Generation of two-mode nonclassical states and a quantum-phase-gate operation in trapped-ion cavity QED

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We propose a scheme to generate nonclassical states of a quantum system, which is composed of the one-dimensional trapped-ion motion and a single-cavity field mode. We show that two-mode SU(2), entangled coherent states, two-mode squeezed vacuum states, and their superposition can be generated. If the vibration mode and the cavity mode are used to represent separately a qubit, a quantum phase gate can be implemented.

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In recent years, there has been much interest in methods of nonclassical state generation by entangling quantum systems. In this field, advances in ion cooling and ion trapping have opened new prospects by controlling the quantized ion motion precisely [1–3]. In particular, motional Fock states, squeezed states, and microscopic quantum interference (Schrödinger-cat) states are in the scope. More recently, various schemes were proposed to control two vibration modes of a single ion, which is confined within a two-dimensional trap [3–6]. Most of these proposals concern the quantized ion vibration by treating the laser field classically. But, the quantization of the laser field brings more possibilities. A number of schemes were proposed for generating various nonclassical states of single-mode cavity fields [7]. An experimental realization of a Schrödinger-cat state in a cavity system was reported [8]. Furthermore, a single trapped ion can be used as a probe for measuring the cavity field intensity [9]. A new possibility for quantum state engineering and quantum information processing has been opened by using a trapped ion in a high-Q cavity. The influence of the field statics on the ion dynamics [10] was investigated as well as the transfer of the coherence between the motional state and the light field [11]. In a recent paper [12] a simple scheme was proposed to generate the whole Bell basis composed of the vibrational mode of a single ion and a cavity field.

In this paper, we propose a scheme to generate two-mode nonclassical states of a quantum system, which is composed of the vibration mode of one trapped ion and a single light field mode. The set of these states includes two-mode SU(2) Schrödinger-cat states [13,14], entangled coherent states [15,16], two-mode squeezed vacuum states, and their superpositions [14]. We also show that quantum phase gates can be implemented by representing the qubits by the vibration mode of a single trapped ion and the cavity mode quantum state restricted on the subspace spanned by the two lowest Fock states. In order to relate our scheme to this physical arrangement, we consider a trapped ion confined in a harmonic trap located inside an optical cavity. The atomic transition between the two internal electronic states $|e\rangle$ and $|g\rangle$ (frequency ω_0) is coupled to a single mode of the cavity field of the frequency ω_{cav} and is also driven by an external classical laser field of frequency ω_A. The cavity is aligned along the x axis, while the laser field is incident from a direction along the y axis. Thus, in a frame rotating at the laser frequency ω_A the system's Hamiltonian is in the form [11]

$$H=\nu b^\dagger b+\delta_{cA}a^\dagger a+\Delta_{oA}\sigma_+\sigma_-+E_A\sigma_++E_A\sigma_- +g_0\sin\eta(b^\dagger+b)(a^\dagger\sigma_-+a\sigma_+). \tag{1}$$

Here $(a,a^\dagger)$ and $(b,b^\dagger)$ are the boson annihilation and creation operators of the cavity field and the quantized atomic vibration. The operator $\sigma_-=|g\rangle\langle e|$ changes the internal electronic state from $|e\rangle$ to $|g\rangle$ and η is the corresponding Lamb-Dicke parameter. The detuning δ_{cA} and Δ_{oA} are given by $\delta_{cA}=\omega_{cav}-\omega_A$ and $\Delta_{oA}=\omega_o-\omega_A$. The quantity E_A is the amplitude of the laser field. The single-photon atom-cavity dipole coupling strength is given by g_0, while the sine function describes the standing-wave structure of the cavity field. We assume that the center of the trap is located at the node of the cavity field. Since the detuning of the laser field from the atomic transition frequency is assumed to be very large $(\Delta_{oA}\gg\nu,\delta_{cA},g_0,E_A)$, the internal atomic dynamics can be adiabatically eliminated. The corresponding Hamiltonian takes the form

$$H=\nu b^\dagger b+\delta_{cA}a^\dagger a-\frac{g_0^2}{\Delta_{oA}}\sin^2\eta(b^\dagger+b)a^\dagger a\sigma_z -\frac{g_0\epsilon_A}{\Delta_{oA}}\sin\eta(b^\dagger+b)(e^{-i\varphi_A}a^\dagger+e^{i\varphi_A}a)\sigma_z, \tag{2}$$

