Boundary of two mixed Bose-Einstein condensates

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(Received 19 December 2001; published 19 July 2002)

We study the boundary between two interpenetrating Bose-Einstein condensates interacting repulsively in the case of their spatial separation. We show that the analytical expressions for the distribution of the condensate density can be obtained in the limiting cases corresponding to the weak and strong separations. Using these expressions, we consider the excitation spectrum of a particle confined in the vicinity of the boundary and the surface waves at the boundary.

DOI: 10.1103/PhysRevA.66.013612 PACS number(s): 03.75.Fi

I. INTRODUCTION

The experimental realization of Bose-Einstein condensation in trapped dilute gases [1–3] has allowed us to investigate a variety of the properties of quantum gases both theoretically and experimentally. In recent years, it has become possible to produce and explore mixtures of Bose-Einstein condensates corresponding to the different internal states [4–8]. The two-component condensates have been extensively studied both analytically [9–12] and numerically [13–15]. The theoretical treatment of the mixtures [10,11] assures that it is possible to observe the spatial separation depending on the relative strength of the interactions inside each condensate and between them. A general discussion of all possible classes of the solutions for the two-component Bose-Einstein condensates has been presented in Ref. [16]. The experimental realization of such systems [4–8] has allowed us to investigate the equilibrium properties and the dynamics of separation. Several papers [17–20] describe collective excitations in the two-component Bose-Einstein condensates.

In the present paper, we explore the boundary between two repulsively interacting condensates at zero temperature in the limiting cases corresponding to the weak and strong separations of the condensates. We show that in the case of weak separation it is possible to derive equations for the densities and, using the iteration method, to solve them analytically. We show that the asymptotic behavior of our solution coincides with those predicted in Ref. [11]. For the sake of completeness, we also present the solution of these equations in the case of strong separation. The importance of these solutions lies in the possibility to explore quantitatively the different types of excitations at the boundary. The structure of the boundary in both cases allows one to consider one-particle excitations as well as surface waves associated with the existence of the boundary. The expressions obtained for the dispersion relation of surface waves can be used to explore the phenomenon experimentally.

The Hamiltonian describing the mixture of two weakly interacting Bose-Einstein condensates can be written in the form

$$H=\sum_{i=1,2}\int d\mathbf{r}\Psi_i^+\left[-\frac{\hbar^2\nabla^2}{2m_i}+V_i(\mathbf{r})+\frac{u_i}{2}\Psi_i^+\Psi_i\right]\Psi_i + u_{12}\int d\mathbf{r}\Psi_1^+\Psi_2^+\Psi_1\Psi_2. \tag{1}$$

Here $u_i=4\pi\hbar^2 a_i/m_i>0$ characterizes the interaction inside each condensate; $u_{12}=2\pi\hbar^2 a_{12}(m_1+m_2)/(m_1m_2)>0$, the intercondensate interaction; m_i is the mass of a particle of each condensate; a_i, a_{12} are the corresponding scattering lengths; and $V_i(\mathbf{r})$ are the external trapping potentials. The theoretical treatment of the mixtures [10,11] has shown that the separation takes place when $u_{12}/\sqrt{u_1u_2}>1$.

Starting with the Hamiltonian (1), we obtain the Gross-Pitaevskii equations for the condensate wave functions:

$$\begin{aligned} i\hbar\frac{\partial\Psi_1}{\partial t}&=\left(-\frac{\hbar^2\nabla^2}{2m_1}+V_1(\mathbf{r})+u_1|\Psi_1|^2+u_{12}|\Psi_2|^2\right)\Psi_1,\\ i\hbar\frac{\partial\Psi_2}{\partial t}&=\left(-\frac{\hbar^2\nabla^2}{2m_2}+V_2(\mathbf{r})+u_2|\Psi_2|^2+u_{12}|\Psi_1|^2\right)\Psi_2. \end{aligned} \tag{2}$$

