Electromagnetically induced transparency and controlled group velocity in a multilevel system

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We analyze the interaction of N laser fields with a $(N+1)$-level quantum system. A general analytic expression for the steady-state linear susceptibility for a probe-laser field is obtained and we show that the system can exhibit multiple electromagnetically induced transparency, with at most $N-1$ transparency windows occurring in the system. The group velocity of the probe-laser pulse can also be controlled.

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For more than a decade there has been intensive interest in the phenomenon of electromagnetically induced transparency (EIT) [1–5] in three-level systems. In this phenomenon, an otherwise opaque medium is rendered transparent to a resonant probe-laser field that couples one of the transitions by the application of a strong, coupling laser field to the other transition. EIT has been observed in atoms [6], rare-earth-ion-doped crystals [7], and semiconductor quantum wells [8]. Potential applications of EIT range from lasing without inversion and enhanced nonlinear optics to quantum computation and communication [1–5]. EIT has also been shown to occur in four-level systems of various configurations [9–14] and some experimental results already exist for these systems [15–19]. Quite recently, McGloin *et al.* [20] have shown how EIT can also be extended to five- and six-level cascade systems.

In this paper, we analyze the interaction of a $(N+1)$-level quantum system in the configuration illustrated in Fig. 1 with N coherent laser fields. We assume that the system is initially prepared in a particular lower level and study the absorption and dispersion properties of a probe-laser field coupling this level to the upper level. To achieve this we use a density-matrix formalism and obtain a general analytical expression for the linear susceptibility of the probe-laser field. We then use this result to show that the system can become transparent to the probe-laser field at $N-1$ different frequencies. In addition, the group velocity of the probe-laser pulse is analyzed. We show that the group velocity can obtain $N-1$ different values at transparency and can be controlled by the coupling laser fields.

Denoting the excited state by $|0\rangle$ and the lower levels by $|1\rangle$, $|2\rangle, \ldots, |N\rangle$ and assuming that each laser pulse drives only one transition, the Hamiltonian of this system in the interaction picture and in the rotating wave and dipole approximations is given by (we use units such that $\hbar=1$)

$$H=\sum_{n=1}^{N} \Omega_n e^{-i\delta_n t}|n\rangle\langle 0| + \text{H.c.} \tag{1}$$

Here, $\Omega_n = -\vec{\mu}_{n0}\cdot\hat{\varepsilon}_n\mathcal{E}_n$ is the Rabi frequency of the transition $|n\rangle \leftrightarrow |0\rangle$, with $\vec{\mu}_{n0}$ being the associated dipole transition-matrix element. Also, $\hat{\varepsilon}_n$ is the polarization vector and $\mathcal{E}_n$ the electric-field amplitude of each laser pulse. Finally, $\delta_n=\omega_0-\omega_n-\overline{\omega}_n$ is the laser field detuning from resonance with the transition $|0\rangle \leftrightarrow |n\rangle$, with the energies of the nth lower level and upper level, respectively, being ω_n and ω_0 and the angular frequency of the laser field being $\overline{\omega}_n$.

We will analyze the system using a density-matrix approach. From the Liouville equation we obtain the following equations for the density-matrix elements:

$$i\dot{\rho}_{00}(t)=\sum_{n=1}^{N}[-i\Gamma_{0n}\rho_{00}(t)+\Omega_n^*\rho_{n0}(t)-\Omega_n\rho_{0n}(t)], \tag{2}$$

$$i\dot{\rho}_{nn}(t)=-i\sum_{m}\Gamma_{nm}\rho_{nn}(t)+i\sum_{k}\Gamma_{kn}\rho_{kk}(t)+\Omega_n\rho_{0n}(t) - \Omega_n^*\rho_{n0}(t), \quad n=1-N, \tag{3}$$

$$i\dot{\rho}_{n0}(t)=-(\delta_n+i\gamma_{n0})\rho_{n0}(t)+\Omega_n\rho_{00}(t) - \sum_{m=1}^{N}\Omega_m\rho_{nm}(t), \quad n=1-N, \tag{4}$$

$$i\dot{\rho}_{nm}(t)=(\delta_m-\delta_n-i\gamma_{nm})\rho_{nm}(t)+\Omega_n\rho_{0m}(t) - \Omega_m^*\rho_{n0}(t), \quad n\neq m, n,m=1-N, \tag{5}$$