with $\sigma_z=|e\rangle\langle e|-|g\rangle\langle g|$ and $E_A=\epsilon_A e^{-i\varphi_A}$. In the following we choose $\varphi_A=\pi/2$. In the Lamb-Dicke region $(\eta\ll1)$ we may write $\sin\eta(b^\dagger+b)\simeq\eta(b^\dagger+b)$. If we choose the detuning between the cavity and the laser field to be $\delta_{cA}=\nu$, the effective interaction Hamiltonian

$$H_1=i\Omega_1(a^\dagger b-ab^\dagger)\sigma_z \tag{3}$$

is obtained in the rotating wave approximation. Here Ω_1 denotes the effective interaction strength. We consider the situation in which the ion's internal state is prepared initially as a superposition $(1/\sqrt{2})(|g\rangle+|e\rangle)$ and the vibration mode of the ion is in the Fock state $|n\rangle_b$. The cavity field is prepared in the vacuum state $|0\rangle_a$:

$$\Psi(0)=\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)\,|0\rangle_a|n\rangle_b. \tag{4}$$

If the laser pulse of the interaction (3) is applied, the system evolves into

$$\Psi(t)=\frac{1}{\sqrt{2}}\exp[\Omega_1 t(a^\dagger b-ab^\dagger)]|e\rangle|0\rangle_a|n\rangle_b + \frac{1}{\sqrt{2}}\exp[-\Omega_1 t(a^\dagger b-ab^\dagger)]|g\rangle|0\rangle_a|n\rangle_b . \quad (5)$$

The carrier transition $|e\rangle\leftrightarrow|g\rangle$ is driven in order to generate a $\pi/2$-pulse on the internal state: $|e\rangle\rightarrow 1/\sqrt{2}(|g\rangle+|e\rangle)$; $|g\rangle\rightarrow 1/\sqrt{2}(|g\rangle-|e\rangle)$. The quantum state evolves into

$$\Psi(t)=\frac{1}{2}\{\exp[\Omega_1 t(a^\dagger b-ab^\dagger)] + \exp[-\Omega_1 t(a^\dagger b-ab^\dagger)]\}|g\rangle|0\rangle_a|n\rangle_b + \frac{1}{2}\{\exp[\Omega_1 t(a^\dagger b-ab^\dagger)]-\exp[-\Omega_1 t(a^\dagger b - ab^\dagger)]\}|e\rangle|0\rangle_a|n\rangle_b . \quad (6)$$

Upon detection of the internal state, this state vector is projected onto the two-mode quantum states, which we denote with

$$\Phi_\pm=\exp[-i\Omega_1 t(a^\dagger b-ab^\dagger)]\,|0\rangle_a|n\rangle_b \pm\exp[i\Omega_1 t(a^\dagger b-ab^\dagger)]\,|0\rangle_a|n\rangle_b . \quad (7)$$

The quantum state Φ_+ is obtained if the internal state $|g\rangle$ of the ion is detected. Otherwise the quantum state Φ_- is generated. These two-mode quantum states (7) are the SU(2)-Schrödinger-cat states of the form $(|\zeta,j\rangle\pm|-\zeta,j\rangle)$ [14], if the SU(2)-coherent states $|\zeta,j\rangle=\exp[\beta J_+-\beta^* J_-]|j,-j\rangle$ [17] are used with the identification $\beta=\Omega_1 t$, $j=n/2$, $\zeta=\tan(\Omega_1 t/2)$.

In order to entangle coherent states of a special form, we again initialize the internal ion state in $(1/\sqrt{2})(|g\rangle+|e\rangle)$ but we prepare the two Bose modes in the coherent states $|\alpha\rangle_a$ and $|\beta\rangle_b$:

$$\Psi(0)=\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)\,|\alpha\rangle_a|\beta\rangle_b. \quad (8)$$

If the laser pulse, which corresponds to the interaction (3), is applied over a time interval $t_1=\pi/4\Omega_1$, the system evolves into

$$\Psi(t_1)=\frac{1}{\sqrt{2}}|e\rangle\,|(\alpha+\beta)/\sqrt{2}\rangle_a\,|(\beta-\alpha)/\sqrt{2}\rangle_b + \frac{1}{\sqrt{2}}|g\rangle\,|(\alpha-\beta)/\sqrt{2}\rangle_a\,|(\alpha+\beta)/\sqrt{2}\rangle_b . \quad (9)$$

Then the atom is subjected to a $\pi/2$ pulse, which is resonant with the transition $|e\rangle\leftrightarrow|g\rangle$. Upon detection of the internal state, the state vector is projected into the two-mode coherent states

$$\Phi_\pm=|(\alpha+\beta)/\sqrt{2}\rangle_a\,|(\beta-\alpha)/\sqrt{2}\rangle_b \pm|(\alpha-\beta)/\sqrt{2}\rangle_a|(\alpha+\beta)/\sqrt{2}\rangle_b . \quad (10)$$

