Engineering entanglement between two cavity modes

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We present a scheme for the generation of entanglement between different modes of radiation field inside high-Q superconducting cavities. Our scheme is based on the interaction of a three-level atom with the cavity field for precalculated interaction times with each mode. This work enables us to generate a complete set of Bell basis states and a quantum entangled Greenberger-Horn-Zeilinger state.

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Quantum-information theory combines quantum mechanics with classical information theory. The Einstein-Podolsky-Rosen– (EPR-)Bell correlations, and quantum entanglement [1,2] in general, form the essential ingredients that distinguish quantum-information theory from its classical counterpart. An entangled state of two or more quantum systems is a state that cannot be factorized [3]. The most familiar example of an entangled state is the Bohm state $(|\uparrow_A,\downarrow_B\rangle + |\downarrow_A,\uparrow_B\rangle)/\sqrt{2}$, which represents the state of two spin-1/2 particles decaying from a spin-zero parent [4]. The two particles are correlated, their spins are always antiparallel, and they remain that way no matter what is the separation between them. Hence entangled systems demonstrate nonlocal quantum effects.

Entanglement is one of the main pillars of quantum compution [5], quantum cryptography [6], quantum teleportation [7], and many other applications of quantum-information technology [8–13]; therefore, generation of entangled states and its further applications are of immense importance. Several schemes have been proposed for the generation of entangled states in atoms, ions, and photons [14–22].

The first evidence of entangled state generation of two cavity fields was seen in the teleportation procedure presented by Davidovich *et al.* [23]. This entangled state of two cavity fields of the same mode occurs as an intermediate step in the teleportation procedure and it requires the presence of one photon in either of the two cavities. The generation of a Greenberger-Horn-Zeilinger (GHZ) state between two cavities has been suggested as well [24]. An entangled state in which the number of photons distributed between two cavities is fixed has also been proposed [25]. All these schemes provide entanglement between radiation fields of the same mode in two cavities. Recently, Ref. [26] reported the existence of a Bohm entangled state between different modes of the radiation field.

In this paper we propose schemes to generate entanglement between different modes of the electromagnetic field and engineer an EPR-Bell basis and a GHZ entangled state [27]. We propagate a three-level atom through a cavity which contains initially field modes in vacuum. We may express the three levels as $|a\rangle$, $|b\rangle$, and $|c\rangle$ with their eigenenergies E_a, E_b, and E_c, as shown in Fig. 1. The dipole transition between the upper two levels $|a\rangle$ and $|b\rangle$ of the atom is forbidden, whereas transitions from the two upper levels to the lower level $|c\rangle$ are allowed. We consider that the frequencies ω_A and ω_B of the two modes A and B, respectively, of the cavity field are in resonance with the transition frequencies, such that $\omega_A=(E_a-E_c)/\hbar$ and $\omega_B=(E_b-E_c)/\hbar$. With the help of a Ramsey field we prepare the upper two levels of the atom in linear superposition before it enters the cavity field. We may express the initial state of the system as

$$|\psi(0)\rangle=\frac{1}{\sqrt{2}}[|a\rangle+e^{i\phi}|b\rangle]|0_A,0_B\rangle, \tag{1}$$

where ϕ is the relative phase between two atomic states.

We write the interaction picture Hamiltonian in the dipole and rotating wave approximation as

$$H=\hbar g_1(a|a\rangle\langle c|+a^\dagger|c\rangle\langle a|)+\hbar g_2(b|b\rangle\langle c|+b^\dagger|c\rangle\langle b|) \tag{2}$$

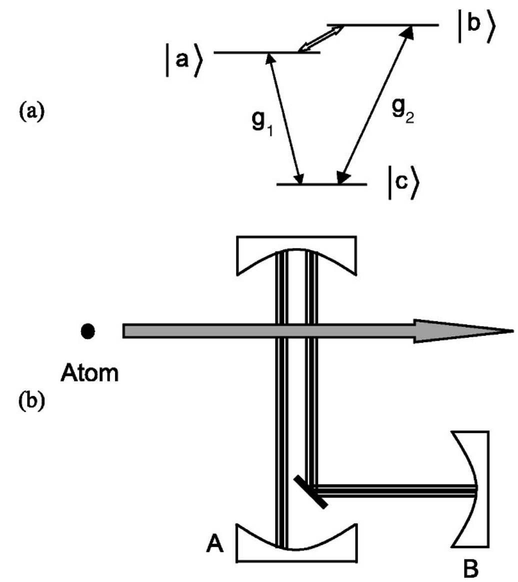

FIG. 1. Schematic diagram for the generation of an entangled state between two cavity modes. We prepare a three-level atom with superposition of the upper two levels by a Ramsey field and let it pass through the cavity carrying two different modes of radiation.

