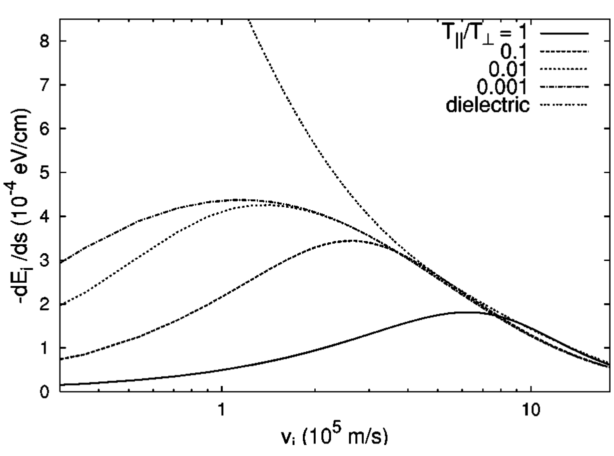

FIG. 12. Energy loss of protons in the present binary collision model for increasing asymmetry $T_\perp / T_\parallel$ of the velocity distribution and in the dielectric theory with a flat velocity distribution, $T_\parallel = 0$ [9]. Standard parameters of Ref. [9]: $n_e = 3\times 10^{14}\ \mathrm{m}^{-3}$, $k_B T_\perp = 1$ eV, $B = 0.07$ T.

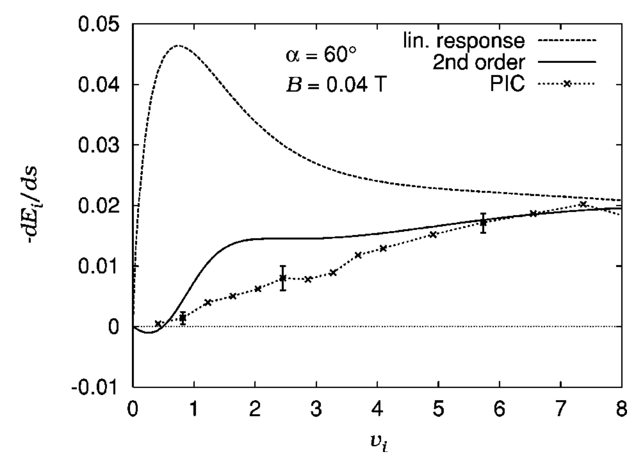

FIG. 13. Energy loss in units of $4\pi(Ze^2)^2 n_e/(m v_{\mathrm{th}\parallel}^2)$ as a function of the longitudinal thermal velocity $v_{\mathrm{th}\parallel}$ for $Z = 10$, $T_\parallel = 10$ K, $T_\perp = 1000$ K, $n_e = 1.55\times 10^{16}\ \mathrm{m}^{-3}$, $\alpha = 60°$ and $B = 0.04$ T. Solid curve: present second-order treatment; dashed curve: linear-response theory [22]. Points: PIC simulation of the nonlinear Vlasov-Poisson equation with error bars indicating the size of the statistical fluctuations [15].

associated with the influence of the $\vec{E}\times\vec{B}$ drift on the c.m. motion. It turns out that the second-order treatment of the Coulomb interaction is both necessary for a velocity transfer that does not vanish due to symmetry and sufficient to fulfill this conservation law. Hard collisions are taken into account by regularizing the integrand leading to the Coulomb logarithm at the lower boundary, a procedure that is exact for Rutherford scattering. For the interaction of an ion with a homogeneous electron beam the first-order terms vanish due to symmetry, but there remain the leading $O(Z^2)$ contributions.

As an application, the energy loss of ions in a magnetized electron plasma was considered. The influence of the magnetic field is ambiguous. The magnetic field reduces the loss of energy if the ion moves along the field lines, but it enhances the energy loss for a transverse motion of the ion. This enhancement in the present binary collision model agrees quite well with CTMC results for binary collisions, which do not involve any cutoff procedures for hard collisions as well with results from PIC simulations, which include the collective response of the electrons. Nevertheless, a more analytical theory of the energy loss of ions in a magnetized electron plasma, accounting for hard collisions, collective response, and a nonflattened electron-velocity distribution is highly desirable. For that purpose it has been proposed to correct the binary collision model by the difference between the dynamic and the static linear response, e.g., the error made in the replacement (4.20) [3,4,19]. Such calculations are in progress.

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APPENDIX: SOME TECHNIQUES FOR TREATING MULTIPLE INTEGRALS

In the calculation of the trajectory correction and the second-order velocity transfer there occur, frequently, multiple integrals that can be done by partial integration according to

$$\int_{-\infty}^{\tau} d\tau' \frac{dg}{d\tau'} \int_{-\infty}^{\tau'} d\tau'' f(\tau'') = g(\tau)\int_{-\infty}^{\tau} d\tau'' f(\tau'') - \int_{-\infty}^{\tau} d\tau' g(\tau') f(\tau'),$$

provided that

$$g(\tau)\int_{-\infty}^{\tau} d\tau'' f(\tau'') \to 0, \quad \tau\to\infty.$$

Double integrals, where both integrands are either even or odd, can be converted into products of simple integrals by symmetrization,

$$\int_{-\infty}^{\infty} d\tau f(\tau) \int_{-\infty}^{\tau} d\tau' g(\tau') \to \frac{1}{2}\int_{-\infty}^{\infty} d\tau f(\tau) \int_{-\infty}^{\infty} d\tau' g(\tau'),$$

$$\tau\to-\tau, \quad \tau'\to-\tau'.$$

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