Generation of entangled photon states by using linear optical elements

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We present a scheme to generate the polarization-entangled two-photon state $1/\sqrt{2}(|H\rangle|V\rangle+|V\rangle|H\rangle)$, which is of much interest in the field of quantum information processing. Furthermore, we demonstrate the capability of this concept in respect of a generalization to entangle N-photon states for interferometry and lithography. This scheme requires single-photon sources, linear optical elements, and a multifold coincidence detection.

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The generation of entangled quantum states plays a prominent role in quantum optics. An experimental realization in this context can be achieved with trapped ions [1], cavity QED [2], or Bose-Einstein condensation [3]. Experiments with polarization-entangled photons opened a whole field of research. Polarization entanglement was used to test the Bell inequality [4] and to implement quantum information protocols such as quantum teleportation [5], quantum dense coding [6], and quantum cryptography [7]. The experimental generation of GHZ states of three or four photons were reported [8]. These polarization-entangled photon states were only being produced randomly, since there was no way of demonstrating that polarization entanglement was generated without measuring and destroying the outgoing state [9]. Some quantum protocols such as error correction were designed for maximally entangled quantum states without random entanglement [10]. Thus, a photon source is needed, which produces maximally polarization-entangled outgoing photons. Remarkably, an efficient quantum computation with linear optics was put forward [11]. Such schemes can be used directly to generate polarization-entangled quantum states. It was suggested to arrange an array of beam splitters in order to implement a basic nondeterministic gate [11]. A feasible linear optical scheme [12] was proposed to generate polarization entanglement by making use of single-photon quantum nondemolition measurements based on an atom-cavity system [13]. There is a potential interest in generating entanglement of optical modes with greater photon numbers. Entangled N-photon states of the form

$$\Psi=\frac{1}{\sqrt{2}}(|0,N\rangle+|N,0\rangle) \tag{1}$$

are of much interest in respect of the phase sensitivity in a two-mode interferometer [14]. They should allow a measurement at the Heisenberg uncertainty limit [15]. Recently it was shown that such states allow subdiffraction limited lithography [16]. In the case of $N=2$ the entangled N-photon state (1) can be generated easily by using linear optical elements. For higher values of N, a scheme was proposed by using nonlinear media [17]. It was assumed that the generation of quantum states of this type is possible by using linear optical schemes. Recently, the first linear optical scheme was proposed to entangle four-photon states [18]. Furthermore, it was shown that any two-mode photon states can, in principle, be generated using a linear optical scheme based on nondetection [19,20]. Kok *et al.* presented a linear optical scheme to generate the state (1), which is based on a N-fold photon coincidence detection.

At first we show that two-photon polarization entanglement can be generated with our scheme, if the initial state is $|H\rangle_1|H\rangle_2$. We use the abbreviations H and V to denote the horizontal and vertical linear polarization of each photon. In Fig. 1 the required symmetric experimental setup of our protocol is shown. Two optical input modes are entangled by a beam splitter BS. In each of these entangled branches a polarization rotator, a single-photon injection block and another rotator is inserted. Finally these branches pass a polarization beam splitter (PBS). This setup looks like an optical interferometer with the possibility to vary the angle of the rotators continuously, to inject a photon and to get classical information with the aid of the detectors D_1 and D_2.

In the following the performance of this arrangement of linear optical devices will be analyzed in detail. The action of the symmetric BS can be described by the unitary operator

