Electron loss from heavy heliumlike projectiles in ultrarelativistic collisions with many-electron atomic targets

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We study single- and double-electron loss from heavy heliumlike projectiles in ultrarelativistic collisions with neutral many-electron target atoms. The simultaneous interaction of the target with two projectile electrons is found to be the dominant process in the double-electron loss provided the atomic number of the projectile, Z_p, that of the target, Z_t, and the collision velocity, v, satisfy the condition $Z_pZ_t/v>0.4$. It is shown that for a wide range of projectile and target atomic numbers the asymptotic double-to-single loss ratio strongly depends on the target atomic number but is nearly independent of the nuclear charge of the projectile. It is also demonstrated that many-photon exchange between the target and each of the projectile electrons considerably influences the double loss in collisions with very heavy targets.

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I. INTRODUCTION

The double-to-single ionization ratio, $R_{\mathrm{He}}=\sigma^{2+}/\sigma^{+}$, for helium atoms colliding with fast nonrelativistic charged particles was extensively discussed in atomic physics literature (see, e.g., Refs. [1,2] and references therein). It was established that for high-velocity collisions with low-charged projectiles, when the projectile charge is much smaller than the collision velocity and both single and double ionization of helium occur via the single-virtual-photon exchange between the helium target and the projectile, this ratio reaches a constant value of $\approx 2.5\times 10^{-3}$. Taking into account that collision velocities cannot exceed the speed of light $c=137$ a.u. it was suggested in Ref. [3] that the highest projectile charge, for which this limit may still be reached, is restricted to projectile charge states not exceeding 13. It was pointed out, however, that the inclusion of relativistic effects into the consideration leads to the conclusion that in ultrarelativistic collisions, when the Lorentz factor $\gamma\rightarrow\infty$, this ratio can be reached, at least in principle, for any projectile charge state [4]. Thus, the value $R_{\mathrm{He}}=2.5\times 10^{-3}$ represents a true and unique high-energy limit for ionization of atomic helium by charged projectiles. In addition, a simple analysis suggests that the corresponding high-energy limit for the double-to-single ionization ratio exists also for other targets and that the value of this limit should depend only on target properties and be independent of the atomic number of a charged projectile.

Collision physics becomes, in general, much richer when not only the target but also the projectile have (active) electrons. Nonrelativistic fast collisions of projectiles, carrying initially one electron, with neutral targets have been studied during several decades and physics of such collisions is rather well understood at present (at least those of them which can be treated within first-order theories, see, e.g., Refs. [1,5] and references therein). Although the processes of projectile-electron excitation and loss in relativistic collisions have also been extensively studied during the last two decades (see, e.g., Ref. [6] and references therein), only very recently the first-order plane-wave Born [7] and semiclassical [8] approximations were rigorously formulated for such processes and applied to calculate electron excitation and loss in collisions of hydrogenlike heavy ions with atomic targets.

In some cases, a heavy projectile may carry several active electrons and more than one projectile electron can be simultaneously excited and/or lost in a collision with a neutral atomic target. During the last two decades some investigations were already devoted to the study of these processes. In particular, the processes of multiple projectile-electron loss and loss excitation in collisions in the domain of lower relativistic energies $E\lesssim 1$ GeV/u were considered in Refs. [9–11] (see also Ref. [12]). One of the simplest and therefore fundamental examples of processes, where more than one projectile electron can be involved, is represented by relativistic collisions of heliumlike heavy ions with neutral atomic targets. In the present paper, we want to address the topic of single- and double-electron loss from (or single and double ionization of) a heavy heliumlike projectile which is initially in the ground state and collides with a neutral many-electron atomic target at asymptotically high energies, where projectile ionization cross sections become practically independent of the collision Lorentz factor (Ref. [13], see also below).

The main goals we pursue here are twofold. First, we wish to obtain the asymptotic high-energy double-to-single ionization ratio and to find out how it depends on the atomic numbers of the projectile and the target. Second, although at the collision energies of interest the so-called binding and polarization effects [14] are negligible, the higher-order terms in the corresponding Born series in the interaction between the projectile electrons and the target (so-called many-photon exchange), may still considerably affect the loss process. Therefore, we are also interested in the study of the influence on the electron loss of the exchange of many virtual photons between the target and each of the projectile electrons. It was shown in Ref. [15] that the deviation from predictions of the first-order consideration can be non-negligible for ionization of hydrogenlike projectiles colliding with heavy atomic targets. In the case of the double-electron

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loss from projectiles such a deviation is expected to be much more pronounced since the latter process occurs at projectile-target distances that are substantially smaller than those typical for single-electron loss.