Since we are interested in studying the stationary solutions of these equations, we assume as usual $\Psi_j\propto\exp(-i\mu_j t)$, where μ_j are the chemical potentials of the condensates, and we obtain two coupled nonlinear equations for the density of gases $n_i(\mathbf{r})=|\Psi_i(\mathbf{r})|^2$:

$$\begin{aligned} \mu_1&=-\frac{\hbar^2}{2m_1}\frac{\nabla^2\sqrt{n_1}}{\sqrt{n_1}}+V_1(\mathbf{r})+u_1n_1+u_{12}n_2,\\ \mu_2&=-\frac{\hbar^2}{2m_2}\frac{\nabla^2\sqrt{n_2}}{\sqrt{n_2}}+V_2(\mathbf{r})+u_2n_2+u_{12}n_1. \end{aligned} \tag{3}$$

These equations are essentially nonlinear. Thus, to find the solutions, we are in need to make some simplifications. We assume that $V_1(\mathbf{r})=V_2(\mathbf{r})$ and consider the case when the size of a boundary between the condensates is much less than the characteristic size of the system. In the case of a parabolic trap potential within the Thomas-Fermi regime, $d\ll R_{TF}$, where d is the size of the boundary and R_{TF} is the Thomas-Fermi radius of the atomic cloud. Physically, this approximation helps to avoid the effect of the potential on the shape of the boundary. To simplify the calculations more, we also suppose that the separation takes place in one direction along the z axis. This allows us to write the system of equations (3) in the following form:

$$\mu_1 = -\frac{\hbar^2}{2m_1\sqrt{n_1}}\frac{d^2}{dz^2}\sqrt{n_1} + u_1 n_1 + u_{12} n_2,$$
$$\mu_2 = -\frac{\hbar^2}{2m_2\sqrt{n_2}}\frac{d^2}{dz^2}\sqrt{n_2} + u_2 n_2 + u_{12} n_1. \quad (4)$$

To find the solution of these equations corresponding to the separation of the condensates, we have to impose external boundary conditions on the densities, $n_1(z)$ and $n_2(z)$. Let us assume that the condensate "1" is on the right-hand side and "2" is on the left-hand side from the boundary. Then, asymptotically we require

$$n_1(z\to+\infty)\to n_{10}, \quad n_1(z\to-\infty)\to 0,$$
$$n_2(z\to-\infty)\to n_{20}, \quad n_2(z\to+\infty)\to 0, \quad (5)$$

where n_{10} and n_{20} are the equilibrium densities of the condensates far from the boundary. Substituting these conditions into Eqs. (4) for the densities and using the equilibrium condition, we obtain

$$\mu_1 = u_1 n_{10}, \quad \mu_2 = u_2 n_{20},$$
$$P_1 = u_1 n_{10}^2/2 = P_2 = u_2 n_{20}^2/2. \quad (6)$$

Here we used the well-known expression for the pressure of a weakly interacting homogeneous Bose gas. This condition presents the relation between the equilibrium densities of the condensates far from the boundary.

To reduce the number of parameters in Eqs. (4), we notice that it is possible to exclude the difference in the masses by replacing

$$u_1^* = u_1 m_1/m_2, \quad u_2^* = u_2 m_2/m_1,$$
$$n_1^* = n_1\sqrt{m_2/m_1}, \quad n_2^* = n_2\sqrt{m_1/m_2},$$
$$\mu_1^* = u_1^* n_{10}^*, \quad \mu_2^* = u_2^* n_{20}^*, \quad (7)$$
$$m^* = \sqrt{m_1 m_2}.$$

After that we obtain the same Eqs. (4) and conditions (5) and (6) but for the asterisks quantities and with the same mass m^*. To simplify the notations, in the following discussion we will omit the asterisks. Note that both parameters u_{12} and Δ are not affected by the above procedure of eliminating the difference in the masses.

