Superfluid pairing in a polarized dipolar Fermi gas

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We calculate the critical temperature of a superfluid phase transition in a polarized Fermi gas of dipolar particles. In this case the order parameter is anisotropic and has a nontrivial energy dependence. Cooper pairs do not have a definite value of the angular momentum and are coherent superpositions of all odd angular momenta. Our results describe prospects for achieving the superfluid transition in single-component gases of fermionic polar molecules.

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I. INTRODUCTION

The recent success in observing quantum degeneracy in ultracold atomic Fermi gases [1–4] stimulates a search for gaseous Fermi systems with an achievable temperature of superfluid phase transition, which is generally very low. The ideas based on Cooper pairing for a short-range Van der Waals interaction between atoms [5,6] require a simultaneous trapping of at least two different fermionic species, with a rather severe constraint on their relative concentrations. Aside from the *s*-wave pairing [5], these ideas also employ the *p*-wave pairing [6] widely discussed in the physics of superfluid ^{3}He (see [7] for review). Recent proposals to reach the transition temperature comparable with the Fermi energy bring in the ideas of pairing via a Feshbach resonance for the interspecies interaction [8,9]. The *p*-wave pairing via a Feshbach resonance for a short-range interaction has been discussed in [10].

Fermi gases of dipolar particles present a different physical picture. Being electrically polarized, these particles interact via long-range anisotropic (partially attractive) dipole-dipole forces. As a result, the orbital angular momentum is not conserved in interparticle collisions. In the ultracold limit, the dipole-dipole scattering amplitude is energy independent for any angular momenta in the incoming and outgoing channels. This follows from the studies of spin relaxation collisions [11,12] in the limit of very low magnetic fields, based on the Born approximation [16]. The energy independence of the amplitude of dipole-dipole elastic scattering has been found and expounded in Refs. [13–15] on the basis of multichannel scattering theory [16].

This opens prospects to achieve the superfluid pairing in a *single-component* Fermi gas, where only scattering with odd orbital momenta (negligible in the case of Van der Waals interactions) is present. These prospects are especially interesting as in single-component fermionic gases the Pauli exclusion principle provides a strong suppression of inelastic collisional rates (see [12]). Hence one can think of achieving higher densities than in Bose gases.

Possible realizations of dipolar Fermi gases include an electrically polarized gas of polar molecules as they have large permanent electric dipoles. The creation of cold clouds of polar molecules has been recently demonstrated in experiments with buffer-gas cooling [17] and in experiments based on deceleration and cooling of polar molecules by time-dependent electric fields [18]. Another option is to create a gas of atoms with electric dipole moments induced by a high dc electric field [13] or by laser coupling of the atomic ground state to an electrically polarized Rydberg state [19].

The *p*-wave Cooper pairing in a *polarized dipolar Fermi gas* has been discussed in [20] for the case of magnetic dipoles and in [14] for field-induced electric dipoles of atoms, and the corresponding critical temperature has been estimated by using the standard BCS approach. In this paper we calculate the value of the critical temperature and find the energy and angular dependence of the order parameter. For this purpose we consider the Cooper pairing for all possible scattering channels. These channels are coupled to each other by the dipole-dipole interaction, and the Cooper pairs prove to be coherent superpositions of contributions of all odd angular momenta. In order to find the preexponential factor for the critical temperature, we perform the calculations to second order in perturbation theory along the lines of the approach of Gor'kov and Melik-Barkhudarov (GM approach) [21].

II. GENERAL EQUATIONS

We consider a spatially homogeneous single-component gas of fermions having a dipole moment **d** oriented along the *z* axis. The Hamiltonian of the system has the form

$$H=\int d\mathbf{r}\hat{\psi}^{\dagger}(\mathbf{r})\left\{-\frac{\hbar^2}{2m}\nabla^2-\mu\right\}\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}d\mathbf{r}'|\hat{\psi}(\mathbf{r})|^2 V_d(\mathbf{r}-\mathbf{r}')|\hat{\psi}(\mathbf{r}')|^2, \qquad (1)$$

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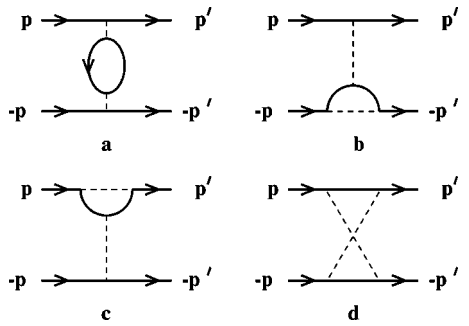

FIG. 1. The lowest order many-body corrections to the effective interparticle interaction.

where $\hat{\psi}(\mathbf{r})$ is the field operator for fermions, $V_d(\mathbf{r}) = (d^2/r^3)(1-3\cos^2\theta_{\mathbf{r}})$ the dipole-dipole interaction, $\theta_{\mathbf{r}}$ is the angle between the interparticle distance $\mathbf{r}$ and the z axis, and μ is the chemical potential. In Eq. (1) we omit the contribution of the p-wave scattering due to the short-range part of the interparticle interaction, since this contribution is small in the dilute ultracold limit.

For a single-component Fermi gas, the Cooper pairing is possible only in the states with an odd angular momentum l. On the other hand, the anisotropic character of the dipole-dipole interaction leads to coupling between Cooper pairs with different values of the angular momentum. Therefore the problem of superfluid pairing requires us to consider states with any odd l.

The critical temperature T_c of the superfluid transition and the order parameter Δ can be found from the gap equation in the momentum representation [7,22]:

$$\Delta(\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} V(\mathbf{p},\mathbf{p}') \frac{\tanh[E(\mathbf{p}')/2T]}{2E(\mathbf{p}')} \Delta(\mathbf{p}'). \tag{2}$$

Here $E(\mathbf{p}) = \sqrt{\Delta^2(\mathbf{p}) + (p^2/2m-\mu)^2}$, and we assume the order parameter to be real. The function $V(\mathbf{p},\mathbf{p}') = V_d(\mathbf{p}-\mathbf{p}') + \delta V(\mathbf{p},\mathbf{p}')$, where $V_d(\mathbf{q})$ is the Fourier transform of the dipole-dipole interaction potential $V_d(\mathbf{r})$:

$$V_d(\mathbf{q}) = \frac{4\pi}{3} d^2[3\cos^2(\theta_{\mathbf{q}}) - 1], \tag{3}$$

with $\theta_{\mathbf{q}}$ being the angle between the momentum $\mathbf{q}$ and the z axis. The quantity $\delta V(\mathbf{p},\mathbf{p}')$ originates from many-body effects and is a correction to the bare interparticle interaction V_d. The leading corrections are second order in V_d and the corresponding diagrams are shown in Fig. 1 (see Ref. [21]). They describe the processes in which one of the two colliding particles polarizes the medium by virtually creating a particle-hole pair. In Fig. 1(a) the particle-hole pair then annihilates due to the interaction with the other colliding particle. In Figs. 1(b)– 1(d) the hole annihilates together with one of the colliding particles. In Figs. 1(b) and 1(c) the particle-hole pair is created due to the interaction of the medium with one of the colliding particles, and the hole annihilates with the other colliding partner. In Fig. 1(d) these creation and annihilation processes involve one and the same colliding particle.

