Phase control of light amplification without inversion in a Λ system with spontaneously generated coherence

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Due to interaction with the vacuum of the radiation field, near-degenerate lower levels in a Λ system have an additional coherence term, the spontaneously generated coherence term. In this paper, we investigate effects of spontaneouly generated coherence on inversionless gain in the presence of a weak probe, a strong coherent field, and an incoherent pump. We find that the inversionless gain stems from both spontaneously generated coherence and dynamically induced coherence, but the former contributes more to the inversionless gain. In particular, we can modulate the inversionless gain just by changing the relative phase between the two fields.

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I. INTRODUCTION

Now it is well understood how the decay of a system with closely lying states induced by interaction with a common bath leads to new types of coherences [1–8]. These coherences modify, among other things, line shapes of spontaneous emission. Systems with near-degenerate levels have been subject of recent studies in connection with the production of quantum beats and probe absorption [3–5]. For a Λ system with near-degenerate levels, Javanainen [7] discussed the possibility of spontaneously generated coherence (SGC) effect. In particular, he examined the response of this system to an external field coupling two transitions simultaneously, and demonstrated that the SGC effect result in the disappearance of the dark state. For the same Λ system, Menon and Agarwal [8] investigated its response to two external fields of arbitary intensity. They found that such coherence brings about quantitative changes in line profiles of absorption and dispersion, and leads to the dependence of line shapes on the relative phase of applied fields. The existence of the SGC effect depends on the nonorthogonality of dipole matrix elements [7,8]. In other words, if dipole matrix elements are orthogonal, the SGC effect will disappear.

It is also well known that strong coherent fields can lead to another kind of coherence, the dynamically induced coherence, which is related to light amplification (lasing) without population inversion (LWI) as well as many other optical phenomena in quantum optics. In the past decade, it has been shown that, in cases where it is difficult or even impossible to create the required population inversion for conventional laser by incoherent pumping, LWI provides us an important and interesting alternative. And it has been identified that the origin of the inversionless gain can be attributed to either inversion between dressed states or coherence among these states [9,10]. Until now, a variety of schemes have been studied either theoretically or experimentally to realize LWI [9–19]. But as we know, in all these schemes, the inversionless gain cannot be adjusted by phases of applied coherent fields, though it is related to amplitudes (or Rabi frequencies) and carrier frequencies (or detunings) of these fields. However, in this paper, we will show that, when an incoherent process is used to pump a Λ system with near-degenerate levels, the inversionless gain also can be related to the relative phase beween the probe and the coherent field. In Ref. [8], in order to keep the SGC effect notable, both applied coherent fields have to be strong enough. While in our paper, due to the existence of the incoherent pump, even if the probe is very weak, the SGC effect could be remarkable.

This paper is organized as follows: in Sec. II, we introduce the Λ system under study, and get the corresponding density-matrix equations; in Sec. III, in the limit of a weak probe, we derive the probe gain coefficient; in Sec. IV, we investigate the dependence of the probe gain on the relative phase between two fields by showing a few graphic results; In Sec. V, we give a brief summary of the results of this paper.

II. THE SYSTEM AND DENSITY-MATRIX EQUATIONS

We consider a Λ-type three-level system (see Fig. 1) driven by a strong coherent field with amplitude (frequency) $\vec{E}_c(\omega_c)$ and an incoherent pump process represented by a rate 2Λ. A weak coherent field with amplitude (frequency) $\vec{E}_p(\omega_p)$ is used to probe the gain on transition $|1\rangle \leftrightarrow |3\rangle$. Since dipole moments $\vec{d}_{12}$ and $\vec{d}_{13}$ are not orthogonal, which is necessary for the existence of the SGC effect, we have to consider an arrangement where each field acts only on one transition. This can be achieved by considering the case shown in Fig. 1, where the probe and the coherent field acts on transitions $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |2\rangle$, respectively. θ represents the angle between the two induced dipole moments. $2\gamma_1$ and $2\gamma_2$ are the spontaneous emission rates from level $|1\rangle$ to levels $|3\rangle$ and $|2\rangle$, respectively.