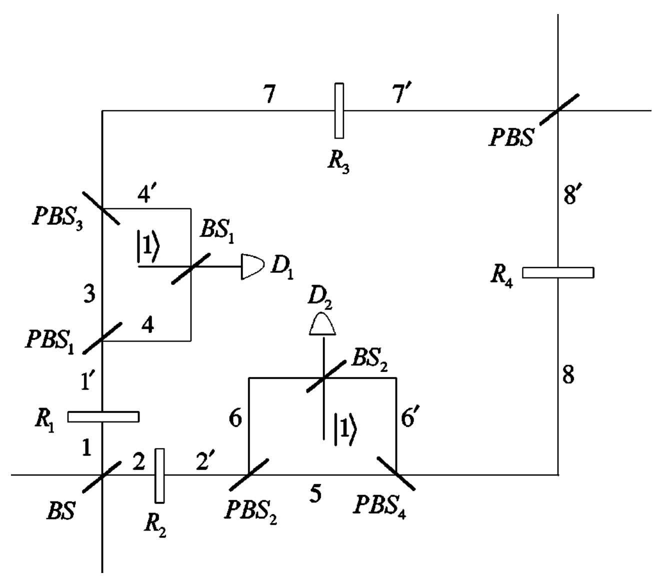

FIG. 1. This figure shows the required experimental setup to generate polarization-entangled two-photon states. The polarization beam splitters [PBS, PBS_i $(i=1,2,3,4)$] transmit H photons and reflect V Photons. Four polarization rotators R_i $(i=1,2,3,4)$ are required. BS and BS_i $(i=1,2)$ denote symmetric beam splitters. The scheme requires three photon-number detectors D_1 and D_2.

$$U_{bs}=\exp\left[\frac{\pi}{4}(a_{1H}a_{2H}^{\dagger}-a_{1H}^{\dagger}a_{2H})\right]. \tag{2}$$

After passing the BS the initial state is transferred into the other mode,

$$\Psi_1=\frac{1}{\sqrt{2}}(|2H\rangle_1|0\rangle_2-|0\rangle_1|2H\rangle_2). \tag{3}$$

The two output modes of the beam splitter pass through two polarization rotators with the rotation angle θ. These linear optical devices can be described by the transformation

$$a_H^{\dagger}\to(\cos\theta)a_H^{\dagger}+(\sin\theta)a_V^{\dagger},$$
$$a_V^{\dagger}\to(\cos\theta)a_V^{\dagger}-(\sin\theta)a_H^{\dagger}. \tag{4}$$

In order to obtain a maximum efficiency of our scheme the rotation angles θ are calibrated to fulfill $\cos\theta=\sqrt{1/3}$. The two-photon state transforms to

$$\Psi_2=\frac{\sqrt{2}}{3}[0.5|2H\rangle_{1'}|0\rangle_{2'}+|HV\rangle_{1'}|0\rangle_{2'}+|2V\rangle_{1'}|0\rangle_{2'}$$
$$-0.5|0\rangle_{1'}|2H\rangle_{2'}-|0\rangle_{1'}|HV\rangle_{2'}-|0\rangle_{1'}|2V\rangle_{2'}]. \tag{5}$$

The two output modes of the rotators pass two polarization beam splitters: PBS_1 and PBS_2. Since the polarization beam splitters transmit only the horizontal polarization component and reflect the vertical component, the state evolves into

$$\Psi_3=\frac{\sqrt{2}}{3}[0.5|2H\rangle_3+|H\rangle_3|V\rangle_4+|2V\rangle_4-0.5|2H\rangle_5$$
$$-|H\rangle_5|V\rangle_6-|2V\rangle_6]. \tag{6}$$

The scheme uses the output modes 4 and 6 to couple separately a single-photon source as the second input port of the symmetric beam splitters BS_1 and BS_2. We assume that these single-photon sources are in the single-photon state $|1\rangle$. If the twofold coincidence detection results in one photon in each detector D_1 and D_2 the quantum state is projected into

$$\Psi_4=\tfrac{1}{2}[|2H\rangle_3-|2V\rangle_{4'}-|2H\rangle_5+|2V\rangle_{6'}]. \tag{7}$$

Conditioned on this outcome the polarization beam splitters PBS_3 and PBS_4 transform the two-photon quantum state into

$$\Psi_5=\tfrac{1}{2}[|2H\rangle_7-|2V\rangle_7-|2H\rangle_8+|2V\rangle_8]. \tag{8}$$

These two output modes 7 and 8 pass through two polarization rotators which are calibrated to fulfill: $\theta_2=\pi/4$. Finally the quantum state

$$\Psi_6=\frac{1}{\sqrt{2}}[|HV\rangle_{7'}-|HV\rangle_{8'}] \tag{9}$$

is incident on a polarization beam splitter in order to obtain the two-photon polarization-entangled quantum state

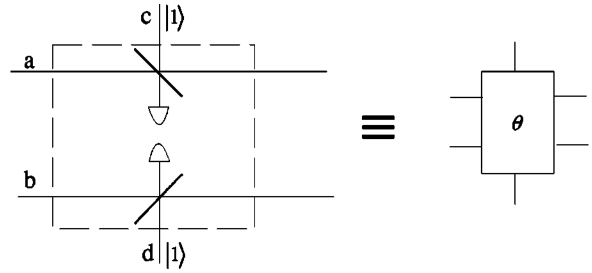

FIG. 2. The basic element to entangle N-photon states. θ denotes the parameter of the beam splitter.