Atomic units (a.u.) are applied throughout, except where otherwise stated. We will use both ''ionization'' and ''loss'' to term the processes in which a projectile loses electron(s). Further, we always denote by Z_p and Z_t the atomic numbers of the projectile ion and the target atom, respectively.

II. THEORETICAL FRAMEWORK

A. Preliminary remarks

At the first glance, a nonperturbative description of ionization of heavy heliumlike projectiles in relativistic collisions with heavy many-electron targets seems to be prohibitory difficult. However, by invoking some reasonable approximations, discussed in detail below, the treatment of this problem can be substantially simplified.

According to first-order theories, ionization of a projectile in collisions with neutral atomic targets can occur either via the interaction with the screened target nucleus, where the state of target electrons remains unchanged (elastic mode), or via the direct electron-electron interaction where both projectile and target electrons make transitions (two-center electron correlation, TCEC) [1,5]. However, if both the projectile ion and the target atom are heavy enough, it is well known that the latter mechanism, which represents a rather soft interaction mode, is of minor importance for the projectile ionization. Therefore, in such collisions the TCEC mode may be safely neglected and this neglect will certainly be a good approximation also for nonperturbative collisions [16]. Then the target effect on the projectile can be very well approximated as caused by a superposition of short-ranged Yukawa-type potentials of the form

$$V_t(r)=Z_t\sum_{j=1}^{3} A_j \frac{e^{-\kappa_j r}}{r}, \tag{1}$$

with certain target-specific parameters A_j, κ_j $(j=1,2,3)$, which are tabulated [17]. Thus, the problem of ionization of a heavy heliumlike projectile in collisions with a many-electron target can be reduced to the projectile ionization by an external potential which is not affected by the collision.

In general, the process of double ionization is substantially more complicated compared to that of single ionization which is usually considered as a basically uncorrelated single-electron process. There are essentially two possibilities to get projectile double ionization. The first is that the target potential (1) simultaneously influences the motion of both projectile electrons and this influence directly leads to their loss. Below, this process will be referred to as the two-step-2 (TS-2) process [18]. The other possibility is that effectively only one projectile electron interacts with the target via the single-photon exchange and is removed by this interaction from the projectile, and the other one is lost either due to electron-electron-correlations within the projectile or due to rearrangement in the projectile final state. These processes can be referred to as the two step 1 (TS-1) and shake off (SO), respectively.

B. Where does the TS-2 dominate?

It seems to be reasonable to assume that double ionization of heavy heliumlike projectiles colliding with heavy targets occurs mainly via the TS-2 mechanism. A similar assumption was used for calculating cross sections for multiple-electron loss [10] and loss excitation [11]. However, to the best of our knowledge, the collision parameter domain, where the TS-2 dominates multiple-electron loss from projectiles, has not been generally established. Below, we attempt to estimate such a domain for the double-electron loss from heliumlike projectiles by using known results for helium double ionization by ion impact and by applying scaling arguments.

(1) We start this procedure with considering not too heavy projectiles and nonrelativistic collision velocities where the Schrödinger equation obviously represents a very good approximation to treat the behavior of the two electrons that move in the projectile field and are subjected to the target field. Using the semiclassical approximation and the projectile frame, where a target atom moves along a straight-line trajectory with a velocity $\mathbf{v}$, the Schrödinger equation reads (in atomic units)

$$i\frac{\partial}{\partial t}\Psi=(H_1+H_2+V_{corr})\Psi. \tag{2}$$

In Eq. (2)

$$H_i=-\frac{\Delta_i}{2}-\frac{Z_p}{r_i}-\frac{Z_t}{|\mathbf{b}+\mathbf{v}t-\mathbf{r}_i|}\times\sum_{j=1}^{3} A_j\exp(-\kappa_j|\mathbf{b}+\mathbf{v}t-\mathbf{r}_i|), \tag{3}$$

where $\mathbf{r}_i$ are the coordinates of the ith projectile electron ($i=1,2$) with respect to the projectile nucleus, $\mathbf{b}$ and t are the collision impact parameter and time, respectively, and A_j and κ_j are the screening parameters. Further,

$$V_{corr}=\frac{1}{r_{12}}, \tag{4}$$

where $r_{12}=|\mathbf{r}_1-\mathbf{r}_2|$, is the ''correlated'' part of the projectile Hamiltonian. Using the scaling transformation $\mathbf{r}_i'=(Z_p/2)\mathbf{r}_i$, $\mathbf{b}'=(Z_p/2)\mathbf{b}$, $t'=(Z_p/2)^2 t$, the Schrödinger equation in the scaled space time is rewritten as