For temperatures just below T_c the order parameter is small and the gap equation is equivalent to the Ginzburg-Landau equation for the spatially homogeneous order parameter. This equation can be obtained by expanding the right-hand side (rhs) of Eq. (2) in powers of the order parameter $\Delta(\mathbf{p})$:

$$\Delta(\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} V(\mathbf{p},\mathbf{p}') \times \left[K(p')\Delta(\mathbf{p}') + \frac{\partial K(p')}{\partial \xi'} \frac{\Delta^3(\mathbf{p}')}{2\xi'} \right], \tag{4}$$

where $K(p) = \tanh(\xi/2T)/2\xi$, and $\xi = p^2/2m - \mu$.

The occurrence of the Cooper pairing is associated with the existence of a nontrivial solution of Eq. (4) for temperatures $T \leqslant T_c$. In order to find the value of the critical temperature one can neglect the second, nonlinear term in the square brackets in the rhs of Eq. (4) because for $T \to T_c$ the order parameter $\Delta \to 0$. The corresponding linearized gap equation also provides us with the momentum dependence of the order parameter, whereas the nonlinear term determines the absolute (temperature dependent) value of Δ.

The integral in Eq. (4) diverges at large momenta. The divergency can be eliminated by expressing the bare interaction V_d in terms of the vertex function (scattering off-shell amplitude) $\Gamma(E,\mathbf{p},\mathbf{p}')$. This is similar to the well-known procedure of renormalization of the scattering length in dilute gases of Bose or Fermi particles interacting via short-range forces [23,24]. One may choose any value of E, and for simplifying our calculations we select $E=0$. Then the vertex function $\Gamma(0,\mathbf{p},\mathbf{p}') = \Gamma_d(\mathbf{p},\mathbf{p}')$ obeys the equation

$$\Gamma_d(\mathbf{p},\mathbf{p}') = V_d(\mathbf{p}-\mathbf{p}') - \int \frac{d\mathbf{q}}{(2\pi\hbar)^3} \Gamma_d(\mathbf{p},\mathbf{q}) \times K_0(q) V_d(\mathbf{q}-\mathbf{p}'), \tag{5}$$

with $K_0(q) = m/q^2$. We will confine ourselves to the second order in perturbation theory with respect to V_d. Omitting higher order corrections, the renormalized linearized gap equation reads

$$\Delta(\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \Gamma_d(\mathbf{p},\mathbf{p}')\{K(p') - K_0(p')\}\Delta(\mathbf{p}') - \int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \delta V(\mathbf{p},\mathbf{p}') K(p') \Delta(\mathbf{p}'). \tag{6}$$

In the dilute ultracold limit only small momenta $\mathbf{p}$ and $\mathbf{p}'$ are important. We thus have to find the scattering amplitude for ultracold particles, in the presence of the dipole-dipole interaction between them.

III. SCATTERING AMPLITUDE IN THE ULTRACOLD LIMIT

The anisotropic and long-range character of the dipole-dipole interaction ($V_d \propto 1/r^3$) ensures that in the ultracold limit all partial waves give an energy independent contribution to the scattering amplitude [16]. For any orbital angular momentum l one has $\Gamma_d \sim d^2 \sim 4\pi\hbar^2 r_*/m$, where the quantity $r_* \sim md^2/\hbar^2$ plays the role of the characteristic radius of interaction for the dipole-dipole potential. For the interparticle separation $r \gg r_*$ the potential $V_d(\mathbf{r})$ does not influence the wave function of the relative motion of two colliding particles and the motion becomes free. The ultracold limit requires particle momenta satisfying the condition

$$pr_*/\hbar \ll 1. \tag{7}$$

The anisotropy of V_d directly couples scattering channels with angular momenta l and $l \pm 2$. Thus, strictly speaking, all even-l (odd-l) channels are coupled to each other, whereas the scattering with odd angular momenta remains decoupled from that with even momenta.

There are two contributions to the scattering amplitude. The long-range contribution comes from distances $r \gtrsim \hbar/p$ and gives $\Gamma_d \sim d^2$ for all angular momenta in the incoming and outgoing channels, allowed by the selection rules. This contribution can be found by using the Born approximation. The short-range contribution comes from distances $r \lesssim r_*$. For the scattering with even l, due to the presence of the s wave, we have again $\Gamma_d \sim d^2$ or even somewhat larger because of the so-called shape resonances [25]. Under the condition (7), the contribution of the s wave to the wave function of the relative motion at distances $r \lesssim r_*$ is independent of p. This leads to an energy independent Γ. However, it depends on a detailed behavior of the interparticle potential at short interparticle distances. Thus for even l one cannot make a general statement on the value of Γ.

In the case of identical fermions only odd orbital angular momenta are present. Then the short-range contribution is much smaller than the long-range one. We will demonstrate this for the p-wave on-shell scattering amplitude, omitting the coupling to the channels with other odd l. For $l=1$ and $m_l=0$ in both incoming and outgoing scattering channels (m_l is the projection of l on the z axis), the dipole-dipole potential $V_d(\mathbf{r})$ averaged over the angle $\theta_\mathbf{r}$ is equal to $\overline{V}_d = -4d^2/5r^3$. In order to analyze the short- and long-range contributions to the scattering amplitude, we consider the relative motion of particles in a truncated potential $V(r) = \overline{V}_d(r)$ for $r < r_0$, and $V(r)=0$ for $r > r_0$.

The truncation radius r_0 is selected such that $r_* \lesssim r_0 \ll \hbar/p$. The Schrödinger equation for the wave function of the relative motion reads

$$\frac{\hbar^2}{m}\left(-\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{2}{r^2}\right)\psi(r) + V(r)\psi(r) = \frac{p^2}{m}\psi(r). \tag{8}$$

At distances $r \ll \hbar/p$ we may put $p=0$ in Eq. (8). Then for $r < r_0$ we use the well-known procedure of finding an analytical solution for $\psi(r)$ in power law potentials [26]. Assuming $r \gg r_*$, this gives

$$\psi(r) \propto \left(\frac{r}{r_*} + \text{const}\right), \tag{9}$$

where the constant term is independent of r_*. At $r > r_0$ the motion is free and $\psi(r)$ depends explicitly on the scattering phase δ. The solution, which for $r \to \infty$ takes the required asymptotic form $(\hbar/pr)\cos(pr/\hbar + \delta)$, at $r \ll \hbar/p$ becomes

$$\psi(r) = -\left(\frac{\sin\delta}{(pr/\hbar)^2} + \frac{(pr/\hbar)\cos\delta}{3}\right). \tag{10}$$

Equalizing the logarithmic derivatives of the wave functions (9) and (10), we immediately obtain $\delta \sim p^3 r_0^2 r_*/\hbar^3$ and find that $\psi(r) \propto p$ for $r \ll \hbar/p$. The scattering amplitude then proves to be $\Gamma_d = \int \psi(r) V(r) d^3r \sim d^2 (pr_0/\hbar)^2$. The short-range contribution to Γ, that is the contribution from distances $r \lesssim r_*$, is obtained from this relation by simply putting $r_0 \sim r_*$.

