Comment on "Photonic tunneling time in frustrated total internal reflection"

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This is a comment on Stahlhofen's paper [Phys. Rev. A **62**, 012112 (2000)]. It is shown by stationary-phase theory that the Goos-Hänchen shift in frustrated total internal reflection (FTIR) is not independent of the group delay (or phase time in the literature). The group delay involves the contribution of Goos-Hänchen shift and is always larger than zero in FTIR. It is also shown that the group delay in the two-dimensional (2D) optical FTIR can be written in the same form as that of the group delay in the 1D quantum tunneling in the sense that the group delay is the derivative of the total phase shift with respect to the angular frequency.

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In a recent paper, Stahlhofen [1] investigated the photonic tunneling time in frustrated total internal reflection (FTIR). Apart from brief discussions on the effect of nonspecular deformations of the transmitted beam, he concluded, by noting that the group delay (also known as phase time in the literature) "allows for vanishing and negative barrier traversal times," that the group delay approach "is not an appropriate tool to determine the tunneling time in FTIR." In addition, he claimed that "the unphysical prediction of the phase time is caused by the neglect of the Goos-Hänchen shift, which provides an independent time scale for the tunneling time in the form of a dwell time."

In this note, we will point out by the stationary-phase theory that the group delay in the two-dimensional (2D) FTIR involves the contribution of the Goos-Hänchen shift. In other words, the time scale based on the Goos-Hänchen shift is not independent of the group delay. The total group delay in FTIR is always larger than zero. At the same time, we will show that the group delay in the 2D optical FTIR can be written in the same form as that of the group delay in the 1D quantum tunneling.

As is shown in Fig. 1, two prisms of the same refractive index n are separated by an air gap of thickness a. A light pulse comes from the left to the prism-air interface at an incidence angle θ that is beyond the critical angle $\sin^{-1}(1/n)$. Without loss of generality, we consider TE polarization. Let the electric field of the Fourier component of the incident pulse be

$$\vec{E}_i(\vec{x}) = A e^{j\vec{k}\cdot\vec{x}}\hat{z}, \quad x<0, \tag{1}$$

(the time-dependent convention is chosen to be $e^{-j\omega t}$), where $A=|A|e^{j\alpha}$, α is the phase of complex amplitude A, $\vec{k}=(k_x,k_y)=(k\cos\theta, k\sin\theta)$, $k=nk_0$, k_0 is the wave number of light in vacuum, and ω is the angular frequency. The total phase of the incident wave being

$$\psi_i = \alpha + \vec{k}\cdot\vec{x} - \omega t,$$

the locus $y_i=y_i(x_i)$ of the incident pulse is determined [2] by $\partial\psi_i/\partial\theta=0$, so that

$$y_i = x_i \tan\theta - \frac{\partial\alpha/\partial\theta}{\partial k_y/\partial\theta}. \tag{2}$$

And the motion equation of the incident pulse is determined by $\partial\psi_i/\partial\omega=0$, so that

$$t_i = \frac{\partial\alpha}{\partial\omega} + \frac{\partial k_x}{\partial\omega}x_i + \frac{\partial k_y}{\partial\omega}y_i. \tag{3}$$

Suppose the electric field of the Fourier component of transmitted pulse is

$$\vec{E}_t(\vec{x}) = F e^{jk_x(x-a)} e^{jk_y y}\hat{z}, \quad x>a, \tag{4}$$

where $F=|F|e^{j\gamma}$. Then the total phase of the transmitted wave is

$$\psi_t = \gamma + k_x(x-a) + k_y y - \omega t.$$

The locus $y_t=y_t(x_t)$ of transmitted pulse is determined by $\partial\psi_t/\partial\theta=0$ to be

$$y_t = (x_t - a)\tan\theta - \frac{\partial\gamma/\partial\theta}{\partial k_y/\partial\theta}, \tag{5}$$

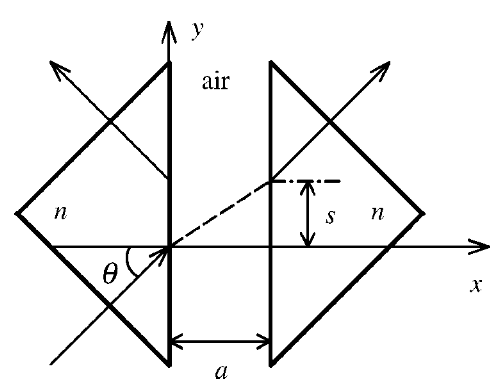

FIG. 1. Schematic diagram of frustrated total internal reflection of light beam.