$$\Psi_7=\frac{1}{\sqrt{2}}(|H\rangle|V\rangle-|V\rangle|H\rangle). \tag{10}$$

In order to present the principal idea of our scheme to generate entangled N-photon states we introduce in Fig. 2 simple building blocks Θ as optical two-mode devices. Two separate single-photon injections and two photon-number detectors are involved in this device. In the following, we will demonstrate the generalization capability of the concept to entangle N-photon states. A detailed estimation of the generalized scheme will be presented in the following. It will be shown how the quantum state (1) can be generated and how the different terms of the N-photon state can be deleted.

We consider an arbitrary N-photon state

$$\Psi_{in}=\sum_{n=0}^{N}C_n|n\rangle_a|N-n\rangle_b \tag{11}$$

to be the input mode of the basic block Θ. In contrast to the previously analyzed two-photon system we label in Fig. 2 the four input modes 1,2,3,4 with the letters a,b,c,d. In order to formulate the functionality of this basic block for arbitrary photon-number states transformation we relate to these modes the annihilation operators a,b,c,d and the creation operators $a^{\dagger},b^{\dagger},c^{\dagger},d^{\dagger}$. The interaction with the beam splitters BS_1 and BS_2 can be formulated with the unitary operators

$$U_1=\exp[\theta(ac^{\dagger}-a^{\dagger}c)], \tag{12}$$

$$U_2=\exp[\theta(bd^{\dagger}-b^{\dagger}d)]. \tag{13}$$

The output of these beam splitters is the four-mode state

$$\Phi=U_1U_2|\Psi_{in}\rangle|1\rangle_c|1\rangle_d$$
$$=\sum_{n=0}^{N}\frac{C_n}{\sqrt{n!(N-n)}}(\cos\theta a^{\dagger}+\sin\theta c^{\dagger})^n$$
$$\times(\cos\theta c^{\dagger}-\sin\theta a^{\dagger})(\cos\theta b^{\dagger}+\sin\theta d^{\dagger})^{N-n}$$
$$\times(\cos\theta d^{\dagger}-\sin\theta c^{\dagger})|0\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d. \tag{14}$$

The function of the building block is based on postselection. A twofold coincidence detection projects the four-mode state

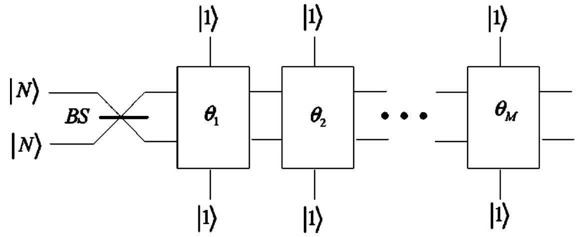

FIG. 3. If M basic blocks are arranged N-photon states can be entangled in a very efficient way. Therefore the choice of the parameter θ_i and a $2M$-fold coincidence detection is needed.

Φ into the two-mode states Ψ. Conditioned on the coincidence of one photon in each detector the (not normalized) quantum state

$$\Psi=\sum_{n=0}^{N} C_n \cos^{N-2}(\cos^2\theta-n\sin^2\theta) \times(\cos^2\theta-(N-n)\sin^2\theta)|n\rangle_a|N-n\rangle_b \qquad (15)$$

is generated. We intend to demonstrate that the term $|i\rangle_a|N-i\rangle_b$ and the term $|N-i\rangle_a|i\rangle_b$ of the input state (11) (with $n=i$ and $n=N-i$) can be deleted by changing the value θ of the beam splitters appropriately. This can be achieved by choosing the parameter θ to satisfy $\tan\theta=\sqrt{i}$.