We then increase r_0 and make it much larger than $\hbar/p$. At distances $r \sim \hbar/p \gg r_*$ the potential $\overline{V}$ is much smaller than the kinetic energy term in the left-hand side (lhs) of Eq. (8). For the contribution of these distances to the scattering amplitude the Born approximation gives $\Gamma_d \sim d^2$. We thus see that the short-range contribution to the scattering amplitude is small compared to the long-range contribution coming from distances $r \sim \hbar/p$. The corresponding ratio is of the order of $(pr_*/\hbar)$.

This has two important consequences. First, a detailed shape of the interaction potential is not important for the scattering amplitude as the latter is determined by the long-range contribution. (The absence of shape resonances in the odd-l scattering channels was recently demonstrated in Ref. [15].) This contribution is obtained in the Born approximation and depends only on the value of the dipole moment. Second, we may include the second order Born correction to the amplitude. This correction is of the order of $d^2(pr_*/\hbar)$ and still greatly exceeds the short-range contribution.

In the second order Born approximation for the off-shell scattering amplitude $\Gamma_d(\mathbf{p},\mathbf{p}')$ we have

$$\Gamma_d(\mathbf{p},\mathbf{p}') = V_d(\mathbf{p}-\mathbf{p}') - \int \frac{d\mathbf{q}}{(2\pi\hbar)^3} V_d(\mathbf{p}-\mathbf{q}) \times K_0(q) V_d(\mathbf{q}-\mathbf{p}'), \tag{11}$$

where the first and second terms in the rhs of Eq. (11) are first and second order in V_d, respectively. The integral for the second order correction to the scattering amplitude in Eq. (11) is formally divergent at large q. This is the same nonphysical divergency as in the case of short-range interactions [23,24], and it will be eliminated in the calculations of the order parameter and critical temperature (see Sec. V).

IV. CRITICAL TEMPERATURE IN THE BCS APPROACH

In a quantum degenerate Fermi gas characteristic momenta of colliding particles are of the order of the Fermi momentum $p_F=(6\pi^2 n)^{1/3}\hbar$ (n is the gas density). Then, with $r_*\sim md^2/\hbar^2$, the condition (7) of the ultracold limit for interparticle collisions can be written as

$$nd^2/\varepsilon_F \ll 1, \tag{12}$$

where $\varepsilon_F=p_F^2/2m$ is the Fermi energy. The lhs of Eq. (12) is the ratio of the mean dipole-dipole interaction energy (per particle) to the Fermi energy. As in the case of short-range interactions [23,24], this is a small parameter of the many-body theory. It is the condition (12) that allows us to omit the contribution of higher order diagrams and use the renormalized gap equation (6).

Generally, in dilute Fermi gases the critical temperature is exponentially small compared to the Fermi energy ε_F. The exponent is inversely proportional to the Fermi momentum p_F and is determined by first order terms in V_d. The account of the second order terms provides us with the preexponential factor.

We first calculate $\Delta(\mathbf{p})$ to first order in V_d and find the correct exponent in the dependence of the critical temperature on the particle density. For this purpose we should keep in Eq. (6) only the terms which are first order in V_d. This is the first term in the rhs of this equation, with $\Gamma_d(\mathbf{p},\mathbf{p}')=V_d(\mathbf{p}-\mathbf{p}')$. Then, Eq. (6) can be rewritten in the form

$$\Delta(\xi,\mathbf{n})=-\int_{-\mu}^{\infty} d\xi'[\tanh(\xi'/2T)/2\xi'] \times\int\frac{d\mathbf{n}'}{4\pi}R(\xi,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}'). \tag{13}$$

Here $\mathbf{n}=\mathbf{p}/p$, and

$$R(\xi,\mathbf{n};\xi',\mathbf{n}')=\nu(\xi')\Gamma_d(p(\xi)\mathbf{n},p(\xi')\mathbf{n}') \times[1-\xi'/(\xi'+\mu)\tanh(\xi'/2T)],$$

where $\nu(\xi)=mp(\xi)/2\pi^2\hbar^3$ is the density of states at energy $\xi+\mu$. The chemical potential μ is equal to the Fermi energy: $\mu=\varepsilon_F$.

The main contribution to the pairing comes from the states near the Fermi surface, where $|\xi|,|\xi'|\ll\varepsilon_F$. In order to single out this contribution in the rhs of Eq. (13), we introduce a characteristic energy $\bar{\omega}$ that obeys the constraint $T\ll\bar{\omega}$, and is of the order of the Fermi energy. We then divide the integral over ξ' in Eq. (13) into two parts: (a) the integration of $R(\xi,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}')$ from $-\bar{\omega}$ to $\bar{\omega}$, and (b) the integration of $[R(\xi,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}')-R(\xi,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}')]$ from $-\bar{\omega}$ to $\bar{\omega}$, plus the integration of $R(\xi,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}')$ from $-\varepsilon_F$ to $-\bar{\omega}$ and from $\bar{\omega}$ to ∞. In part (a) we use the asymptotic formula

$$\int_{-\bar{\omega}}^{\bar{\omega}} d\xi'[\tanh(\xi'/2T)/2\xi']\approx\ln\frac{2\exp(\gamma)\bar{\omega}}{\pi T},$$

where $\gamma=0.5772$ is the Euler constant. In part (b) we replace $\tanh(\xi'/2T)$ by the step function (omitting the unimportant contribution from a narrow interval $|\xi'|\lesssim T\ll\bar{\omega}$) and integrate in parts. As a result, Eq. (13) takes the form

$$\Delta(\xi,\mathbf{n})=-\ln\left[\frac{2\exp(\gamma)\bar{\omega}}{\pi T}\right]\int\frac{d\mathbf{n}'}{4\pi}R(\xi,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}') +\frac{1}{2}\int_{-\varepsilon_F}^{\infty}d\xi'\ln\frac{|\xi'|}{\bar{\omega}}\frac{d}{d|\xi'|} \times\int\frac{d\mathbf{n}'}{4\pi}\{R(\xi,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}')\}, \tag{14}$$

where the first and second terms in the rhs come from parts (a) and (b), respectively.