$$i\frac{\partial}{\partial t'}\Psi'=(H_1'+H_2'+V_{corr}')\Psi', \tag{5}$$

where now

$$H_i' = -\frac{\Delta_i'}{2} - \frac{2}{r_i'} - \frac{Z_t'}{|\mathbf{b}' + \mathbf{v}'t' - \mathbf{r}'_i|} \times \sum_{j=1}^{3} A_j \exp(-\kappa_j' |\mathbf{b}' + \mathbf{v}'t' - \mathbf{r}'_i|), \tag{6}$$

with $\mathbf{v}' = (2/Z_p)\mathbf{v}$, $Z_t' = (2/Z_p)Z_t$, $\kappa_j' = (2/Z_p)\kappa_j$, and

$$V'_{corr} = \frac{2/Z_p}{r'_{12}}. \tag{7}$$

According to Eqs. (5)–(7), the relative importance of the electron–electron interaction (7) in a collision-free ($Z_t = 0$) heliumlike ion with a nuclear charge Z_p is reduced by a factor of $2/Z_p$ compared to helium atom. If one assumes that the interaction (7) does not substantially influence the properties of the initial and final states of a heliumlike ion [19], then the scaling consideration of the collision suggests that, in the case of a heliumlike ion, the relative importance of both the TS-1 and SO contributions to double ionization is reduced roughly by $(2/Z_p)^2$ compared to the case of helium. At the same time the relative importance of the TS-2 contribution is not changed much, since it is approximately determined by the factor $Z_t'/v' = Z_t/v$.

For double ionization of helium in nonrelativistic collisions with bare ions of charge q at velocity v the TS-2 mechanism is known to be the dominant ionization process, if $q/v > 0.2$ (see Refs. [20,21]). Using the scaling arguments one can conclude that in nonrelativistic collisions of heavy heliumlike ions with an atomic number Z_p with pointlike charges the TS-2 would predominate the double ionization of the ions if $q/v > 0.4/Z_p$. Roughly speaking, the TS-2 mechanism dominates in double ionization if the effect of the external perturbation is stronger than that arising from the electron-electron correlation. Taking this into account it is possible to use a similar criterion, $Z_t/v > 0.4/Z_p$, for collisions with neutral atoms since in that case projectile ionization at any collision velocity occurs at rather small impact parameters where the averaged action exerted by the short-range potential of a neutral atom with atomic number Z_t is substantially stronger than that which would be produced by the long-range Coulomb potential of an equivelocity pointlike charge Z_t [22].

(2) The next step of the scaling argumentation is to note an interesting fact that in certain cases the application of the nonrelativistic Schrödinger equation (2) can be a very reasonable approximation for considering projectile ionization even in relativistic and ultrarelativistic collisions. Such a situation, as was pointed out in Ref. [12], is realized for not very heavy projectiles colliding with *neutral* targets. Viewing such collisions in the projectile frame it is not difficult to see that the motion of projectile electrons in both initial and final projectile states is nonrelativistic [23]. Therefore, the magnetic component of the target electromagnetic field is of minor importance for the ionization process. In addition, in the case of not too heavy projectiles the retardation effects, which may be of paramount importance for relativistic collisions with an unscreened charge, are essentially cut off because of the short-range nature of the screened potential of a neutral target. Thus, the conclusion about the relative importance of the TS-1, SO, and TS-2 mechanisms, reached in the previous paragraph, is directly applicable for ionization of projectiles with say $Z_p \lesssim 30$–40 by neutral targets also in collisions at $v \to c$.

(3) Finally, it is quite obvious that the tendency of the decreasing role of the electron-electron correlation in projectile double ionization with increasing Z_p will also hold for very heavy ions colliding with many-electron targets at nonrelativistic and relativistic velocities where the scaling arguments obtained with the use of the Schrödinger equation are not directly applicable. Thus, one can conclude that double ionization of heliumlike heavy ions in both relativistic and nonrelativistic collisions with neutral atoms will occur predominantly via the TS-2 mechanism provided the condition

$$\frac{Z_t Z_p}{v} > 0.4 \tag{8}$$

is fulfilled.

In summary, we have argued that the TS-2 is mainly responsible for the double ionization of heavy projectiles colliding with heavy neutral atomic targets and established the collision parameter domain, defined by Eq. (8), where this is the case. As a consequence, in the domain (8) the projectile ionization can be dealt with within the independent electron model (IEM), which has been proved to be quite successful in describing cross sections in cases of strong external perturbations (see, e.g., Refs. [1,12], and references therein).