We can solve the equations for the densities (4) with conditions (5) and (6) analytically in the two limiting cases: for the weak separation when $\Delta = u_{12}/\sqrt{u_1 u_2} - 1 \ll 1$ and for the strong separation when $\Delta \gg 1$.

II. WEAK SEPARATION

Let us consider the weak separation when the condition $\Delta = u_{12}/\sqrt{u_1 u_2} - 1 \ll 1$ is satisfied. We expect that in the simplest case, when $u_1 = u_2$, the total density of a gas $n(z) = n_1(z) + n_2(z)$ is approximately constant. Then, instead of n_1 and n_2 it is natural to introduce other quantities and solve Eqs. (4) using the small parameter Δ. Let us consider the functions

$$\rho = (u_1/u_2)^{1/4} n_1 + (u_2/u_1)^{1/4} n_2,$$
$$g = [(u_1/u_2)^{1/4} n_1 - (u_2/u_1)^{1/4} n_2]/\rho. \quad (8)$$

Conditions (5) and (6) give us the simple asymptotic behavior of these functions:

$$\rho(z\to\pm\infty)\to\rho_0 = \sqrt{n_{10} n_{20}},$$
$$g(z\to\pm\infty)\to\pm 1. \quad (9)$$

The densities of condensates 1 and 2 are easily obtained if functions ρ and g are known:

$$n_1 = (u_2/u_1)^{1/4}\rho[1+g]/2,$$
$$n_2 = (u_1/u_2)^{1/4}\rho[1-g]/2. \quad (10)$$

It is straightforward to derive the equations for ρ and g:

$$\frac{\sqrt{\rho}''}{\sqrt{\rho}} - \frac{\rho}{\rho_0}\left[1 + \alpha g + \frac{\Delta}{2}(1-g^2)\right] = \frac{g'^2}{4(1-g^2)} - 1 - \alpha g,$$
$$\frac{g''}{1-g^2} + \frac{2\sqrt{\rho}' g'}{\sqrt{\rho}(1-g^2)} + \frac{g g'^2}{(1-g^2)^2} + 2\alpha \quad (11)$$
$$= \frac{\rho}{\rho_0}[2\alpha + \Delta(\alpha - g)].$$

Here $f' = \xi_0(df/dz)$ and we also introduced $\alpha = (\sqrt{u_1} - \sqrt{u_2})/(\sqrt{u_1} + \sqrt{u_2})$ and $1/\xi_0^2 = m(u_1 u_2)^{1/4}(\sqrt{u_1} + \sqrt{u_2})\rho_0/\hbar^2$.

To find the asymptotic solutions of Eqs. (11) in the case of $\Delta \ll 1$, we use the iteration method. Let us assume that the terms with derivatives of ρ are much smaller than the others. We will justify this assumption at the end of our calculations.

Neglecting the terms with derivatives in ρ, we get from Eqs. (8),

$$\rho/\rho_0 = 1 - \frac{g'^2 + 2\Delta(1-g^2)^2}{4(1-g^2)(1+\alpha g)},$$
$$\frac{g''}{1-g^2} + \frac{2g + \alpha(1+g^2)}{2(1-g^2)(1+\alpha g)} g'^2 + \frac{\Delta(1-\alpha^2) g}{1+\alpha g} = 0. \quad (12)$$

The equation for g can be solved by substituting $g' = f$. Taking into account conditions (9), we get that g is the solution of the equation

$$g' = \sqrt{\Delta(1-\alpha^2)}\,\frac{(1-g^2)}{\sqrt{1+\alpha g}}. \quad (13)$$

At this point it is clear that the assumptions we used were correct. Namely, we obtain $\sqrt{\rho}' \propto \Delta^{3/2}$, $\sqrt{\rho}'' \propto \Delta^2$, and $g' \propto \sqrt{\Delta}$, so the terms we neglected are by Δ times less than the others.