One can easily see that the ratio of the second to the first term in Eq. (14) is as small as $1/\ln[2\exp(\gamma)\bar{\omega}/\pi T]$. Therefore the second term is only important for the preexponential factor in the expression for the critical temperature and will be omitted in this section. This is equivalent to the commonly used BCS approach where the kernel $R(\xi,\mathbf{n};\xi',\mathbf{n}')$ is replaced by $R(0,\mathbf{n};0,\mathbf{n}')$ for $|\xi|,|\xi'|\leqslant\bar{\omega}$ and by zero otherwise. Putting $\xi=0$ in Eq. (14) we obtain the following equation for finding the critical temperature:

$$\Delta(0,\mathbf{n})=-\ln\left[\frac{2\exp(\gamma)\bar{\omega}}{\pi T}\right]\int\frac{d\mathbf{n}'}{4\pi}R(0,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}'). \tag{15}$$

The anisotropic character of the scattering amplitude leads to a nontrivial angular dependence of the order parameter $\Delta(0,\mathbf{n})$. In order to analyze the possibility of pairing we expand $\Delta(0,\mathbf{n})$ in terms of a complete set of eigenfunctions $\phi_s(\mathbf{n})$ of the integral operator with the kernel $R(0,\mathbf{n};0,\mathbf{n}')$:

$$\Delta(0,\mathbf{n})=\sum_{s=0}^{\infty}\Delta_s\phi_s(\mathbf{n}), \tag{16}$$

$$\int\frac{d\mathbf{n}'}{4\pi}R(0,\mathbf{n};0,\mathbf{n}')\phi_s(\mathbf{n}')=\lambda_s\phi_s(\mathbf{n}),\quad s=0,1,\ldots. \tag{17}$$

The functions $\phi_s(\mathbf{n})$ are normalized by the condition $\int(d\mathbf{n}/4\pi)\phi_s^2(\mathbf{n})=1$, and they are labeled by the index s in such a way that the eigenvalues $\lambda_s<\lambda_{s+1}$. Then Eq. (15) reduces to a set of equations

$$\Delta_s\left(1+\lambda_s\ln\frac{2\exp(\gamma)\bar{\omega}}{\pi T}\right)=0.$$

The appearance of a nontrivial solution for $\Delta(0,\mathbf{n})$ below a certain critical temperature requires the presence of at least one negative eigenvalue λ_s. For a single eigenvalue $\lambda_{s*}<0$, the critical temperature immediately follows from the condition $\{1+\lambda_{s*}\ln[2\exp(\gamma)\bar{\omega}/\pi T_c]\}=0$, and we have $\Delta_{s*}\neq0$ and $\Delta_s=0$ for $s\neq s^*$. In the case of several negative

eigenvalues $\lambda_s<0$, one has to choose the solution that corresponds to the lowest eigenvalue as it gives the highest critical temperature.

Using Eq. (3) one finds that negative λ_s corresponds to eigenfunctions ϕ_s which are independent of the azimuthal angle φ. This means that only spherical harmonics with zero projection m of the angular momentum l appear in their decomposition. For these functions the kernel $R(0,\mathbf{n};0,\mathbf{n}')$ can be reduced to its average over the azimuthal angles φ and φ'. Using Eq. (3) for $\Gamma_d(p_F\mathbf{n},p_F\mathbf{n}')$, we obtain

$$R(0,\cos\theta;0,\cos\theta')=2\pi\frac{nd^2}{\varepsilon_F}\left(\frac{3}{2}|\cos\theta-\cos\theta'|-1\right), \tag{18}$$

where θ and θ' are the polar angles for the vectors $\mathbf{n}$ and $\mathbf{n}'$, and n is the gas density. Note that the first multiple in the rhs of Eq. (18) is a small parameter of the theory, given by Eq. (12) and representing the ratio of the mean dipole-dipole interaction energy to the Fermi energy ε_F.

Keeping in mind that due to the Pauli principle only odd angular momenta are present, we obtain the solutions of Eq. (16):

$$\phi_s(\mathbf{n})=\sqrt{2}\sin\left(\frac{\pi}{2}(1+2s)\cos(\theta)\right),$$
$$\lambda_s=-\frac{nd^2}{\varepsilon_F}\frac{12}{\pi(1+2s)^2}. \tag{19}$$

The lowest eigenvalue is $\lambda_0=-12nd^2/\pi\varepsilon_F$. Therefore the angular dependence of the order parameter will be characterized by the function $\phi_0(\mathbf{n})$ (see Sec. VI for details). The critical temperature is then given by

$$T_c=\frac{2\exp(\gamma)\bar{\omega}}{\pi}\exp\left(-\frac{1}{|\lambda_0|}\right). \tag{20}$$

In the BCS approach the preexponential factor ($\bar{\omega}$) remains undetermined. One can only argue that it is of the order of ε_F. We thus have

$$T_c^{\mathrm{BCS}}\sim\varepsilon_F\exp\left(-\frac{\pi\varepsilon_F}{12nd^2}\right). \tag{21}$$

In Ref. [14] the exponent in the expression for T_c^{BCS} is only expressed in terms of the scattering amplitude which should be found from the solution of a set of coupled equations. The estimate for this exponent in Ref. [20] takes into account only the $p-p$ scattering channel and contains a numerical error.

In order to find the preexponential factor one has to include the contribution from the second term in Eq. (14), together with the second order corrections to the eigenvalue λ_0. These corrections originate from the second order many-body effects and from the second order corrections to the scattering amplitude, described by the second terms in Eqs. (6) and (11), respectively.

V. GM APPROACH. THE CALCULATION OF THE PREEXPONENTIAL FACTOR

We now proceed with the calculation of the preexponential factor in the expression (20) for the critical temperature. We first consider the contribution of the second term in the rhs of Eq. (14), which is logarithmically small compared to the already calculated first term. For this purpose we specify the value of $\bar{\omega}$ by the condition

$$\int\frac{d\mathbf{n}}{4\pi}\phi_0(\mathbf{n})\int_{-\varepsilon_F}^{\infty}d\xi'\ln\frac{|\xi'|}{\bar{\omega}}\frac{d}{d|\xi'|}\times\int\frac{d\mathbf{n}'}{4\pi}\{R(0,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}')\}=0. \tag{22}$$

Then, using Eqs. (16) and (17) we obtain the following expression for $\bar{\omega}$:

$$\ln\bar{\omega}=-\frac{1}{\lambda_0}\int\frac{d\mathbf{n}}{4\pi}\phi_0(\mathbf{n})\frac{1}{2}\int_{-\varepsilon_F}^{\infty}d\xi'\ln|\xi'|\frac{d}{d|\xi'|}\times\int\frac{d\mathbf{n}'}{4\pi}\left\{R(0,\mathbf{n};\xi',\mathbf{n}')\frac{\Delta(\xi',\mathbf{n}')}{\Delta_0}\right\}. \tag{23}$$