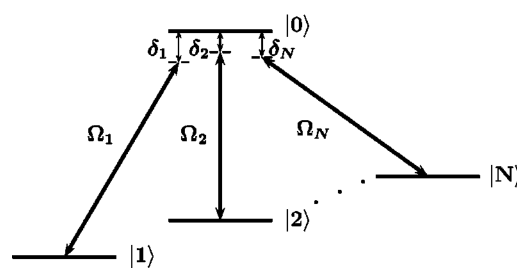

FIG. 1. Schematic diagram of studied system. It consists of N lower levels and a single upper level. The lower states are coupled near resonantly to the excited state by N laser fields.

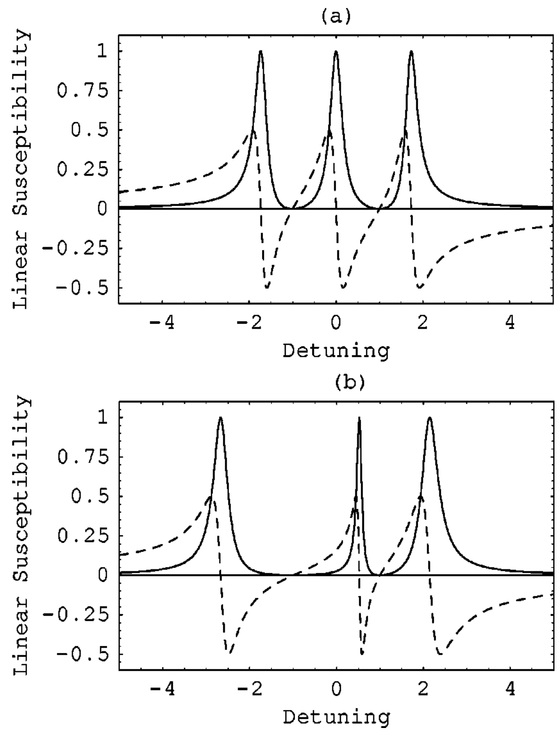

FIG. 2. The absorption (solid curves) and dispersion spectra (dashed curves), in arbitrary units, for a four-level system ($N=3$) with parameters $\delta_2=-1$, $\delta_3=1$, $\gamma_{1n}=0$ with $n=2,3$ and (a) $\Omega_2=\Omega_3=1$, (b) $\Omega_2=1$, $\Omega_3=2$. All parameters are in units of γ_{10}.