Now we can write down the solutions for both ρ and g in the parametric form,

$$1-\frac{\rho}{\rho_0}=\frac{\Delta}{4}\,\frac{1-g^2}{(1+\alpha g)^2}[3-\alpha^2+2\alpha g],$$

$$\frac{z-z_0}{\xi_0/\sqrt{\Delta(1-\alpha^2)}}=\frac{\sqrt{1+\alpha}}{2}\ln\left|\frac{\sqrt{1+\alpha g}+\sqrt{1+\alpha}}{\sqrt{1+\alpha g}-\sqrt{1+\alpha}}\right| -\frac{\sqrt{1-\alpha}}{2}\ln\left|\frac{\sqrt{1+\alpha g}+\sqrt{1-\alpha}}{\sqrt{1+\alpha g}-\sqrt{1-\alpha}}\right|. \tag{14}$$

Here the second equation is the solution of Eq. (13). In these expressions an arbitrary constant z_0 defines the position of the boundary and, in the case of the finite geometry with the given average number of particles in the condensates, it can be obtained from the condition of equal pressures (6). The symmetries $z-z_0 \to z_0-z$, $g \to -g$, $\alpha \to -\alpha$ are also physically obvious.

Finally, the densities of condensates have the following form:

$$n_1=\frac{n_{10}}{2}\left(1-\frac{\Delta}{4}\,\frac{1-g^2}{(1+\alpha g)^2}[3-\alpha^2+2\alpha g]\right)[1+g],$$

$$n_2=\frac{n_{20}}{2}\left(1-\frac{\Delta}{4}\,\frac{1-g^2}{(1+\alpha g)^2}[3-\alpha^2+2\alpha g]\right)[1-g]. \tag{15}$$

Together with the parametric equation for g these densities are the main result of the paper. Using the method described, it is possible to derive the expressions for the densities in next orders in small parameter Δ in the form of an asymptotic series. As follows from the above estimations of the neglected terms in Eqs. (11), next order is proportional to Δ^2. The typical dependence of the densities on the distance from the boundary is shown in Figs. 1 and 2.

We notice that the total density $n=n_1+n_2$ at some values of the parameters has a hollowlike behavior at the boundary as shown in Fig. 1. In the case of the weak separation we see that the hollow is broad and shallow. Provided the interaction between particles of different species is larger than that of each species, that is $\Delta>1$, we expect the probability of a particle to be close to the boundary to decrease, meaning a lower density. The existence of such a hollow allows us to consider the possibility of confining a particle of another sort in the vicinity of the boundary. This is discussed below. Although we cannot further simplify solutions (14), we can derive the asymptotic behavior of these functions in the specific limits:

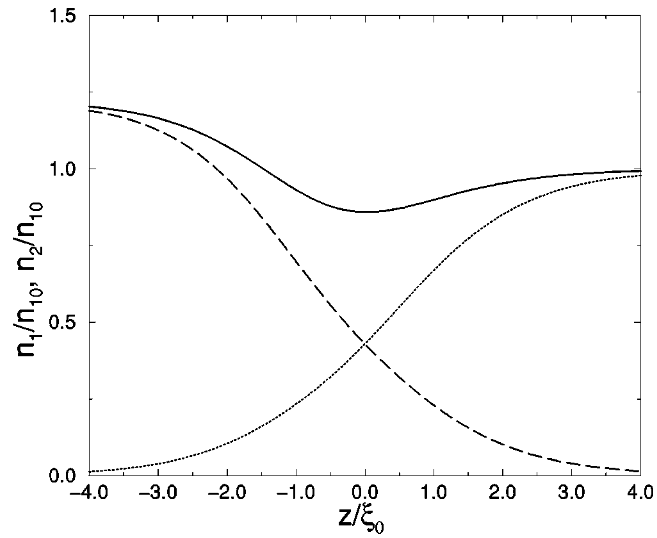

FIG. 1. Distribution of the densities in the case of weak separation: $\Delta=0.3$, $\alpha=0.1$. Dotted and dashed lines correspond to species 1 and 2. The solid line is the total density.