This definition of $\bar{\omega}$ immediately leads to Eq. (20) for the critical temperature and allows us to rewrite Eq. (14) in the form

$$\Delta(\xi,\mathbf{n})=\frac{1}{\lambda_0}\int\frac{d\mathbf{n}'}{4\pi}R(\xi,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}')-\int\frac{d\mathbf{n}'}{4\pi}\cdot\frac{1}{2}\int_{-\varepsilon_F}^{\infty}\frac{d\xi'}{|\xi'|}\left\{R(\xi,\mathbf{n};\xi',\mathbf{n}')\Delta(\xi',\mathbf{n}')-\frac{R(\xi,\mathbf{n};0,\mathbf{n}')}{\lambda_0}\frac{\Delta(0,\mathbf{n}')}{\Delta_0}\int\frac{d\mathbf{n}_1}{4\pi}\times\int\frac{d\mathbf{n}_2}{4\pi}\phi_0(\mathbf{n}_1)R(0,\mathbf{n}_1;\xi',\mathbf{n}_2)\Delta(\xi',\mathbf{n}_2)\right\}, \tag{24}$$

where the second term in the rhs is proportional to the small parameter of the theory nd^2/ε_F and can thus be considered as a perturbation. This follows from the fact that the bracket in this term vanishes for $\xi'\to0$. As a result, in contrast to the first term of the rhs, the second term does not contain the large logarithm $\ln(\bar{\omega}/T)\sim\lambda_0^{-1}\sim(\varepsilon_F/nd^2)$.

The leading contribution to the angular dependence of the order parameter on the Fermi surface comes from the term with $s=0$ in Eq. (16): $\Delta(0,\mathbf{n})=\Delta_0\phi_0(\mathbf{n})$. Therefore, to the leading order in nd^2/ε_F, the solution of Eq. (24) is

$$\Delta(\xi,\mathbf{n})\approx\frac{1}{\lambda_0}\int\frac{d\mathbf{n}'}{4\pi}R(\xi,\mathbf{n};0,\mathbf{n}')\Delta_0\phi_0(\mathbf{n}'). \tag{25}$$

After substituting this expression into Eq. (23) and performing a numerical integration, we obtain

$$\bar{\omega} \approx \exp\left(-\frac{1}{\lambda_0}\int \frac{d\mathbf{n}}{4\pi}\phi_0(\mathbf{n})\frac{1}{2}\int_{-\varepsilon_F}^{\infty} d\xi' \ln|\xi'|\frac{d}{d|\xi'|} \times \int \frac{d\mathbf{n}'}{4\pi}\left\{ R(0,\mathbf{n};\xi',\mathbf{n}')\frac{\Delta(\xi',\mathbf{n}')}{\Delta_0}\right\}\right) = 0.42\varepsilon_F . \tag{26}$$

Corrections to Eqs. (25) and (26) are related to the terms with $s \neq 0$ in Eq. (16), and from Eq. (24) we find that the quantities $\Delta_{s\neq 0} \sim \Delta_0(nd^2/\varepsilon_F)$. These corrections lead to the relative contribution of the order of nd^2/ε_F to the preexponential factor for the critical temperature and hence will be neglected.

We now calculate the contributions from the second terms in Eqs. (6) and (11). As one can see from Eq. (16), these terms result in the correction for the eigenvalue λ_0:

$$\delta\lambda_0 = \nu(0)\int \frac{d\mathbf{n}}{4\pi}\int \frac{d\mathbf{n}'}{4\pi}\phi_0(\mathbf{n})\left\{ \delta V(p_F\mathbf{n}, p_F\mathbf{n}') - \int \frac{d\mathbf{q}}{(2\pi\hbar)^3} V_d(\mathbf{p}-\mathbf{q})K_0(q)V_d(\mathbf{q}-\mathbf{p}')\right\}\phi_0(\mathbf{n}'). \tag{27}$$

The first term in the integrand of Eq. (27) originates from many-body effects, and the quantity $\delta V(\mathbf{p},\mathbf{p}')$ is shown diagrammatically in Fig. 1. The analytical expressions for the diagrams in Figs. 1(a)–1(d), read:

$$\delta V_a(\mathbf{p},\mathbf{p}') = \int \frac{d\mathbf{q}}{(2\pi)^3}\frac{N(\mathbf{q}+\mathbf{p}_-/2)-N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2}-\xi_{\mathbf{q}-\mathbf{p}_-/2}}V_d^2(\mathbf{p}_-),$$

$$\delta V_b(\mathbf{p},\mathbf{p}') = -\int \frac{d\mathbf{q}}{(2\pi)^3}\frac{N(\mathbf{q}+\mathbf{p}_-/2)-N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2}-\xi_{\mathbf{q}-\mathbf{p}_-/2}} \times V_d(\mathbf{p}_-)V_d(\mathbf{q}-\mathbf{p}_+/2),$$

$$\delta V_c(\mathbf{p},\mathbf{p}') = -\int \frac{d\mathbf{q}}{(2\pi)^3}\frac{N(\mathbf{q}+\mathbf{p}_-/2)-N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2}-\xi_{\mathbf{q}-\mathbf{p}_-/2}} \times V_d(\mathbf{p}_-)V_d(\mathbf{q}+\mathbf{p}_+/2),$$

$$\delta V_d(\mathbf{p},\mathbf{p}') = -\int \frac{d\mathbf{q}}{(2\pi)^3}\frac{N(\mathbf{q}+\mathbf{p}_+/2)-N(\mathbf{q}-\mathbf{p}_+/2)}{\xi_{\mathbf{q}+\mathbf{p}_+/2}-\xi_{\mathbf{q}-\mathbf{p}_+/2}} \times V_d(\mathbf{q}-\mathbf{p}_-/2)V_d(\mathbf{q}+\mathbf{p}_-/2).$$

Here $\mathbf{p}_\pm = \mathbf{p} \pm \mathbf{p}'$, and $N(\mathbf{p})$ is the Fermi-Dirac distribution at zero temperature. The integrals related to the first term in the rhs of Eq. (27), with $\delta V(\mathbf{p},\mathbf{p}') = \Sigma_{\alpha=a,\ldots,d}\delta V_\alpha(\mathbf{p},\mathbf{p}')$, were calculated numerically by using the Monte Carlo method. Each of the terms $\delta V_\alpha(\mathbf{p},\mathbf{p}')$ provides a correction $\delta\lambda_0^{(\alpha)} = [\nu(0)4\pi d^2/3]^2\eta_\alpha$. For the coefficients η_α we find $\eta_a = 0.19$, $\eta_b = \eta_c = -0.08$, and $\eta_d = 0.42$. Thus the first term in the rhs of Eq. (27) gives the correction

$$\delta\lambda_0^{(1)} = 0.45\left(\nu(0)\frac{4\pi d^2}{3}\right)^2.$$

The second term in the integrand of Eq. (27) comes from the second order correction to the scattering amplitude Γ_d. For the correction to λ_0, originating from this term, our numerical calculation gives

$$\delta\lambda_0^{(2)} = -0.86\left(\nu(0)\frac{4\pi d^2}{3}\right)^2.$$

Note that the function $\phi_0(\mathbf{n})$ is odd with respect to $\cos\theta$. For this reason, the integration over $d\mathbf{n}$ and $d\mathbf{n}'$ eliminates the formal divergency of the integral over $d\mathbf{q}$ at large q.