In the interaction picture, the semiclassical interaction Hamiltonian of this Λ system in a rotating-wave frame can be written as

$$H_I = \Delta_p|1\rangle\langle 1| + (\Delta_p - \Delta_c)|2\rangle\langle 2| - [G_c|1\rangle\langle 2| + G_p|1\rangle\langle 3| + \text{c.c}]. \tag{1}$$

Then by using the Weisskopf-Wigner theory of spontane-

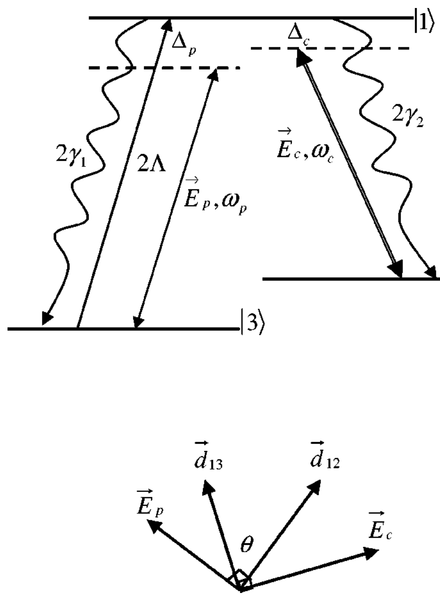

FIG. 1. Schematic diagram of a three-level Λ system driven by a strong coherent field, a weak coherent field, and an incoherent pump. The field polarizations are chosen so that one field drives only one transition.

ous emission [20], we get the density-matrix equations in the rotating-wave approximation and the dipole approximation as follows:

$$\begin{aligned}
\dot{\rho}_{22} &= 2\gamma_2\rho_{11} + iG_c^*\rho_{12} - iG_c\rho_{21},\\
\dot{\rho}_{33} &= 2\gamma_1\rho_{11} - 2\Lambda\rho_{33} + iG_p^*\rho_{13} - iG_p\rho_{31},\\
\dot{\rho}_{12} &= -(\gamma_1+\gamma_2+i\Delta_c)\rho_{12} + iG_p\rho_{32} - iG_c(\rho_{11}-\rho_{22}),\\
\dot{\rho}_{13} &= -(\gamma_1+\gamma_2+\Lambda+i\Delta_p)\rho_{13} + iG_c\rho_{23} - iG_p(\rho_{11}-\rho_{33}),\\
\dot{\rho}_{23} &= -(\Lambda+i\Delta_p-i\Delta_c)\rho_{23} + 2\sqrt{\gamma_1\gamma_2}\cos\theta\,\eta\rho_{11} + iG_c^*\rho_{13}\\
&\quad - iG_p\rho_{21}.
\end{aligned} \tag{2}$$

The above equations are constrained by $\rho_{ij}=\rho_{ji}^*$ and $\rho_{11}+\rho_{22}+\rho_{33}=1$. Here $\Delta_c=\omega_{12}-\omega_c$ and $\Delta_p=\omega_{13}-\omega_p$ designate the detuning of the probe and the coherent field, respectively. $G_c=\vec{d}_{12}\cdot\vec{E}_c=G_c^0\sin\theta$ and $G_p=\vec{d}_{13}\cdot\vec{E}_p=G_p^0\sin\theta$ are the coupling coefficients, i.e., the Rabi frequencies. $2\sqrt{\gamma_1\gamma_2}\cos\theta\eta\rho_{11}$ represents the quantum interference effect resulting from the cross coupling between spontaneous emissions $|1\rangle\rightarrow|2\rangle$ and $|1\rangle\rightarrow|3\rangle$, i.e., the SGC effect. If levels $|2\rangle$ and $|3\rangle$ lies so closely that the SGC effect has to be taken into account, then $\eta=1$, otherwise $\eta=0$. We should note that only for small energy spacing between the two lower levels is the SGC effect remarkable; as for large energy spacing, the rapid oscillation in ρ_{23} will average out such effect [8].

III. THE ANALYTICAL SOLUTIONS

Usual systems with well seperated levels only depend on amplitudes and detunings of applied fields but not on their phases, so Rabi frequencies can be treated as real parameters. However, due to the existence of the SGC effect, i.e., the existence of $2\sqrt{\gamma_1\gamma_2}\cos\theta\rho_{11}$, this Λ system becomes quite sensitive to phases of the probe and the coherent field, thus we have to treat Rabi frequencies as complex parameters. If we define ϕ_p and ϕ_c as phases of the probe and the coherent field, then we get $G_p=g_pe^{i\phi_p}$ and $G_c=g_ce^{i\phi_c}$. Redefining atomic variables in Eq. (2) as $\tilde{\rho}_{ii}=\rho_{ii}$, $\tilde{\rho}_{12}=\rho_{12}e^{i\phi_c}$, $\tilde{\rho}_{13}=\rho_{13}e^{i\phi_p}$, and $\tilde{\rho}_{23}=\rho_{23}e^{i\Phi}$, where $\Phi=\phi_p-\phi_c$, we obtain equations for the redefined density-matrix elements $\tilde{\rho}_{ij}$ which are found to be identical to Eq. (2) except that η is replaced by $\eta_\Phi=\eta e^{i\Phi}$, G_p is replaced by g_p, and G_c is replaced by g_c. In the following, we define $p=2\sqrt{\gamma_1\gamma_2}\cos\theta\eta_\Phi$, and treat g_p and g_c as real parameters.