$$1-g(z\to+\infty)\propto\exp\left[-\frac{2z\sqrt{\Delta}}{\xi_2}\right],$$

$$1+g(z\to-\infty)\propto\exp\left[\frac{2z\sqrt{\Delta}}{\xi_1}\right],$$

$$g(z\to z_0)\to\frac{(z-z_0)\sqrt{\Delta(1-\alpha^2)}}{\xi_0}. \tag{16}$$

Here $\xi_i=\hbar/\sqrt{2m_i\mu_i}$ are the correlation lengths of the condensates determined by the chemical potentials μ_1 and μ_2. We used Eq. (7) to include the difference in the masses. The asymptotic behavior far from the boundary of each condensate is governed by its correlation length. As we see, the size

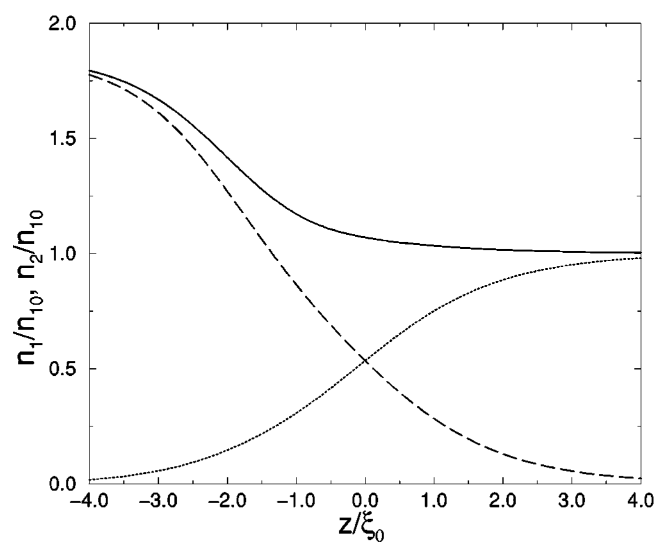

FIG. 2. Distribution of the densities in the case of weak separation: $\Delta=0.3$, $\alpha=0.3$. Dotted and dashed lines correspond to species 1 and 2. The solid line is the total density.

of the boundary can be approximated as $d \approx (\xi_1 + \xi_2)/(2\sqrt{\Delta})$ and for $\Delta \ll 1$ it appears to be much larger than the correlation lengths. We notice that the limiting behavior of densities (16) coincides with the result of Ref. [11]. The solutions have the simplest form when $\alpha = 0$:

$$n_1(z) = \frac{n_0}{2}\left(1 - \frac{3\Delta}{4\cosh^2\left[\frac{z\sqrt{\Delta}}{\xi_0}\right]}\right)\left(1 + \tanh\left[\frac{z\sqrt{\Delta}}{\xi_0}\right]\right),$$

$$n_2(z) = \frac{n_0}{2}\left(1 - \frac{3\Delta}{4\cosh^2\left[\frac{z\sqrt{\Delta}}{\xi_0}\right]}\right)\left(1 - \tanh\left[\frac{z\sqrt{\Delta}}{\xi_0}\right]\right). \quad (17)$$

Here we chose $z_0 = 0$.

The case of the weak separation has been first observed in the experiment with the spinor Bose-Einstein condensates [6] with $\Delta \simeq 10^{-4}$. The size of the boundary is of the same order of the magnitude as the size of the condensate.