This equation demonstrates the generation of entangled coherent states of a special type. Entangled coherent states of another type can be generated, if the interaction time is chosen to be twice as long: $\Omega_1 t_1=\pi/2$. In this example the initial state (8) evolves into

$$\Psi(t_1)=\frac{1}{\sqrt{2}}|e\rangle|\beta\rangle_a|-\alpha\rangle_b+|g\rangle|-\beta\rangle_a|\alpha\rangle_b . \quad (11)$$

If the atom's internal state is subjected to a $\pi/2$ pulse, as described above, the detection of the ground state $|g\rangle$ or the excited state $|e\rangle$ projects into the entangled coherent states

$$\Phi_\pm=|\beta\rangle_a|-\alpha\rangle_b\pm|-\beta\rangle_a|\alpha\rangle_b . \quad (12)$$

There is also the possibility of choosing the detuning between cavity field and laser field to be $\delta_{cA}=-\nu$. In this case, we can obtain the corresponding effective interaction Hamiltonian

$$H_2=i\Omega_2(ab-a^\dagger b^\dagger)\sigma_z . \quad (13)$$

Here, Ω_2 denotes the effective interaction strength. The interaction (12) is a parametric amplifier, which leads to a two-mode squeezed vacuum state, if the two Bose modes are at the beginning in the vacuum state and the ion's vibration ground state. Again we initialize the ion's internal state in the state $1/\sqrt{2}(|g\rangle+|e\rangle)$. Thus, the system's initial state

$$\Psi(0)=\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)\,|0\rangle_a|0\rangle_b \quad (14)$$

evolves under the interaction (13) into

$$\Psi(t)=\frac{1}{\sqrt{2}}\exp[\Omega_2 t(a^\dagger b^\dagger-ab)]\,|e\rangle|0\rangle_a|0\rangle_b + \frac{1}{\sqrt{2}}\exp[-\Omega_2 t(a^\dagger b^\dagger-ab)]\,|g\rangle|0\rangle_a|0\rangle_b . \quad (15)$$

After subjecting the atom's internal state to a $\pi/2$ pulse and detecting the internal state, a projection onto the squeezed cat states results in

$$\Phi_{\pm}=\exp[\Omega_2 t(a^\dagger b^\dagger-ab)]|0\rangle_a|0\rangle_b \pm\exp[-\Omega_2 t(a^\dagger b^\dagger-ab)]|0\rangle_a|0\rangle_b. \quad (16)$$

Finally we consider the case $\omega_0-\omega_{cav}=\nu$. In the Lamb-Dicke limit we obtain after discarding the rapidly oscillating terms in Hamiltonian (1) the interaction Hamiltonian [12]

$$H_3=\Omega_3(ab\sigma_+ + a^\dagger b^\dagger\sigma_-). \quad (17)$$

This was used to generate Bell states of a single-ion vibration mode and the cavity field state [12]. When the ion's internal state is initially in the ground state $|g\rangle$, the interaction (17) is applied with $\Omega_3 t_1=\pi$. We obtain

$$|0\rangle_a|0\rangle_b|g\rangle\rightarrow|0\rangle_a|0\rangle_b|g\rangle,$$

$$|0\rangle_a|1\rangle_b|g\rangle\rightarrow|0\rangle_a|1\rangle_b|g\rangle,$$

$$|1\rangle_a|0\rangle_b|g\rangle\rightarrow|1\rangle_a|0\rangle_b|g\rangle,$$

$$|1\rangle_a|1\rangle_b|g\rangle\rightarrow-|1\rangle_a|1\rangle_b|g\rangle, \quad (18)$$

where the states to the left of the arrows represent initial states $\Psi(0)$ and the states to the right are the states $\Psi(t_1)$. Equation (18) shows that a quantum-phase-gate operation can be implemented if the vibration mode and the cavity mode are used to represent separately a qubit.

In summary, we presented a scheme to generate nonclassical two-mode states of a single trapped-ion vibration mode and the light field state. This quantum state generation scheme makes the generation of two-mode SU(2)-Schrödinger-cat states, entangled coherent states, two-mode squeezed vacuum states, and their superposition possible. We also showed that a quantum phase gate can be implemented if the two-qubit states are identified with the two-mode Fock states with the quantum numbers 0 and 1. These results demonstrate the usefulness of this trapped-ion cavity system with respect to the nonclassical state generation.

Note added. Recently, we became aware of two different proposals to implement quantum logic gates in a trapped-ion cavity QED system [18].

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