Under the steady-state condition, in the limit of a weak probe, solutions for $\tilde{\rho}_{ij}$ to the zero order of g_p, but to all orders of g_c, are given as follows:

$$\begin{aligned}
\tilde{\rho}_{11}^{(0)} &= \frac{\Lambda(\gamma_1+\gamma_2)g_c^2}{D},\\
\tilde{\rho}_{22}^{(0)} &= \frac{\Lambda(\gamma_1+\gamma_2)g_c^2+\Lambda\gamma_2[(\gamma_1+\gamma_2)^2+\Delta_c^2]}{D},\\
\tilde{\rho}_{33}^{(0)} &= \frac{\gamma_1(\gamma_1+\gamma_2)g_c^2}{D},\\
\tilde{\rho}_{12}^{(0)} &= \frac{\gamma_2\Lambda g_c(\Delta_c+i\gamma_1+i\gamma_2)}{D},\\
\tilde{\rho}_{13}^{(0)} &= \frac{ipg_c\tilde{\rho}_{11}^{(0)}}{(\gamma_1+\gamma_2+\Lambda)(\Lambda-i\Delta_c)+g_c^2},\\
\tilde{\rho}_{23}^{(0)} &= \frac{p(\gamma_1+\gamma_2+\Lambda)\tilde{\rho}_{11}^{(0)}}{(\gamma_1+\gamma_2+\Lambda)(\Lambda-i\Delta_c)+g_c^2},\\
D &= (\gamma_1+\gamma_2)(2\Lambda+\gamma_1)g_c^2+\Lambda\gamma_2[(\gamma_1+\gamma_2)^2+\Delta_c^2]
\end{aligned} \tag{3}$$

With Eq. (3), solutions for $\tilde{\rho}_{11}$ and $\tilde{\rho}_{31}$ to the first order of g_p, while to all orders of g_c, are derived as follows:

$$\tilde{\rho}_{11}^{(1)} = \tilde{\rho}_{11}^{(0)} + \frac{ig_pg_c[\Delta_c\Lambda(\tilde{\rho}_{23}^{(0)}-\tilde{\rho}_{32}^{(0)})+(\gamma_1+\gamma_2)[g_c(\tilde{\rho}_{31}^{(0)}-\tilde{\rho}_{13}^{(0)})-i\Lambda(\tilde{\rho}_{23}^{(0)}+\tilde{\rho}_{32}^{(0)})]]}{2D}, \tag{4}$$

$$\tilde{\rho}_{31}^{(1)} = \frac{ig_p[(\Lambda-i\Delta_p+i\Delta_c)(\tilde{\rho}_{11}^{(0)}-\tilde{\rho}_{33}^{(0)})-ig_c\tilde{\rho}_{12}^{(0)}]-ip^*g_c\tilde{\rho}_{11}^{(1)}}{(\gamma_1+\gamma_2+\Lambda-i\Delta_p)(\Lambda+i\Delta_c-i\Delta_p)+g_c^2}. \tag{5}$$

The probe gain coefficient on transition $|1\rangle \leftrightarrow |3\rangle$ is related to the imaginary part of $\tilde{\rho}_{31}^{(1)}$. From Eq. (5), we find that $\tilde{\rho}_{31}^{(1)}$ is contributed by three terms: the population difference term (proportional to $\tilde{\rho}_{11}^{(0)} - \tilde{\rho}_{33}^{(0)}$), the dynamically induced coherence term (proportional to $\tilde{\rho}_{12}^{(0)}$), and the spontaneously generated coherence term (proportional to $\tilde{\rho}_{11}^{(1)}$), an additional term compared with the first two terms that are usual in conventional systems for LWI. According to above equations, when Λ is not very large, except the first term, both the second and the last term could have positive contribution to the probe gain, so the probe gain is inversionless gain, and it originates from both dynamically induced coherence and spontaneously generated coherence.

With $\Lambda = 0$, Eq. (5) can be simplified into

$$\tilde{\rho}_{31}^{(1)} = \frac{-g_p(\Delta_p - \Delta_c)}{-[i(\gamma_1 + \gamma_2) + \Delta_p](\Delta_p - \Delta_c) + g_c^2}. \qquad (6)$$

Obviously, in this case, $\tilde{\rho}_{31}^{(1)}$ is independent of p, i.e., independent of the SGC effect, because the probe field is so weak that all population is reserved in the ground state $|3\rangle$, thus no spontaneous emission takes place, no SGC effect [8]. So, it can be concluded that the incoherent pump plays a very important role in reserving the SGC effect in the case of a weak probe.