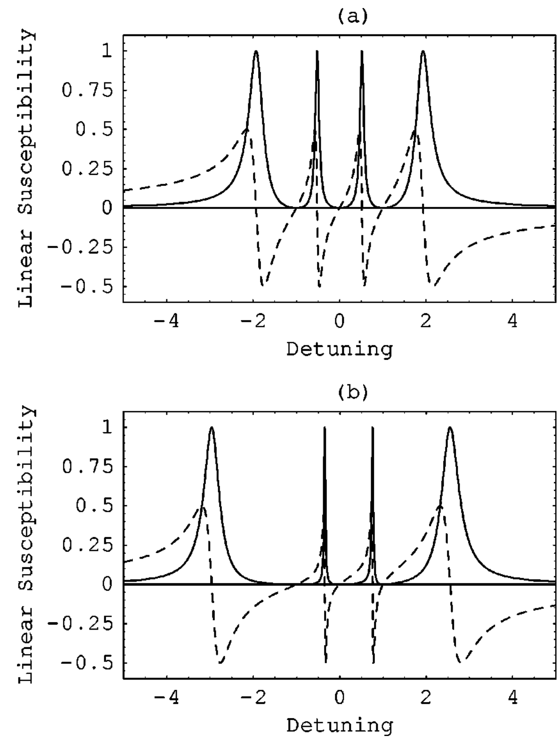

FIG. 3. The absorption (solid curves) and dispersion spectra (dashed curves), in arbitrary units, for a five-level system ($N=4$) with parameters $\delta_2=-1$, $\delta_3=0$, $\delta_4=1$, $\gamma_{1n}=0$ with $n=2-4$ and (a) $\Omega_2=\Omega_3=\Omega_4=1$, (b) $\Omega_2=1$, $\Omega_3=\sqrt{2}$, $\Omega_4=2$. All parameters are in units of γ_{10}.

with $\sum_{n=0}^{N}\rho_{nn}(t)=1$ and $\rho_{nm}(t)=\rho_{mn}^{*}(t)$. We have assumed a closed system, i.e., there is no decay to levels outside the $(N+1)$-level manifold we study. We denote by Γ_{nm} the radiative decay rate of the populations from level $|n\rangle$ to level $|m\rangle$ and by γ_{nm} the coherence decay rate between states $|n\rangle$ and $|m\rangle$, with

$$\gamma_{nm}=\frac{1}{2}\sum_{k}\Gamma_{nk}+\frac{1}{2}\sum_{l}\Gamma_{ml}+\gamma'_{nm}, \tag{6}$$

where k,l denote the states $|k\rangle$ and $|l\rangle$ that states $|n\rangle$ and $|m\rangle$, respectively, decay to. Also, γ'_{nm} describes the decay due to inhomogeneous broadening in this medium. Examples of inhomogeneous broadening include collisions in atomic and molecular systems or electron-electron scattering, interface roughness, and phonon scattering in semiconductor quantum well systems. We will not consider the effects of Doppler broadening in the system in this medium.

We assume that the system is in its ground state $|1\rangle$ for time $t=0$, i.e., $\rho_{11}(0)=1$. In order to investigate the absorption and dispersion properties of a weak probe-laser field coupling states $|1\rangle$ and $|0\rangle$ we calculate the steady-state linear susceptibility, with absorption (dispersion) determined by the imaginary (real) part of the susceptibility. In our case the steady-state linear susceptibility can be expressed as

$$\chi(\delta_1)=-\frac{4\pi\mathcal{N}|\vec{\mu}_{10}|^2}{\Omega_1}\rho_{10}(t\to\infty), \tag{7}$$

with $\mathcal{N}$ being the medium density. The coherence $\rho_{10}(t)$ is obtained by solving Eqs. (2)–(5) using perturbation theory. We assume that the probe laser is weak so that $\rho_{00}(t)\approx 1$ for all times. We apply this approximation to Eqs. (2)—(5), take the steady-state limit and solve for ρ_{10} to first order in Ω_1. The linear susceptibility then reads

$$\chi(\delta_1)=4\pi\mathcal{N}|\vec{\mu}_{10}|^2\frac{1}{\delta_1+i\gamma_{10}-\sum_{n=2}^{N}|\Omega_n|^2/(\delta_1-\delta_n+i\gamma_{1n})}. \tag{8}$$

In the case that $\gamma_{1n}=0$, for $n=2-N$, which is a condition that we impose for the rest of this paper, the susceptibility goes to zero when $\delta_1=\delta_n$, for $n=2-N$. Therefore, if all the detunings are different then this $(N+1)$-level system will become transparent at $N-1$ different frequencies of the probe field.