The total correction to the eigenvalue λ_0 is then

$$\delta\lambda_0 = \delta\lambda_0^{(1)} + \delta\lambda_0^{(2)} = -0.41\left(\nu(0)\frac{4\pi d^2}{3}\right)^2. \tag{28}$$

On the basis of Eqs. (20), (26), and (28), we obtain the final expression for the critical temperature:

$$T_c = \frac{2\exp(\gamma)}{\pi}\times 0.42\,\varepsilon_F \exp(-1/|\lambda_0+\delta\lambda_0|) \approx 1.44\,\varepsilon_F \exp(-\pi\varepsilon_F/12nd^2). \tag{29}$$

It is worth noting that if we include only the p-p scattering channel the exponent in Eq. (29) will be larger by a factor of $10/\pi^2$. The preexponential factor becomes then larger by a factor of 1.1. This shows that the main contribution to the pairing comes from the p-wave scattering channel.

VI. ANISOTROPIC ORDER PARAMETER

In order to find the temperature dependence of the order parameter for $T \leqslant T_c$, we have to include the nonlinear term in the gap equation (15). This term can be written as

$$\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3}V_d(\mathbf{p}-\mathbf{p}')\left[\frac{1}{\cosh^2(\xi'/2T)} - \frac{\tanh(\xi'/2T)}{\xi'/2T}\right]\frac{\Delta^3(\mathbf{p}')}{8\xi'^2\,T},$$

where we neglect the many-body correction to the interparticle interaction. The expression in the square brackets vanishes as $|\xi'|^{-3}$ for $|\xi'|\to\infty$. Therefore the main contribution to the integral comes from the region of small ξ', i.e., from $\mathbf{p}'$ close to the Fermi momentum p_F. This allows us to write

$$\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} V_d(\mathbf{p}-\mathbf{p}') \left[\frac{1}{\cosh^2(\xi'/2T)} - \frac{\tanh(\xi'/2T)}{\xi'/2T} \right] \frac{\Delta^3(\mathbf{p}')}{8\xi'^2\, T}$$

$$\approx \int \frac{d\mathbf{n}'}{4\pi} V_d(\mathbf{p}-p_F\mathbf{n}')\Delta^3(p_F\mathbf{n}') \times \int \frac{p'^2\, dp'}{2\pi^2\hbar^3} \left[\frac{1}{\cosh^2(\xi'/2T)} - \frac{\tanh(\xi'/2T)}{\xi'/2T} \right] \frac{1}{8\xi'^2\, T}$$

$$= -\frac{7\zeta(3)}{8\pi^2 T^2} \int \frac{d\mathbf{n}'}{4\pi} R(\xi,\mathbf{n};0,\mathbf{n}')\Delta^3(0,\mathbf{n}'),$$

where $\zeta(z)$ is the Riemann zeta function. As a result, to first order in V_d the nonlinear gap equation reads

$$\Delta(\xi,\mathbf{n}) = -\ln\frac{2\gamma\overline{\omega}}{\pi T} \int \frac{d\mathbf{n}'}{4\pi} R(\xi,\mathbf{n};0,\mathbf{n}')\Delta(0,\mathbf{n}') - \frac{7\zeta(3)}{8\pi^2 T^2} \int \frac{d\mathbf{n}'}{4\pi} R(\xi,\mathbf{n};0,\mathbf{n}')\Delta^3(0,\mathbf{n}'). \quad (30)$$

With the order parameter $\Delta(0,\mathbf{n})$ from Eq. (16), where now $\Delta_s = \Delta_s(T)$ and $\Delta_s(T) \to 0$ for $T \to T_c$, Eq. (30) takes the form

$$\sum_{s=0}^{\infty} \Delta_s \left(1 + \lambda_s \ln\frac{2\gamma\overline{\omega}}{\pi T} \right) \phi_s(\mathbf{n}) - \frac{7\zeta(3)}{8\pi^2 T^2} \sum_{s=0}^{\infty} \lambda_s \phi_s(\mathbf{n}) \times \left(\sum_{\{s_i\}} C^s_{s_1 s_2 s_3} \Delta_{s_1} \Delta_{s_2} \Delta_{s_3} \right) = 0. \quad (31)$$

The coefficients $C^s_{s_1 s_2 s_3}$ follow from the relation

$$\phi_{s_1}(\mathbf{n})\phi_{s_2}(\mathbf{n})\phi_{s_3}(\mathbf{n}) = \sum_s C^s_{s_1 s_2 s_3} \phi_s(\mathbf{n}).$$

For temperatures below T_c, satisfying the inequality $(T_c - T)/T_c \ll 1$, Eq. (31) can be rewritten as

$$\sum_{s=0}^{\infty} \Delta_s \left(\frac{\lambda_0 - \lambda_s}{\lambda_0} + \lambda_s \frac{T_c - T}{T_c} \right) \phi_s(\mathbf{n}) - \frac{7\zeta(3)}{8\pi^2 T_c^2} \sum_{s=0}^{\infty} \lambda_s \phi_s(\mathbf{n}) \times \left(\sum_{\{s_i\}} C^s_{s_1 s_2 s_3} \Delta_{s_1} \Delta_{s_2} \Delta_{s_3} \right) = 0, \quad (32)$$

where we neglect higher powers of $(T_c - T)/T_c$. It can be easily seen from Eq. (32) that for $T \to T_c$ one has $\Delta_0 \sim (T_c - T)^{1/2}$, and that Δ_s with $s > 0$ are either equal to zero or proportional to $(T_c - T)^{3/2}$. Therefore the equation for the leading coefficient Δ_0 is

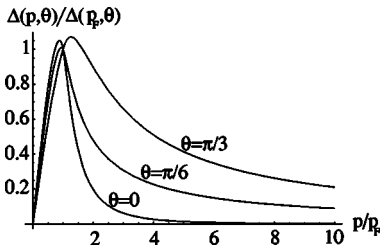

FIG. 2. The order parameter $\Delta(p,\theta)$ [in units of $\Delta(p_F,\theta)$] as a function of the momentum p (in units of p_F) for various values of the polar angle θ.