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and the motion equation of transmitted pulse is determined by $\partial\psi_t/\partial\omega=0$ to be

$$t_t=\frac{\partial\gamma}{\partial\omega}+\frac{\partial k_x}{\partial\omega}(x_t-a)+\frac{\partial k_y}{\partial\omega}y_t. \tag{6}$$

It is seen from Eqs. (2) and (5) that the lateral shift of transmitted pulse at $x_t=a$ relative to the incident pulse at $x_i=0$ is given by

$$s=-\frac{\partial(\gamma-\alpha)/\partial\theta}{\partial k_y/\partial\theta}. \tag{7}$$

And the time delay of transmitted pulse at point $[a, -(\partial\gamma/\partial\theta)/(\partial k_y/\partial\theta)]$ relative to the incident pulse at point $[0, -(\partial\alpha/\partial\theta)/(\partial k_y/\partial\theta)]$ is given, as can be seen from Eqs. (3) and (6), by

$$\tau=\frac{\partial(\gamma-\alpha)}{\partial\omega}+\frac{\partial k_y}{\partial\omega}s. \tag{8}$$

Similarly, let the electric field of the Fourier component of reflected pulse be

$$\vec{E}_r(\vec{x})=Be^{-jk_xx}e^{jk_yy}\hat{z}, \quad x<0, \tag{9}$$

where $B=|B|e^{j\beta}$. Then the total phase of reflected wave is

$$\psi_r=\beta-k_xx+k_yy-\omega t.$$

The locus $y_r=y_r(x_r)$ of the reflected pulse is determined by $\partial\psi_r/\partial\theta=0$ to be

$$y_r=-x_r\tan\theta-\frac{\partial\beta/\partial\theta}{\partial k_y/\partial\theta}, \tag{10}$$

and the motion equation of reflected pulse is determined by $\partial\psi_r/\partial\omega=0$ to be

$$t_r=\frac{\partial\beta}{\partial\omega}-\frac{\partial k_x}{\partial\omega}x_r+\frac{\partial k_y}{\partial\omega}y_r. \tag{11}$$

According to Eqs. (2) and (10), we see that the shift of the reflected pulse relative to the incident pulse at interface $x=0$ is

$$s'=-\frac{\partial(\beta-\alpha)/\partial\theta}{\partial k_y/\partial\theta}. \tag{12}$$

And according to Eqs. (3) and (11), the time delay of the reflected pulse relative to the incident pulse at interface $x=0$ is

$$\tau'=\frac{\partial(\beta-\alpha)}{\partial\omega}+\frac{\partial k_y}{\partial\omega}s'. \tag{13}$$

Denoting $\gamma-\alpha$ by ϕ, which means the phase of the complex transmission coefficient $\phi=\arg(F/A)$, we have for the lateral shift of transmitted pulse

$$s=-\frac{\partial\phi/\partial\theta}{\partial k_y/\partial\theta}. \tag{14}$$

It is noted that ϕ defined above is not the phase shift of transmitted wave at point $[a, -(\partial\gamma/\partial\theta)/(\partial k_y/\partial\theta)]$ relative to the incident wave at point $[0, -(\partial\alpha/\partial\theta)/(\partial k_y/\partial\theta)]$. In fact, such a phase shift is, as can be seen from Eqs. (1) and (4),

$$\phi_t=\phi+k_ys.$$

With this phase shift and noting the fact that the spatial lateral shift s is not an explicit function of angular frequency ω, the time delay (8) of transmitted pulse can be written as