III. STRONG SEPARATION

To analyze the case of strong separation $\Delta = u_{12}/\sqrt{u_1 u_2} \gg 1$ (we use the same notation but for another quantity), we start from Eqs. (4) for the density. In this case we expect that the density at the boundary will be approximately zero because the interparticle interactions make it almost impossible for one condensate to penetrate inside the other. To estimate the density of the condensates at the boundary, we can use the fact that second derivatives of the wave functions should be approximately zero there. This transforms the system of Eqs. (4) into a set of two linear equations with the solution

$$n_{1B} = \frac{n_{10}}{\Delta + 1} \approx \frac{n_{10}}{\Delta} \ll n_{10},$$

$$n_{2B} = \frac{n_{20}}{\Delta + 1} \approx \frac{n_{20}}{\Delta} \ll n_{20}. \quad (18)$$

This allows us, in zero approximation, to use the simple conditions for the densities $n_1(z \leqslant 0) = n_2(z \geqslant 0) = 0$. Then Eqs. (4) have the simple form:

$$\mu_1 = -\frac{\hbar^2}{2m_1\sqrt{n_1}}\frac{d^2}{dz^2}\sqrt{n_1} + u_1 n_1 \quad \text{for} \quad z \geqslant 0,$$

$$\mu_2 = -\frac{\hbar^2}{2m_2\sqrt{n_2}}\frac{d^2}{dz^2}\sqrt{n_2} + u_2 n_2 \quad \text{for} \quad z \leqslant 0. \quad (19)$$

The solutions are easily obtained:

$$n_1(z \geqslant 0) = n_{10}\tanh^2\left[\frac{z}{\sqrt{2}\xi_1}\right],$$

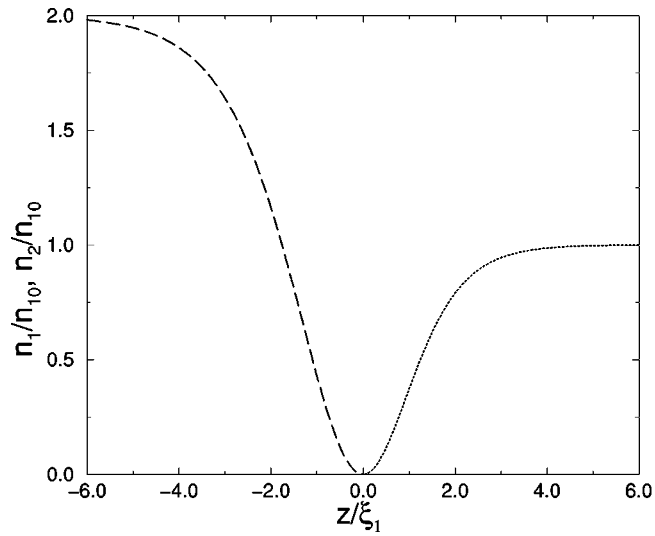

FIG. 3. Distribution of the densities in the case of strong separation when $u_1/u_2 = 4$. Dotted and dashed lines correspond to species 1 and 2.

$$n_2(z \leqslant 0) = n_{20}\tanh^2\left[\frac{z}{\sqrt{2}\xi_2}\right]. \quad (20)$$

Here n_{10} and n_{20} are connected by the condition of equal pressures (6) and we choose the position of the boundary at $z = 0$.

The size of the boundary in this case is approximately $d \approx 2\sqrt{2}(\xi_1 + \xi_2)$. The dependence of the densities on the distance from the boundary is shown in Fig. 3. As in the preceding section, we can see that again there is a hollow in the total density but in the case of the strong separation it becomes narrower and deeper in comparison with that for the weak separation.