$$\frac{T_c - T}{T_c}\Delta_0 - \frac{7\zeta(3)}{8\pi^2 T_c^2} C^0_{000}\Delta_0^3 = 0,$$

where the coefficient C^0_{000} is equal to 3/2. We thus obtain the following expression for the order parameter on the Fermi surface ($\xi = 0$):

$$\Delta(0,\mathbf{n}) = \frac{4\pi}{\sqrt{21\zeta(3)}} T_c \sqrt{\frac{T_c - T}{T_c}} \phi_0(\mathbf{n}) = 2.5\, T_c \sqrt{\frac{T_c - T}{T_c}} \phi_0(\mathbf{n}), \quad \frac{T_c - T}{T_c} \ll 1. \quad (33)$$

For $\xi \neq 0$, i.e., $p \neq p_F$, the order parameter can be calculated by using Eq. (25). Figure 2 shows the numerically calculated dependence of the order parameter on the modulus of the momentum $\mathbf{p}$ for various values of the angle θ between the vector $\mathbf{p}$ and the direction of dipoles. Note that for both *s*- and *p*-wave pairing due to a short-range interaction, the order parameter is momentum independent for p satisfying the condition of the ultracold limit and rapidly decays at larger p. The momentum dependence of the order parameter for dipolar gases results in a nonuniform energy gap for single-particle excitations and can, for example, manifest itself in processes with a large (of the order of p_F) momentum transfer.

In Eq. (33) the anisotropy of the order parameter in the momentum space is described by the function $\phi_0(\mathbf{n}) = \sqrt{2}\sin[(\pi/2)\cos\theta]$. The order parameter is an odd function of $\cos\theta$ and is negative for $\pi/2 < \theta \leqslant \pi$. This does not cause any problems as all physical quantities are determined by $|\Delta|$. The maximum value of $|\Delta(0,\mathbf{n})|$ is reached in the direction of the dipoles, i.e., for $\theta = 0$ ($\phi_0 = \sqrt{2}$). In the direction perpendicular to the dipoles ($\theta = \pi/2$) the order parameter vanishes.

If we consider only the p-p scattering channel, the angular dependence of the order parameter will be determined by the function $\sqrt{3}\cos\theta$ instead of $\phi_0(\mathbf{n})$. The coefficient C^0_{000} is then equal to 9/4, and the result for the order parameter reads

$$\Delta_{pp}(0,\mathbf{n}) = \frac{4\pi}{3}\sqrt{\frac{2}{7\zeta(3)}}T_c\sqrt{\frac{T_c-T}{T_c}}\sqrt{3}\cos\theta$$
$$= 2.5\,T_c\sqrt{\frac{T_c-T}{T_c}}\sqrt{2}\cos\theta, \quad \frac{T_c-T}{T_c}\ll 1. \tag{34}$$

The angular dependence of $|\Delta_{pp}(0,\mathbf{n})|$ is qualitatively similar to that of the true order parameter $|\Delta(0,\mathbf{n})|$. The maximum value of $|\Delta_{pp}(0,\mathbf{n})|$ is also reached in the direction of the dipoles and it is exactly equal to the maximum value of $|\Delta(0,\mathbf{n})|$. Also, $|\Delta_{pp}(0,\mathbf{n})|$ vanishes in the perpendicular direction. However, for intermediate values of θ the quantity $|\Delta_{pp}(0,\mathbf{n})|$ can be up to 40% smaller than $|\Delta(0,\mathbf{n})|$.

The anisotropy of the order parameter provides a major difference of the properties of the superfluid dipolar Fermi gas from those of the (two-component) fermionic gas with the *s*-wave pairing due to short-range intercomponent interaction. This anisotropy ensures the anisotropic momentum dependence of the gap in the spectrum of single-particle excitations, which appears below the transition temperature T_c. For example, excitations with momenta in the direction of the dipoles acquire the largest gap. In contrast to this, the eigenenergies of excitations with momenta perpendicular to the dipoles remain unchanged. The properties of collective excitations are also expected to have a nontrivial dependence on the direction of their momenta. Therefore the response of the dipolar superfluid Fermi gas to small external perturbations will have a pronounced anisotropic character.

The Fourier transform of $\Delta(p,\theta)$ shows that the momentum-space anisotropy of the order parameter transforms into its spatial anisotropy. The order parameter is maximum in the direction of the dipoles, and is equal to zero in the direction perpendicular to the dipoles.

Another distinguished feature of the superfluid dipolar Fermi gas is related to the temperature dependence of the specific heat. Well below the critical temperature the single-particle contribution to the specific heat is proportional to T^2, rather than being exponentially small as in the case of the *s*-wave pairing. This follows from the fact that the energy ε of single-particle excitations has a line of zeros on the Fermi surface: $\varepsilon(p_F)=0$ for the angles at which $\Delta(p_F,\mathbf{n})=0$, i.e., for $\theta=\pi/2$ and an arbitrary azimuthal angle φ. As a consequence, the density of states in the vicinity of the Fermi energy is $\nu(\varepsilon)\sim\varepsilon$ for $\varepsilon\ll\Delta_0$. Therefore, at temperatures $T\ll\Delta_0\sim T_c$, the temperature dependent part of the energy of the system is proportional to T^3, and the specific heat is hence proportional to T^2. This contribution is much larger than the one of collective modes which is $\propto T^3$ and is dominant in the case of the *s*-wave pairing.

It should also be mentioned that the properties of the superfluid dipolar fermionic gas are different from the properties of the gas with the *p*-wave pairing originating from a short-range attractive interaction in the *p*-wave channel. The reason is that in the latter case the order parameter is isotropic, similar to the *B*-phase of superfluid ^{3}He. The order parameter of dipolar gases is also different from the order parameter of the *A* phase of ^{3}He where the gap vanishes only at two points on the Fermi sphere, i.e., at the poles of the sphere $\theta=0$ and $\theta=\pi$.

The anisotropy of the order parameter of dipolar gases is similar to that in the polar phase of superfluid liquid ^{3}He, not realized in experiments as it has higher energy than experimentally observed *A* and *B* phases (see, e.g., Ref. [27]). For the polar phase the order parameter is also equal to zero on the equator of the Fermi sphere ($\theta=\pi/2$ and an arbitrary φ). The situation where the order parameter is zero on one or several lines on the Fermi surface is encountered in heavy-fermion compounds (for a review of possible superconducting phases of heavy-fermion compounds belonging to different crystalline groups see, e.g., Ref. [28]). In these cases the temperature dependence of the specific heat is also $\propto T^2$ (see, e.g., Ref. [29]). However, from a general point of view, one would expect a different physical behavior of dipolar gases, for example, with regard to the frequency and angular dependence of the response. This is because of the different types of symmetry groups: continuous rotational group for dipolar gases and discrete crystalline group in the case of heavy-fermion materials (see Ref. [28] for more details).