C. Transition amplitudes and cross sections

Within the IEM the single and double ionization cross sections, respectively, read

$$\sigma^{(1)} = \int d^2\mathbf{b}\;\; P_1(\mathbf{b}) = 2\int d^2\mathbf{b}\;\; p(\mathbf{b})[1 - p(\mathbf{b})], \tag{9}$$

$$\sigma^{(2)} = \int d^2\mathbf{b}\;\; P_2(\mathbf{b}) = \int d^2\mathbf{b}[p(\mathbf{b})]^2. \tag{10}$$

Here, $P_1(\mathbf{b})$ and $P_2(\mathbf{b})$ are the impact parameter dependent probabilities for single- and double-electron loss, respectively, and the one-electron transition probability $p(\mathbf{b})$ is given by

$$p(\mathbf{b}) = \int d^3\mathbf{k}\;\; |a_{0\to\mathbf{k}}(\mathbf{b})|^2, \tag{11}$$

where $a_{0\to\mathbf{k}}$ denotes the amplitude for the collision-induced transition from the ground state ψ_0 to a continuum state $\psi_\mathbf{k}$ of a hydrogenlike ion with an effective nuclear charge Z_{eff}. It is convenient to calculate $a_{0\to\mathbf{k}}$, $p(\mathbf{b})$ and the corresponding cross sections in the projectile reference frame and this frame will be used below.

The semiclassical first-order treatment [8] results in the following transition amplitude:

$$a_{0\to\mathbf{k}}^{pert}(\mathbf{b})=\langle\psi_{\mathbf{k}}(\mathbf{r})|(1-\beta\alpha_z)\exp\left(i\frac{\omega_{k0}}{v}z\right)\frac{2iZ_t}{v}\times\sum_{j=1}^{3}A_jK_0(B_{k,j}|\mathbf{b}-\mathbf{r}_\perp|)|\psi_0(\mathbf{r})\rangle. \quad (12)$$

Here v denotes the collision velocity, $\beta=v/c$, $\gamma=1/\sqrt{1-\beta^2}$, ω_{k0} is the energy difference between the final and initial electron state, and $B_{k,j}=\sqrt{(\omega_{k0}^2/v^2\gamma^2)+\kappa_j^2}$. Further, $\mathbf{r}=(\mathbf{r}_\perp,z)$ is the coordinate of the projectile electron with respect to the projectile nucleus with z being the coordinate along the collision (target) velocity, α_z is the Dirac matrix, and K_0 is the modified Bessel function.

The Dirac equation for an electron that is subjected to the fields of a binding center and a pointlike charge that moves at the speed of light can be solved exactly [24]. In the case of collisions with the short-ranged potential (1) at the light velocity ($\gamma\to\infty$) it is also possible to determine analytically the exact amplitude for the transition from the ground state ψ_0 to some excited state ψ_n induced by the target potential (1) [15]. The result is

$$a_{0\to\mathbf{k}}^{\infty}(\mathbf{b})=\langle\psi_{\mathbf{k}}(\mathbf{r})|(1-\alpha_z)\exp\left(i\frac{\omega_{k0}}{c}z\right)\times\exp\left(\frac{2iZ_t}{c}\sum_j A_jK_0(\kappa_j|\mathbf{b}-\mathbf{r}_\perp|)\right)|\psi_0(\mathbf{r})\rangle. \quad (13)$$

Using this expression for calculating $\sigma^{(1)}$ and $\sigma^{(2)}$ one obtains "exact" single and double ionization cross sections for collisions with $\gamma\to\infty$.

As it follows from Eq. (12) in ultrarelativistic collisions ($\beta\approx1$) the first-order amplitude becomes independent of the collision energy (per nucleon) for high enough values of γ when $B_{k,j}\simeq\kappa_j$, i.e., when

$$\gamma^2\gg\frac{\omega_{k0}^2}{c^2\kappa_j^2}. \quad (14)$$

It is natural to define the region, where the condition (14) is fulfilled, as the region of asymptotically high collision energies or of asymptotically high values γ. In this region the amplitude (12) can be considered as a first-order approximation of the "exact" amplitude (13). In general, different projectile-target pairs enter the asymptotic region at different values of γ since the criterion (14) depends on the atomic number of the projectile and target via the transition energies ω_{k0} and the screening parameters κ_j. Simple estimates show that for projectiles with $Z_p\sim20$–30 the asymptotic region is reached already at $\gamma\sim10$–20. Clearly, for high enough values of γ, collisions of any projectile-target pair will be in the asymptotic region. Estimates show that this situation is essentially reached at $\gamma>\sim150$–200, i.e., starting with collision energies of 150–200 GeV/u.

TABLE I. Cross sections (in kb) for single and double ionization of heliumlike projectiles in ultrarelativistic collisions with neutral targets. The projectiles are initially in their ground states. The fourth and fifth columns display cross sections obtained within the perturbative treatment. The sixth, seventh, and eighth columns contain the nonperturbative results.