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where g_1 and g_2 are vacuum Rabi frequencies of the two modes while a ($a^\dagger$) and b ($b^\dagger$) are the annihilation (creation) operators of the two cavity modes A and B, respectively. The atom-field state vector after the atom field interaction can be written as

$$|\psi_{at}^{(t)}(A,B)\rangle = C_{a,0,0}|a,0,0\rangle + C_{b,0,0}|b,0,0\rangle + C_{c,1,0}|c,1,0\rangle + C_{c,0,1}|c,0,1\rangle \qquad (3)$$

where $C_{a,n,m}$, $C_{b,n,m}$, and $C_{c,n,m}$ represent the time dependent probability amplitudes for the atom to be in the states $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively, with n the number of photons in mode A and m photons in mode B. The rate equations of these probability amplitudes can be obtained by the Schrödinger equation as

$$\frac{d}{dt}C_{a,0,0} = -ig_1 C_{c,1,0}, \qquad (4)$$

$$\frac{d}{dt}C_{c,1,0} = -ig_1 C_{a,0,0}, \qquad (5)$$

$$\frac{d}{dt}C_{b,0,0} = -ig_2 C_{c,0,1}, \qquad (6)$$

$$\frac{d}{dt}C_{c,0,1} = -ig_2 C_{b,0,0}. \qquad (7)$$

Solving these differential equations in the presence of the initial conditions mentioned in Eq. (1), we find the atom-field entangled state as

$$|\psi^{(t)}(A,B)\rangle = \frac{1}{\sqrt{2}}[\cos(g_1 t)|a,0,0\rangle - i\sin(g_1 t)|c,1,0\rangle + e^{i\phi}\cos(g_2 t)|b,0,0\rangle - ie^{i\phi}\sin(g_2 t)|c,0,1\rangle]. \qquad (8)$$

For the generation of a maximally entangled field state between two cavity modes such that if one mode has one photon then the other will be in vacuum, the atom after its interaction with the cavity fields is required to be detected in the ground state $|c\rangle$. This leads to the condition that the probability amplitudes of the states $|c,1,0\rangle$ and $|c,0,1\rangle$ are equal, i.e.,

$$\sin(g_1 t) = \sin(g_2 t). \qquad (9)$$

The total probability of detecting the atom in the ground state is determined as

$$P_c = \frac{1}{2}\{\sin^2(g_1 t) + \sin^2(g_2 t)\}. \qquad (10)$$

This probability becomes maximum when the time of interaction of the atom with mode A and mode B is $m\pi/2g_1$ and $n\pi/2g_2$, respectively. Here, m and n are odd integer numbers. Hence, in order to generate two-mode entanglement the time of interaction of the atom with the cavity is an odd integer multiple of one-half of the Rabi cycle. This ensures that the cavity will obtain one photon in either of the two modes when the atom is detected in the ground state after its propagation through the cavity.

The interaction times of the atom with the two modes of the cavity field will be different because of the different coupling constants of each mode of the radiation field. These interaction times of the atom in the cavity can be controlled by using a velocity selector before the cavity and then applying Stark field adjustment so that the atom becomes resonant with the cavity field modes only for the suggested amount of time in each mode of the cavity field [23].

Hence the atom passing in the superposition of levels $|a\rangle$ and $|b\rangle$ interacts with the two cavity modes A and B for interaction times $m\pi/2g_1$ and $n\pi/2g_2$, respectively. As a result the atom leaves the cavity in the ground state and develops an entangled state between the two cavity modes, viz.,

$$|\psi(A,B)\rangle = \frac{-i}{\sqrt{2}}[|0_A,1_B\rangle + e^{i\phi}|1_A,0_B\rangle]. \qquad (11)$$

We may adjust the interaction time of the atoms with the cavity fields, such that

$$|\psi(A,B)\rangle = \frac{-i}{\sqrt{2}}[|0_A,1_B\rangle - e^{i\phi}|1_A,0_B\rangle]. \qquad (12)$$

In order to generate the other two Bell bases we prepare the three-level atom in level $|a\rangle$ without considering the superposition of the upper two levels $|a\rangle$ and $|b\rangle$. We let the excited atom pass through two cavities successively, which are prepared initially in the vacuum state. The transition from level $|a\rangle$ to $|c\rangle$ is again in resonance with the cavity mode A, whereas the transition from $|b\rangle$ to $|c\rangle$ is in resonance with the cavity mode B.