VII. CONCLUDING REMARKS

Our results show prospects for achieving the BCS transition in single-component trapped gases of dipolar particles, in particular for (electrically polarized) fermionic polar molecules. As has been shown in Refs. [20,30,31], the BCS transition temperature T_c in trapped Fermi gases is very close to that for the uniform gas of density n equal to the maximum density in the trap. This result is valid if T_c is much larger than the trap frequencies, which is generally the case for achievable temperatures. As we consider temperatures T_c significantly lower than the Fermi energy (Fermi temperature) $\varepsilon_F=(6\pi^2 n)^{2/3}\hbar^2/2m$, the density profile of the trapped gas is already independent of the temperature and is given by the well-known Thomas-Fermi relation. Thus, for estimating T_c in the trapped case, we will use Eq. (29) where the quantity n is now the temperature-independent maximum density in the trap.

We first compare our equations (21) and (29) with the well-known BCS formula (see, e.g., [23]) for the two-component Fermi gas with short-range attractive intercomponent interaction. In the latter case the exponent is expressed in terms of the *s*-wave scattering length a and is equal to $\pi\hbar/2p_F|a|$. We then see that our dipole-dipole scattering with odd orbital angular momenta is equivalent to having the *s*-wave scattering length

$$a_d = -\frac{2md^2}{\pi^2\hbar^2}. \tag{35}$$

Accordingly, Eq. (29) takes the form

$$T_c = 1.44\varepsilon_F\exp\left\{-\frac{\pi\hbar}{2p_F|a_d|}\right\}, \tag{36}$$

where $p_F=\hbar(6\pi^2 n)^{1/3}$.

Qualitatively, the result of Eq. (35) is more or less expected, since $|a_d|$ turns out to be of the order of the characteristic radius of the dipole-dipole interaction, $r_* \sim md^2/\hbar^2$, introduced in Sec. III. Our approach assumes a small ratio T_c/ε_F following from Eq. (36). In the present stage, it is not clear to which temperatures one can cool the gas of fermionic polar molecules [for example, in current studies of atomic Fermi gases one has severe limitations, although technical, to reach below $0.2\varepsilon_F$ (see [2,32])]. Below, giving the numbers for absolute values of T_c and n we will keep in mind the ratio $T_c/\varepsilon_F \sim 0.1$.

For most polar molecules the electric dipole moment ranges from 0.1 to 1 Debye. For example, the dipole moment of fermionic ammonia molecules $^{15}ND_3$[1] is $d=1.5$ D, and we have the effective scattering length $a_d=-1450$ Å. This is larger than the scattering length for the intercomponent interaction in the widely discussed case of two-species fermionic gas of ^{6}Li. From Eq. (36) we find that the BCS transition temperature for the single-component ND_3 dipolar gas approaches 100 nK at densities exceeding $n \sim 10^{12}$ cm^{-3}. Another interesting example is a linear fermionic molecule HCN which has dipole moment $d=2.98$ D, and the corresponding effective scattering length $a_d=-7400$ Å.

Remarkably, in ultracold single-component fermionic gases inelastic decay processes will be strongly suppressed due to the Pauli exclusion principle. For two identical fermions with momentum p of the relative motion, the pair correlation function behaves as $(pr/\hbar)^2$ at interparticle distances r smaller than the de Broglie wavelength $\hbar/p$. Generally, inelastic processes occur at short interparticle distances R_0 which in the ultracold limit are much smaller than $\hbar/p$. Therefore, two-body inelastic collisions will be suppressed as $(pR_0)^2$ compared to the bosonic case where the pair correlation function is of order unity at any r outside the region of interparticle interaction. As a result, in a nondegenerate gas of fermions the inelastic rate decreases with temperature and is suppressed as T/ε_0, where the energy $\varepsilon_0=\hbar^2/mR_0^2$. In a quantum degenerate Fermi gas a characteristic momentum of particles is of the order of p_F and the suppression factor is $\sim(\varepsilon_F/\varepsilon_0)$. The suppression of two-body inelastic collisions in fermionic gases was first found for spin relaxation in atomic deuterium [12]. For the rate of three-body recombination we expect even a stronger suppression, i.e., by a factor of $(T/\varepsilon_0)^2$ for the nondegenerate gas and by a factor of $(\varepsilon_F/\varepsilon_0)^2$ in the regime of quantum degeneracy ($T<\varepsilon_F$). On the other hand, the dipole-dipole elastic scattering occurs at distances of the order of the de Broglie wavelength of particles and is not suppressed by the reduction of the pair correlation function at small distances. As discussed above, the amplitude of dipole-dipole elastic scattering remains constant in the ultracold limit. Therefore at temperatures above ε_F the elastic collisional rate and, hence, the rate of forced evaporative cooling are proportional to $\sqrt{T}$, similar to the case of Bose gases. Together with the suppression of inelastic rates, this is important for cooling dipolar fermionic gases at temperatures above ε_F.

At temperatures significantly lower than ε_F the Pauli blocking will suppress the rates of elastic collisions and forced evaporative cooling (see [32]). Nevertheless, the above discussed suppression of inelastic rates can help to reach lower temperatures and/or higher densities than in the common case of two-component atomic Fermi gases. In particular, it will reduce the recently predicted [33] effect of heating induced by the creation of holes in the single-particle distribution. These holes appear due to inelastic losses of particles and the corresponding heating rate is proportional to the loss rate.

Interestingly, in gases of atoms with induced dipole moments one can obtain the effective scattering length $|a_d| \sim 100$ Å [34,35]. By using a high dc electric field ($\sim 10^6$ V/cm) [13] one can induce permanent atomic dipole moments close to $0.1D$. One obtains the same or even larger values of d for the time averaged dipole moment of an atom, induced by a stroboscopic laser coupling of the ground atomic state to a Rydberg state [19]. Then one can think of achieving the BCS transition in such single-component fermionic gases at temperatures ~ 100 nK and atomic densities in between 10^{14} and 10^{15} cm^{-3}.

[1] Bosonic ($^{14}ND_3$) and fermionic ($^{15}ND_3$) ammonia molecules were recently trapped at temperatures of around 30 mK in the Rijnhuizen experiment [18].

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[34] In the case of atoms with large magnetic moments one can also think of efficient elastic scattering due to the magnetic dipole interaction. For chromium atoms, the magnetic moment is $6\mu_B$, where μ_B is the Bohr magneton. This is equivalent to having the dipole moment $d=0.056$ D, and Eq. (35) leads to $a_d \approx -5$ Å. Equation (36) then shows that even at nanokelvin temperatures the BCS transition requires extremely high densities at which the gas will be unstable because of recombination processes.
[35] For fermionic $^{13}C^{16}O$ molecules in the ground electronic state, which have dipole moment $d=0.1$ D and effective scattering length $a_d \approx -9$ Å, the situation is similar to that for chromium atoms. On the other hand, the dipole moment of the metastable molecule $^{13}C^{16}$ O in the first electronically excited $a^3\Pi$ state is equal to 1.37 D, and the effective scattering length is $a_d \approx -1670$ Å. This opens interesting possibilities to (optically) manipulate the permanent dipole moment d and the scattering length a_d in CO molecular gases.