We now assume that $L-1$ of the detunings δ_n are equal to δ, with $2<L<N$, and the remaining $N-L$ are different than δ. (To simplify the notation we will take $\delta_2=\delta_3=\cdots=\delta_L=\delta$). The susceptibility then becomes

$$\chi(\delta_1)=4\pi\mathcal{N}|\vec{\mu}_{10}|^2\frac{1}{\delta_1+i\gamma_{10}-\sum_{n=L+1}^{N}|\Omega_n|^2/(\delta_1-\delta_n)-\sum_{n=2}^{L}|\Omega_n|^2/(\delta_1-\delta)}. \quad (9)$$

Therefore, there are $N-L+1$ transparency windows in the $(N+1)$-level system. Finally, if all the detunings δ_n with $n\neq 1$ are equal to δ then the susceptibility reduces to

$$\chi(\delta_1)=4\pi\mathcal{N}|\vec{\mu}_{10}|^2\frac{(\delta_1-\delta)}{(\delta_1+i\gamma_{10})(\delta_1-\delta)-\sum_{n=2}^{N}|\Omega_n|^2}, \quad (10)$$

which means that the susceptibility reduces to a form similar to that of a three-level Λ-type system [1–5] with the only difference being that the magnitude squared of the Rabi frequency of the coupling laser field is now replaced with the sum of the magnitude squared of the Rabi frequencies of the coupling fields.

The value of the group velocity of the probe-laser pulse is also of interest, and is given by [21]

$$v_g=c/\left[1+\frac{1}{2}\mathrm{Re}(\chi)+\frac{\overline{\omega}_1}{2}\frac{\partial\mathrm{Re}(\chi)}{\partial\overline{\omega}_1}\right], \quad (11)$$

with the derivative of the real part of the susceptibility being evaluated at the carrier frequency of the probe-laser field. When none of the detunings are the same the group velocity at the nth transparency window approximates $v_g\approx c|\Omega_n|^2/2\pi\overline{\omega}_1\mathcal{N}|\vec{\mu}_{10}|^2$, $n=2-N$, therefore the group velocity of the probe-laser pulse may be significantly reduced, similar to a Λ-type atom [21–27]. Now, however, the group velocity can be controlled via the intensity of the coupling laser fields and the probe-laser field can propagate with $N-1$ different group velocities in the medium.

When $L-1$ of the detunings are equal, in the manifold of states with unequal detunings the group velocity of the pulse approximates $v_g\approx c|\Omega_n|^2/2\pi\overline{\omega}_1\mathcal{N}|\vec{\mu}_{10}|^2$, $n=(L+1)-N$ near the nth transparency window and $v_g\approx c\Sigma_{n=2}^{L}|\Omega_n|^2/2\pi\overline{\omega}_1\mathcal{N}|\vec{\mu}_{10}|^2$ around detuning δ. Finally if all detunings are equal, then the group velocity approximates $v_g\approx c\Sigma_{n=2}^{N}|\Omega_n|^2/2\pi\overline{\omega}_1\mathcal{N}|\vec{\mu}_{10}|^2$ around the single transparency window.

We will now give a few examples of absorption and dispersion spectra that could occur in $(N+1)$-level systems. In