Now we propose the generation of the maximally entangled N-photon state (1) by arranging the building blocks Θ as it is shown in Fig. 3. For simplicity, we consider the $2N$-photon state $1/\sqrt{2}(|0,2N\rangle+|2N,0\rangle)$, but the same scheme can also be used to generate the entangled $(2N+1)$-photon state $1/\sqrt{2}(|0,2N+1\rangle+|2N+1,0\rangle)$. As the input mode of the symmetric BS we use N photons in the mode 1 and N photons in the mode 2. The output of this first beam splitter is the entangled $2N$-photon state,

$$\Psi=\frac{1}{2^N}\sum_{m=0}^{N}(-1)^m\frac{\sqrt{(2m)!(2N-2m)!}}{m!(N-m)!} \times|2N-2m\rangle_{1'}|2m\rangle_{2'}. \qquad (16)$$

In order to generate the maximally entangled state (1) M basic elements are required $\{M=N/2$, if N is even; $M=(N-1)/2$, if N is odd$\}$. Conditioned on the $2M$-fold coincidence detection the state $\Psi=1/\sqrt{2}(|0,2N\rangle+|2N,0\rangle)$ can be generated, if the parameters are chosen appropriately: $\tan\theta_i=\sqrt{2i}$, $(i=1,\ldots,M)$. The probability of this outcome is $(2N)![(2N-2)!!]^2[(N-1)!!]^{2N}/2^{2N-1}(N!)^4(N!!)^{2N}$, if N is odd. Otherwise the probability of this outcome will be $(2N)![(2N-2)!!]^2[N!!]^{2N}/2^{2N-1}((N-1)!)^2[(N+1)!]^2[(N+1)!!]^{2(N)}$.

In order to demonstrate, how entangled $(2N+1)$-photon states can be generated, we require that the input of the symmetric beam splitter is in a quantum state with $2N+1$ photons in the mode 1 and 0 photons in the mode 2. The spatially separated output photons of the beam splitter are incident on N basic elements whose parameter is chosen to be $\tan\theta_i=\sqrt{i}$, $(i=1,\ldots,N)$. Based on the $2N$-fold photon coincidence detection, the maximally entangled $(2N+1)$-photon state (1) will be obtained. The probability of success is $[(2N)!]^2/4^N(N+1)^{2N+1}[N!(N+1)!]^2$.

In summary, we have suggested a feasible scheme to prepare polarization-entangled quantum states and entangled N-photon states by using linear optical devices. In the case of the polarization-entangled two-photon states the probability of the outcome will be 1/18. This is a slightly smaller value than 1/16, which the scheme with the nondeterministic gate [11] makes possible. But the experimental setup of our scheme is simpler. Instead of four detectors only a twofold coincidence detection is required. Thus, the number of detectors is reduced to the half. In the case of entangled $2N$-photon states it is shown that not more than N detectors are required, which is definitely less than other schemes [19–21] need. An entangled six-photon state can be generated with the probability of 9.7%. This probability of the outcome is three times larger than that of the scheme given in Ref. [19] and much larger than that of the schemes given in Refs. [20,21].

A generalization to a multimode block can be made easily, because the building blocks, which are introduced with the Fig. 2, do not couple the input modes. The multiphoton coincidence detection requires only the classical information, which each detector provides. This is a main difference to the building blocks, which Dowling and co-workers suggested in their paper [21]. In our scheme, entanglement is only generated in the first beam splitter, which is shown in Fig. 3. Thus, four-mode entangled photon states in the form of $\frac{1}{2}(|N\rangle|N-m\rangle|0\rangle|0\rangle+|N-m\rangle|N\rangle|0\rangle|0\rangle+|0\rangle|0\rangle|N\rangle|N-k\rangle+|0\rangle|0\rangle|N-k\rangle|N\rangle)$ can be generated within the generalized concept. These quantum states were employed to create two-dimensional patterns on a suitable substrate in quantum optical lithography [16].

One of the difficulties of our scheme with respect to an experimental demonstration consists in the requirement of the sensitivity of the detectors. Other schemes [18,21], which are based on a multifold coincidence detection, pose the same requirements on the capability of the detectors. Recently, the experimental techniques for single-photon detection made tremendous progress. A photon detector based on a visible-light photon counter was reported, which can distinguish between a single-photon incidence and the two-photon incidence with high quantum efficiency, good time resolution, and low bit-error rate [22]. Another difficulty is the availability of photon-number sources. Currently available triggered single-photon sources operate by means of fluorescence from a single molecule [23] or a single quantum dot [24,25] and they exhibit very good performance. The generation of entangled photon states by means of our scheme requires the synchronized arrival of photons on the beam splitter input ports. This will be experimentally challenging. However, the generated entangled-photon states can act as a kind of valuable source for quantum computation and quantum communication.

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