$$\tau=\frac{\partial\phi_t}{\partial\omega}. \tag{15}$$

The same reasoning will give for the lateral shift and time delay of reflected pulse

$$s'=-\frac{\partial\phi'/\partial\theta}{\partial k_y/\partial\theta}, \tag{16}$$

$$\tau'=\frac{\partial\phi_r}{\partial\omega}, \tag{17}$$

respectively, where $\phi'=\beta-\alpha$ is the phase of complex reflection coefficient $\phi'=\arg(B/A)$, and $\phi_r=\phi'+k_ys'$ is the phase shift of reflected wave at point $[0, -(\partial\beta/\partial\theta)/(\partial k_y/\partial\theta)]$ relative to the incident wave at point $[0, -(\partial\alpha/\partial\theta)/(\partial k_y/\partial\theta)]$.

From above discussions, we see that in the 2D optical tunneling, the phase shifts of reflected and transmitted waves relative to the incident wave depend on their lateral shifts. Equations (15) and (17) show that the group time delay is the partial derivative of the total phase shift with respect to the angular frequency. Such a conclusion in the 2D optical tunneling situation is the same as in the 1D quantum tunneling situation [3,4].

In Stahlhofen's discussions, the group time delay of transmitted pulse in 2D situation is defined as

$$\tau_s=\frac{\partial\phi}{\partial\omega}. \tag{18}$$

This definition looks like the same as that of the group delay in the 1D quantum tunneling [3],

$$\tau_q=\hbar\frac{\partial\phi_q}{\partial E}, \tag{19}$$

where E is the energy of the incident particle. But they are physically different. The group time delay (19) is [3,4] the derivative of the phase shift ϕ_q across the barrier, whereas the ϕ in Eq. (18) is not equal to the phase shift ϕ_t, as discussed above. Furthermore, the asymptotic behavior of τ_s in the opaque limit $a\to\infty$ is also different from that of τ_q. In the 1D quantum tunneling through a square potential barrier,

τ_q approaches a nonzero constant as $a \rightarrow \infty$ [2,3], while τ_s approaches zero in the same limit as was shown by Stahlhofen.

According to Eq. (8), the group time delay τ consists of two parts, τ_s and the lateral-shift contribution, $\tau = \tau_s + (\partial k_y / \partial \omega) s$. Although τ_s allows for vanishing and negative values as is shown by Stahlhofen, the total group delay τ is always larger than zero and is dominated by the lateral-shift contribution in the opaque limit. In fact, detailed calculation gives, neglecting the ω dependence of refractive index n,

$$\tau = \frac{2a}{k_x \omega f^2} \left(k_y^2 \frac{\sinh 2\kappa a}{2\kappa a} - k_0^2 \cos 2\delta \cos^2 \delta \right) \tag{20}$$

and

$$\frac{(\partial k_y / \partial \omega) s}{\tau_s} = \frac{k_y^2}{\kappa^2} \left(\frac{\sinh 2\kappa a / 2\kappa a}{\cos 2\delta \cos^2 \delta} - 1 \right), \tag{21}$$

where $f^2 = \sinh^2 \kappa a + \sin^2 2\delta$, $\kappa = (k_y^2 - k_0^2)^{1/2}$ is the decaying constant of the evanescent wave in the air gap, and $\delta = \tan^{-1}(\kappa / k_x)$. Since $k_y > k_0$ when the incidence angle is larger than the critical angle $\sin^{-1}(1/n)$, Eq. (20) shows that τ is larger than zero. Equation (21) shows that when $\kappa a \gg 1$, we have $|(\partial k_y / \partial \omega) s / \tau_s| \gg 1$. That is to say, the Goos-Hänchen shift plays the main role in the total group delay.

At last, let us mention that recent optical experiments performed by Wang *et al.* [5] measured negative group delays of light pulse traveling through an anomalous dispersive medium. And the negative group delay of particles passing through a potential well [6] is tested by a microwave analogy experiment [7]. So the negative feature of group delay may not deny its applicability.

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