IV. ONE-PARTICLE EXCITATIONS ON THE BOUNDARY

The existence of a hollow in the total density allows us to consider the confinement of a particle of another sort in the vicinity of the boundary. The general property of a quantum-mechanical motion in a one-dimensional well is an existence of a confined state. As an example, we consider the simplest case when $\alpha = 0$ and a particle of another sort interacts with both the condensates repulsively with the same constant λ. The Schrödinger equation for the wave function of a particle with the mass M has the form

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dz^2} + \lambda n(z)\right]\phi = E\phi. \quad (21)$$

We can solve Eq. (21) for the weak and strong separation cases simultaneously. It has the universal form

$$\frac{d^2\phi}{dz^2} + \frac{2M}{\hbar^2}\left(\epsilon + \frac{U_0}{\cosh^2[\beta z]}\right)\phi = 0, \quad (22)$$

where $\epsilon = E - \lambda n_0$, and $U_0 = 3\Delta\lambda n_0/4$, $\beta = \sqrt{\Delta}/\xi_0$ for the weak separation; and $U_0 = \lambda n_0$, $\beta = 1/(\sqrt{2}\xi_0)$ for the strong separation. The spectrum of energy ϵ is well known:

$$\epsilon_j = -\mu \frac{\Delta m}{4M}\left[-(1+2j)+\sqrt{1+3\frac{M\lambda}{mu}}\right]^2 \quad \text{weak},$$
$$\epsilon_j = -\mu \frac{m}{8M}\left[-(1+2j)+\sqrt{1+8\frac{M\lambda}{mu}}\right]^2 \quad \text{strong}, \tag{23}$$

where $\mu = un_0$ is the chemical potential and $j=0,1,\ldots$. The condition that an expression in $[\cdots]$ in Eq. (23) is positive determines the upper limit for j. There is always at least one state with $j=0$.

V. SURFACE WAVES AT THE BOUNDARY

There is also another type of excitation associated with the boundary. As we can see, the spatially inhomogeneous distribution of the condensate densities gives rise to nonzero surface tension previously analyzed in Ref. [11]. Here we use the expression for the surface tension derived there:

$$\sigma = \frac{1}{2}\int_{-\infty}^{+\infty} dz \sum_{i=1,2} \frac{\hbar^2}{2m_i}\left(\frac{d\sqrt{n_i}}{dz}\right)^2. \tag{24}$$

Substituting expressions (15) for the densities in the case of the weak separation and taking into account only first nonzero order in Δ, we obtain for the surface tension

$$\sigma_w = \frac{P\sqrt{\Delta}}{6}\left[(\xi_1+\xi_2)\frac{2\alpha^2-1+\sqrt{1-\alpha^2}}{\alpha^2} - (\xi_1-\xi_2)\frac{1-\sqrt{1-\alpha^2}}{\alpha}\right], \tag{25}$$

where $\alpha = (m_1\sqrt{u_1}-m_2\sqrt{u_2})/(m_1\sqrt{u_1}+m_2\sqrt{u_2})$ and we used Eq. (7) to involve the difference in the masses. Here ξ_1, ξ_2 are the correlation lengths of condensates and P is the pressure given by Eq. (6). When $\alpha=0$, $\sigma_w = P\sqrt{\Delta}(\xi_1+\xi_2)/4$.

In the case of the strong separation we use expressions (20) in order to get the surface tension

$$\sigma_s = \frac{P\sqrt{2}}{3}(\xi_1+\xi_2). \tag{26}$$

Let us note that the expressions (25) and (26) differ from those obtained in Ref. [11]. Although in a qualitative sense our expressions coincide with those of Ref. [11], using our method of solution, we can get the general expression applicable for a variety of parameters and retrieve the correct numerical factor for the case considered in Ref. [11]. As follows from the general expression for the surface tension (24), we should know the behavior of the densities not only far but also in the vicinity of the boundary. That is why, the approximate character of the expressions for the densities in Ref. [11] could give only a qualitative answer for the surface tension.