Target	Z_t	Projectile	$\sigma_{pert}^{(1)}$	$\sigma_{pert}^{(2)}$	$\sigma_\infty^{(1)}$	$\sigma_\infty^{(2)}$	$\sigma_\infty^{(2)}/\sigma_\infty^{(1)}$
Kr^0	36	Kr^{34+}	62.5	1.15	62.4	1.13	1.8%
Kr^0	36	Xe^{52+}	29.1	0.497	29.1	0.495	1.7%
Kr^0	36	Pb^{80+}	12.5	0.184	12.5	0.184	1.5%
Xe^0	54	Kr^{34+}	125	5.29	123	4.77	3.9%
Xe^0	54	Xe^{52+}	59.6	2.42	58.9	2.19	3.7%
Xe^0	54	Pb^{80+}	25.1	0.903	24.9	0.817	3.3%
Au^0	79	Kr^{34+}	234	23.0	224	16.3	7.3%
Au^0	79	Xe^{52+}	113	10.6	110	7.63	6.9%
Au^0	79	Pb^{80+}	49.2	3.98	48.9	2.94	6.0%

III. RESULTS AND DISCUSSION

In this section, we discuss some results of the application of the approach described in the preceding section to single and double ionization of heliumlike ions in collisions at asymptotically high γ. For definiteness we consider collisions of Kr^{34+}, Xe^{52+}, and Pb^{80+} with neutral Kr, Xe, and Au. These nine collision pairs cover all possible situations, where the projectile ion is "light heavy," "intermediate heavy," or "very heavy" and collides with "light heavy," "intermediate heavy," or "very heavy" target atom, and, thus, are quite representative. A detailed description of how one can perform the calculation of the necessary one-electron transition amplitudes (12) and (13) can be found in Ref. [15] and will not be repeated here. We only note that we used relativistic (Coulomb-) Dirac wave functions and took the "full" projectile nuclear charge Z_p as the effective projectile charge Z_{eff} for calculating both single and double ionization [25].

Below we refer to calculations that are performed using the IEM with the first-order one-electron transition amplitude (12) and, thus, take into account only the one-photon exchange between the target and each of the projectile electrons, as to *perturbative* calculations (treatment, consideration) and to results, obtained in this way, as to *perturbative* results. The *nonperturbative* calculations, which for the asymptotically high collision energies (14) fully account for the many-photon exchange between the target and the projectile electrons, are based on the IEM and Eq. (13). Results of the latter calculations will be termed as *nonperturbative*. Table I shows the resulting single and double ionization cross sections.

A. High-energy limit for the double-to-single loss ratio

(1) We observe the interesting fact that the calculated ratios are very weakly dependent on the atomic number of the projectile ion and, thus, are essentially determined only by the target atom. An additional calculation using S^{14+} as a projectile confirms this result: here we found ratios of 1.8%, 3.7%, and 7.1% for collisions with Kr, Xe, and Au, respec-

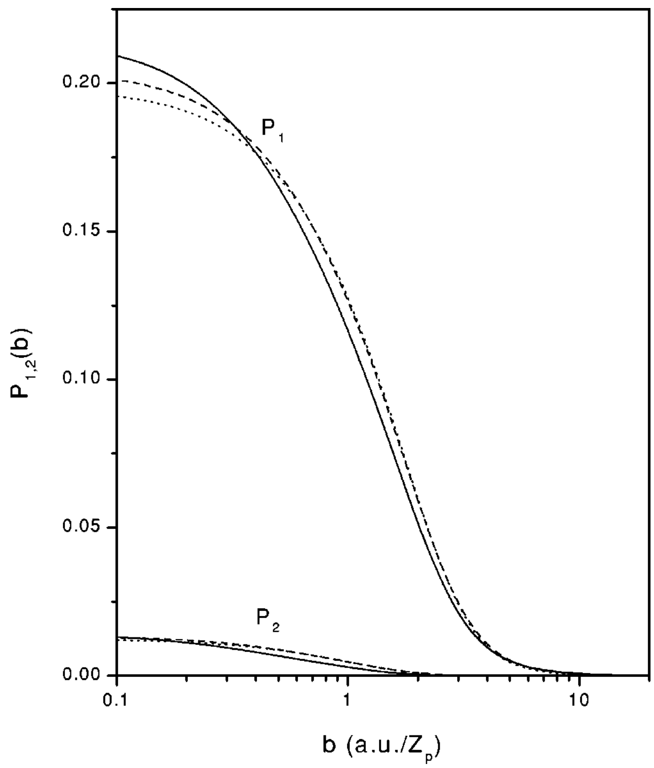

FIG. 1. Nonperturbative asymptotic high-energy probabilities $P_1(b)$ and $P_2(b)$ for single and double ionization of heliumlike Pb, Xe, and Kr ions by impact on neutral Kr atoms. Solid lines, results for Pb^{80+}; dashed lines, results for Xe^{52+}; dotted lines, results for Kr^{34+}.