We adjust the interaction time of the atom with the first cavity field such that it experiences a $\pi/2$ pulse. Hence, there occurs equal probability of finding the atom in ground state $|c\rangle$, after contributing one photon to the cavity mode A, and of finding the atom in the excited state $|a\rangle$, leaving the cavity mode in the vacuum state. As a result we find an atom-field entanglement such that

$$|\psi_{at}(A,B)\rangle = \frac{1}{\sqrt{2}}[|a,0_A\rangle + |c,1_A\rangle]\otimes|0_B\rangle. \qquad (13)$$

Before the atom enters the next cavity, we apply a laser field resonant with the atomic transition from $|b\rangle$ to $|c\rangle$. The width of the beam is adjusted such that the atom exiting from the first cavity field in the ground state $|c\rangle$ is pumped to the excited state $|b\rangle$ with unit probability. However, if the exiting atom is in excited state $|a\rangle$ after interacting with the cavity mode A, the laser field will provide no excitation to the atom.

After passing through the laser field, the atom interacts with the cavity mode B, which is initially in the vacuum state. The interaction time of the atom with the field is adjusted such that the atom in the excited state $|b\rangle$ will be detected in the ground state $|c\rangle$ with unit probability, adding a photon to the cavity mode B. However, if the atom enters the cavity in the excited state $|a\rangle$, it will contribute no photon and will exit in the same atomic state, leaving the cavity mode B in the vacuum state. Therefore, we find the entanglement of the two modes of the radiation field with the atomic states as

$$|\psi_{at}(A,B)\rangle=\frac{1}{\sqrt{2}}[|a,0_A,0_B\rangle+|c,1_A,1_B\rangle]. \tag{14}$$

For a time larger than the lifetime of the propagating atom, the three-party entangled state will collapse to a two-party entangled state, given as

$$|\psi(A,B)\rangle=\frac{1}{\sqrt{2}}[|0_A,0_B\rangle+|1_A,1_B\rangle], \tag{15}$$

which describes the entanglement of the two cavity field modes.

The interaction times of the atoms with the cavity field mode A, laser field, and cavity field mode B are found to be $m\pi/4g_1$, π/Ω, and $n\pi/2g_2$, respectively, where m and n are odd integers. Here Ω is the Rabi frequency of the laser field which interacts with the atom between the two cavities. If the relative difference of interaction times of atoms with the two cavities is taken to be π, we may generate the entangled state

$$|\psi(A,B)\rangle=\frac{1}{\sqrt{2}}[|0_A,0_B\rangle-|1_A,1_B\rangle]. \tag{16}$$

Hence, we can obtain the complete set of Bell bases by controlling the interaction times of the atom with the cavities in both the schemes.

In order to generate a multimode entangled state we may repeat the same process again as suggested above. We provide another laser pulse which is in resonance with the transition from level $|c\rangle$ to another higher level $|b_1\rangle$. The atomic interaction with the laser pulse occurs for a time $\pi/2\Omega_1$, where Ω_1 is the Rabi frequency of the laser pulse. This pulse causes the atom to be in the excited state $|b_1\rangle$ with unit probability. The atom in the excited state $|b_1\rangle$ then interacts with the cavity mode B_1, initially in the vacuum state, for an interaction time $m\pi/2g_3$, where g_3 is the vacuum Rabi frequency of the B_1 mode of the cavity field. Hence if the atom is in the ground state $|c\rangle$ after interacting with the second cavity then it will contribute one photon to the cavity mode B_1 after this interaction, whereas if it is in the excited state $|a\rangle$ after the second cavity then the field will not interact with it because of the detuning. By repeating the process using various different excited states we may develop a GHZ entangled state [27] as

$$|\psi(A,B,\ldots,N)\rangle=\frac{1}{\sqrt{2}}[|0_A,0_B,\ldots,0_N\rangle+|1_A,1_B,\ldots,1_N\rangle], \tag{17}$$

which indicates entanglement between N modes.

In order to measure any component of the Bell basis we may make use of a quantum catalog [28]. We let an atom interact with two cavities, one containing a known standing wave field and the other a known Bell basis. We consider the interacting atom as far detuned from the cavity fields. We store atomic interference pattern corresponding to each Bell basis, therefore developing a catalog of interference patterns. Later, we may measure any unknown Bell basis by comparing its interference pattern with the interference patterns stored in the quantum catalog.

In order to realize our suggested scheme in a laboratory experiment within the microwave region, we may consider slow Rb atoms in higher Rydberg states, which have lifetimes of the order of a few milliseconds [23]. These slow atoms, initially pumped to a high Rydberg state, pass through a high-Q superconducting cavity of dimension of a few centimeters with a velocity of around 400 m/s [15,23,26]. The interaction times of the atom with cavities come out to be of the order of a few tens of microseconds, which is far smaller than the cavity lifetime. High-Q cavities of lifetime of the order of milliseconds are being used in recent experiments [26]. The interaction time of the atom with different cavities can be controlled by using a velocity selector and applying Stark field adjustment in different cavities in order to make the atom resonant with the field for the right amount of time [23]. The atomic decay rates, interaction times, and cavity lifetime ensure that the atom does not decay spontaneously. As this entanglement remains only for the cavity lifetime period, any application regarding this entangled state should be accomplished during this period.

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