For the velocities smaller than the speed of sound, we can consider the gas to be incompressible, so that it is possible to use the hydrodynamic equations and to find the dispersion relation of the surface waves at the boundary due to the presence of the surface tension. Suppose that one has the condensates in a "box" and the position of the boundary is determined by the distances L_1 and L_2 from the box walls where labels "1" and "2" correspond to the side with the same condensate. The box sizes in the other directions are L_x and L_y. Taking into account the fact that the velocities vanish at the walls, we get the dispersion relation

$$\omega^2(k) = \frac{\sigma k^3}{\rho_{10}\coth(kL_1)+\rho_{20}\coth(kL_2)}, \tag{27}$$

where σ is the surface tension. Here $\rho_{10}=m_1n_{10}$, $\rho_{20}=m_2n_{20}$ are the mass densities, the wave vector along the boundary is $k=\sqrt{(\pi n_x/L_x)^2+(\pi n_y/L_y)^2}$, where $n_x, n_y = 0,1,\ldots$, not $n_x=n_y=0$, and L_1, L_2 are the sizes of the condensates in the direction perpendicular to the boundary. We assume that L_1 and L_2 are much larger than the size of the boundary d.

For the dispersion relation, we consider two limiting cases corresponding to the wavelengths $\lambda \ll L_i$ $(kL_i \gg 1)$ and $\lambda \gg L_i$ $(kL_i \ll 1)$. Let us note that the second case is possible only if $L_i \ll L_{x,y}$.

In the first case, we obtain

$$\omega(k) = \left(\frac{\sigma}{\rho_{10}+\rho_{20}}\right)^{1/2} k^{3/2}. \tag{28}$$

In the long-wavelength limit, we get

$$\omega(k) = \left(\frac{\sigma L_1 L_2}{\rho_{10}L_2+\rho_{20}L_1}\right)^{1/2} k^2. \tag{29}$$

As we see that for the weak separation $\omega \propto \Delta^{1/4}$, and the surface waves are relatively "soft;" in this case, we can consider them as a dissipative channel for other condensate excitations.

VI. CONCLUSION

We presented here the analysis of the boundary of two overlapping Bose-Einstein condensates interacting repulsively in the limiting cases corresponding to the weak and strong separations at zero temperature.

For the weak separation, we obtained solutions (15) of two coupled nonlinear Gross-Pitaevskii equations using the small parameter Δ. The solutions show that the penetration depth of the condensate "i" inside the other is estimated as $\xi_i/(2\sqrt{\Delta})$, i.e., the size of the boundary $d \approx (\xi_1+\xi_2)/(2\sqrt{\Delta})$ is much larger than the correlation lengths. This is observed experimentally [6]. In addition, there is a hollow in the full density profile, resulting from the wave-function behavior near the boundary. On the whole, the method proposed for obtaining density distributions for the case of the weak separation can be extended to obtain the expressions for next orders in Δ.

We also considered the case of the strong condensate separation, but restricted ourselves to zero-order approximation. In this case the size of the boundary is $d \approx 2\sqrt{2}(\xi_1$

$+\xi_2)$, and the total density of gases tends to vanish at the boundary. The existence of a hollow in the density profile at some parameters allowed us to consider one-particle excitations at the boundary. Using the expressions for the densities, we found the excitation spectrum of a particle in the simplest case when the constants of the particle-condensate interaction are the same and the distribution of the densities at the boundary corresponds to $\alpha=0$. The generalization to other cases can be done with the use of expressions (15) and (20) for the density distributions. We notice that the existence of a potential well depends on the interaction constants of a particle with the condensates as well as on the relationship between the interaction constants of the condensates. The observation of such confined states can be possible only if the temperature is smaller than the energy of a bound state.

It was shown that there also exist collective excitations associated with the surface tension. The expressions for the surface tension were obtained for the both weak and strong separations. The dispersion relation for surface waves is analyzed in the case when the condensates fill the finite volumes. The dispersion relation has different forms in the cases corresponding to the short- and long-wavelength limits. In the case of weak separation, the soft surface modes can represent a dissipative channel for other condensate excitations.

ACKNOWLEDGMENTS

The author gratefully acknowledges valuable discussions with Yu. M. Kagan and the other members of the Theoretical Division of Russian Research Center ''Kurchatov Institute,'' Moscow.

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