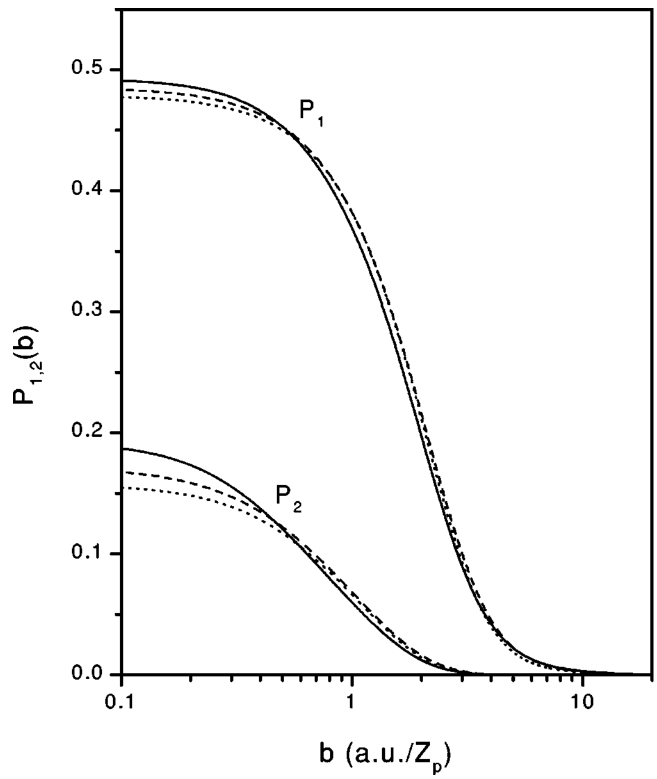

FIG. 2. Same as Fig. 1, but for impact on neutral Au atoms.

tively. Figures 1 and 2 offer some insight why the ratios are nearly independent of the projectile atomic number Z_p. They show the ionization probabilities as a function of the impact parameter, which is given in units of $1/Z_p$. Surprisingly, the curves for different projectiles look rather similar. This means that both the single and double ionization cross sections scale approximately like Z_p^{-2}. Thus, as Figs. 1 and 2 clearly indicate, the influence of the projectile nucleus charge on the ionization process is, basically, to set the length scale. This, however, does not affect much the ratio.

(2) The above predictions, that the double-to-single loss ratio is strongly dependent on the atomic number Z_t of the ionizing agent and is nearly independent of the nuclear charge Z_p of the electron binding center, are in sharp contrast to what is expected in the high-energy limit for the double-to-single ionization ratio in the case of ionization of helium and heliumlike positive ions in collisions with charged particles. In the latter case the expected features of the ratio are: (i) the strong dependence on the atomic number of the binding center (helium, heliumlike ion), and (ii) the independence of the charge of the ionizing agent (a pointlike charged particle). It is the fundamental difference between the influence of a short-range potential in the case of ionization by a neutral atom and that of the long-range Coulomb potential in the case of ionization by a charged particle which is mainly responsible for this contrast.

To emphasize some consequences of this difference we note that our asymptotic double-to-single ionization ratios are of the same order as the values found experimentally in Refs. [20,21] for the ionization of He by fast highly charged ions. There, ratios of 1.5% and 3% were measured for ionization of helium by 1 GeV/u Kr^{36+} impact and 420 MeV/u U^{90+} impact, respectively [26]. It may appear surprising that the calculated asymptotic double-to-single ionization ratios for the much heavier Kr^{34+}, Xe^{52+}, and Pb^{80+} ions are so close to the measured values for helium ionization. However, one should keep in mind that the latter are due to charged-particle impact whereas the former originate from collisions with neutral atoms. In our case the interaction potential (1) is short ranged, whereby the contribution from larger impact parameters is cut off. Since in the case of ionization by a charged particle larger values of b would be much more important for single ionization than for double ionization, this cut off reduces $\sigma^{(1)}$ relatively much stronger than $\sigma^{(2)}$. This explains, why in collisions with neutral targets the double-to-single ionization ratio is greatly enhanced compared to collisions with charged particles.

B. Nonperturbative behavior of the loss process

(1) One usually assumes that theories of the first order in the projectile-target interaction give a good description of the collision dynamics at high enough collision energies. However, in collisions with neutral atomic targets mainly small impact parameters contribute to the ionization cross section. At those small distances the interaction can be strong even at $v \rightarrow c$, which may lead to the failure of the first-order approach.