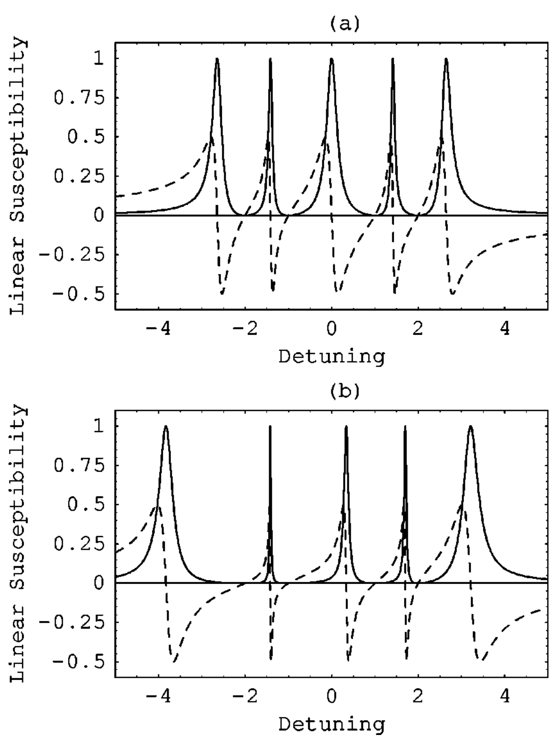

FIG. 4. The absorption (solid curves) and dispersion spectra (dashed curves), in arbitrary units, for a six-level system ($N=5$) with parameters $\delta_2=-2$, $\delta_3=-1$, $\delta_4=1$, $\delta_5=2$, $\gamma_{1n}=0$ with $n=2-5$ and (a) $\Omega_2=\Omega_3=\Omega_4=\Omega_5=1$, (b) $\Omega_2=1$, $\Omega_3=\sqrt{2}$, $\Omega_4=\sqrt{3}$, $\Omega_5=2$. All parameters are in units of γ_{10}.

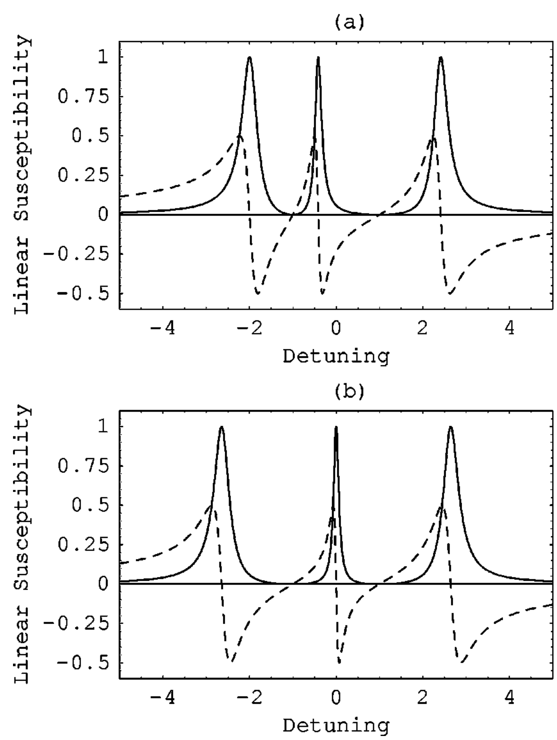

FIG. 5. The absorption (solid curves) and dispersion spectra (dashed curves), in arbitrary units, for a six-level system ($N=5$) with parameters $\delta_2=\delta_3=\delta_4=\delta=-1$, $\delta_5=1$, $\gamma_{1n}=0$ with $n=2-5$ and (a) $\Omega_2=\Omega_3=\Omega_4=\Omega_5=1$, (b) $\Omega_2=\Omega_3=\Omega_4=1$, $\Omega_5=\sqrt{3}$. All parameters are in units of γ_{10}.

Fig. 2 we plot the linear absorption and dispersion spectra for a four-level system. The absorption and dispersion are either symmetric or asymmetric and their shapes depend critically on the system parameters. The double transparency and the control of group velocity is clearly demonstrated here. These spectra are similar to those obtained in Ref. [14]. We also plot spectra for the case of five- (Fig. 3) and six- (Fig. 4) level systems. In this case three and four transparency windows occur, respectively, for the five- and six-level system, and significant control of the group velocity (as can be seen from the slopes of the dispersion spectra) is possible. Finally, in Fig. 5 we give an example with three equal detunings in a six-level system. In this case only two transparency windows appear in the spectrum and the group velocity can obtain either different [Fig. 5(a)] or same [Fig. 5(b)] value depending on the Rabi frequencies of the coupling laser fields.

In summary, we have studied the interaction of N laser fields with a $(N+1)$-level quantum system. A general analytic expression for the steady state linear susceptibility for a probe-laser field has been obtained. We have shown that the system can exhibit multiple transparency windows. At most $N-1$ transparency windows can occur and, in general, the group velocity of the probe-laser pulse can obtain at most $N-1$ different values at transparency. These group velocities can be controlled by varying the Rabi frequencies of the coupling laser fields.

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