Considering electron loss from hydrogenlike Pb ions we found in Ref. [15] that the inclusion of the many-photon exchange into the consideration changes the loss cross section by about 5% in collisions with neutral Au at the asymptotically high energies. Correspondingly, for hydrogenlike Kr projectiles in collisions with neutral Au we get roughly 10%, which still is small but non-negligible.

The results of the present work now clearly display, that—even for collision velocities practically equal to the speed of light—strong deviations from first-order results can take place. We have already seen that the TS-2 mechanism dominates, provided the condition (8) is fulfilled, which is the case for all projectile-target pairs given in Table I. The TS-2, however, involves the exchange of at least two photons (one photon per one electron) and, thus, is not a first-order mechanism. Moreover, the influence of many-photon processes, where the target exchanges with each of the projectile electrons more than one virtual photon, can also be substantial.

According to the definition of "perturbative" and "nonperturbative," given in the beginning of this section, one can generally refer to the difference between results, obtained with the first order (12) and "exact" (13) transition amplitudes as to the *nonperturbative behavior* of the loss process.

(2) The double ionization cross sections for Pb^{80+}, Xe^{52+}, and Kr^{34+} ions impinging on Au atoms, calculated within the IEM with the first-order transition amplitude (12), are by 35–42 % larger than the corresponding nonperturbative results. Figure 3 shows the corresponding ionization probabilities as a function of the impact parameter. Clearly, the region of very small impact parameters is responsible for the pronounced nonperturbative behavior.

(3) Regarding single ionization of heliumlike Kr, Xe, and Pb ions by impact on neutral Au, the influence of the many-photon exchange on the cross section is much weaker, of course. In particular, this influence turned out to be even smaller than that we found for the electron loss from the corresponding hydrogenlike projectiles. This observation can easily be explained if one notes that within the IEM the identity $\sigma^{(1)}=2(\sigma_{loss}-\sigma^{(2)})$ holds, where σ_{loss} denotes the cross section for the electron loss from the corresponding hydrogenlike projectile. Hence the weaker signs of the nonperturbative behavior in single ionization are due to a partial compensation of the contributions of the many-photon exchanges to the electron loss and double ionization. As the collision system Pb+Au shows, by accident this compensation can be almost complete.

(4) In the set of neutral targets, presented in the Table I, Kr is the lightest one. At the asymptotically high energies the field of Kr represents already a quite weak perturbation (since the ratio Z_t/v is rather small: $Z_t/v\approx 0.26$). Therefore in this case, as the results for both single and double loss indicate, the application of first-order perturbation theory to calculate the one-electron transition amplitude $a_{0\to\mathbf{k}}(\mathbf{b})$ is justified.

(5) The perturbative calculations predict that for $30\lesssim Z_p \lesssim 90$ the double ionization cross sections should approximately scale according to Z_t^4 dependence. Such a dependence would be the exact scaling law within the IEM for collisions with bare target nuclei. Thus, according to the perturbative treatment, the screening of the target nucleus by the target electrons does not affect much the double ionization even for not very heavy projectile ions. This suggests that the double ionization mainly occurs at impact parameters where the projectile-electrons interact, in essence, with the unscreened target nucleus. In the nonperturbative treatment the scaling Z_t^4 gets lost and the double ionization cross sections increase slower with Z_t than it would follow from the perturbative consideration. It is very likely, however, that this slower increase is a signature of the many-photon exchange, which for strong interactions are known to reduce ionization cross sections compared to perturbative results, rather then some indications to a greater role played by the screening effects in the nonperturbative treatment. Such a conclusion is directly supported by the curves plotted in Fig. 3 where the main difference between the perturbative and "exact" results appears at very small impact parameters where the screening effects are of minor importance.

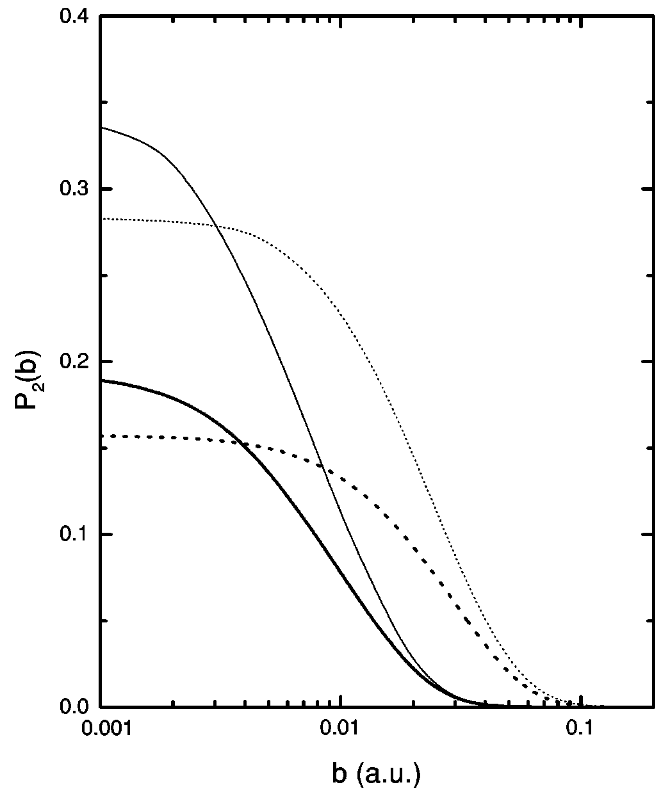

FIG. 3. Perturbative vs nonperturbative probabilities $P_2(b)$ for double ionization of heliumlike Pb and Kr ions by ultrarelativistic impact on neutral Au atoms. Thick solid line, nonperturbative result for Pb^{80+}; thin solid line, perturbative result for Pb^{80+}; thick dotted line, nonperturbative result for Kr^{34+}; thin dotted line, perturbative result for Kr^{34+}.

In the conclusion of this section, we would like to point out the following. It is rather obvious that, within the IEM, cross sections for multiple-electron loss from heavy projectiles, having several electrons, may be even more sensitive to the form of the single-electron transition probability $p(\mathbf{b})$ than the cross section for double-electron loss from heavy heliumlike projectiles. Therefore, as the results of the present paper show and contrary to usual statements (see e.g. Ref. [12], p. 204), the application of perturbation theory for obtaining the one-electron transition probability $p(\mathbf{b})$ might result in considerable errors in calculated cross sections for multiple-electron loss from projectiles.

IV. CONCLUSION

We have considered single and double ionization of heavy heliumlike ions in ultrarelativistic collisions with heavy neutral atomic targets. Our consideration was based on the assumption that the two-center electron-electron correlations are of minor importance for the projectile-electron loss. This assumption seems to be well justified if both the projectile and the target are heavy enough. In addition, for such collision pairs the TS-2 mechanism was shown, by invoking scaling arguments, to be the dominant one in the projectile double ionization.

Four main conclusions can be drawn from the present study. First, even in the asymptotic region $\gamma\to\infty$ the many-photon exchange between the target and each of the projectile electrons plays an important role in the projectile double ionization by heavy targets. Second, the double ionization of very heavy and even not very heavy heliumlike projectiles occurs mainly in collisions at so small impact parameters, where the screening effects of the target nucleus by the target electrons are already rather weak. Third, the double-to-single ionization ratio is strongly dependent on the atomic number Z_t of the target. Fourth, this ratio was found to be nearly independent of the nuclear charge Z_p of the projectile.

The last two predictions can be directly tested in an experiment. In particular, after upgrading the accelerator facilities at the GSI (Darmstadt, Germany), that will allow to accelerate ions to collision energies corresponding to the Lorentz factor $\gamma\lesssim 20$, these predictions could be experimentally verified for heavy heliumlike projectiles with $Z_p\lesssim 30$.

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[18] The target interaction with each of the projectile electrons in a single collision event can involve both single- and many-photon exchanges. Therefore the TS-2, as it is defined in the present paper, represents a more general process compared to that which is usually called the "TS-2" in the literature. In the latter only the two-photon exchange between the target and projectile (or, more exactly, one photon per one electron) is taken into account. The TS-2 of the present paper can be reduced to the latter only in the case of not too strong interactions.
[19] This assumption is certainly true for heavy heliumlike ions but also represents a not very bad approximation even for a helium atom where the electron-electron interaction changes the ground-state energy by a factor $\approx 1.7^2/4 = 0.72 \sim 1$. (One could add that such an assumption would be certainly not valid in the case of H^- where the electron-electron interaction has a much more profound effect on the characteristics of H^- compared to the case of He.)
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[22] The latter observation also means that the criterion $Z_t/v > 0.4/Z_p$ represents a stricter condition for the dominance of

the TS-2 mechanism in double ionization by a neutral atom compared to the case of ionization by a charged particle.

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[26] Both these collision examples are still very far from being in the asymptotic high-energy limit for helium ionization by a charged particle. Therefore, the TS-2 (and not TS-1 and SO) is responsible for helium double ionization in these collisions. This fact brings a certain similarity between the helium ionization and the electron loss from heliumlike projectiles considered in the present paper.