Note, in this paper, parameters g_c, g_p, Δ_c, Δ_p, Λ, and γ_2 are scaled by γ_1.

IV. PHASE-DEPENDENT GAIN LINE SHAPES

When the incoherent pump is added on transition $|1\rangle \leftrightarrow |3\rangle$, as shown by Eq. (5), $\tilde{\rho}_{31}^{(1)}$ becomes a periodical function of the relative phase Φ, for p depends on Φ. In this section, with a small incoherent pump, we focus on the phase-dependent effect of the probe gain in the presence of the SGC effect.

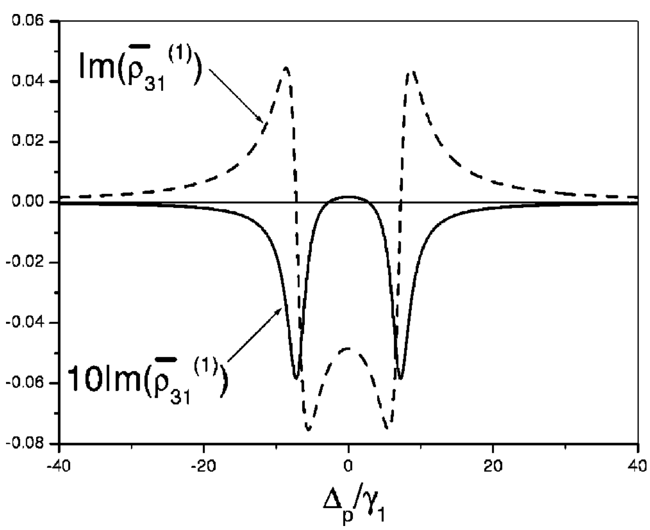

FIG. 2. Dependence of the probe gain Im($\tilde{\rho}_{31}^{(1)}$) upon the probe detuning Δ_p/γ_1 with $\gamma_1 = \gamma_2 = 1$, $\Lambda = 0.5$, $\Delta_c = 0$, $\theta = \pi/4$, $g_p = 0.1 \sin\theta$, and $g_c = 10 \sin\theta$. The solid curve is with $\eta = 0$, while the dashed curve is with $\eta = 1$ and $\Phi = 0$.

In Fig. 2, with $\Lambda = 0.5$, $\gamma_1 = \gamma_2 = 1$, $\theta = \pi/4$, $\Delta_c = 0$, $g_p = 0.1 \sin\theta$, and $g_c = 10 \sin\theta$, we plot the probe gain (or absorption) Im($\tilde{\rho}_{31}^{(1)}$) against the probe detuning Δ_p. Here, according to Eq. (3), $\Lambda = 0.5\gamma_1$ is enough to guarantee that there is no population inversion between level $|1\rangle$ and level $|3\rangle$, so the gain in Fig. 2 is inversionless gain. It is shown that, when no SGC effect exists in this system ($\eta = 0$), the probe is amplified only around $\Delta_p = 0$ with a very small amplitude due to the dynamically induced coherence resulting from the coherent field g_c. While when the SGC effect is considered ($\eta = 1$), the probe is amplified elsewhere with a much larger amplitude. This means that, in the case of $\eta = 1$, the spontaneously generated coherence contributes much more to the probe gain without inversion than the dynamically induced coherence.

In Fig. 3, with different Φ and the same other parameters as used in Fig. 2, we display the dependence of the probe gain Im($\tilde{\rho}_{31}^{(1)}$) upon the detuning of the probe Δ_p. It is found that the probe gain is quite sensitive to the relative phase Φ. With different Φ, we get different gain profiles. When $\Phi = \pi/2$ or $3\pi/2$, much larger probe gain can be achieved at $\Delta_p = -g_c$ or $\Delta_p = g_c$, which corresponds to the dressed-state sublevels $|-\rangle = (1/\sqrt{2})(|1\rangle - |2\rangle)$ or $|+\rangle = (1/\sqrt{2})(|1\rangle + |2\rangle)$ of level $|1\rangle$, respectively. The eigenvalues of the two sublevels are $E_- = -g_c$ and $E_+ = g_c$. While when $\Phi = \pi$, the probe can be amplified with relative smaller amplitudes in a much larger spectrum range: $\delta\omega_p = 2g_c$, the energy spacing between the two dressed-state sublevels $|+\rangle$ and $|-\rangle$.

In Fig. 4, in order to investigate the behavior of the inversionless gain in the case of $\Delta_c \neq 0$, with $\Delta_c = 10$, we plot the probe gain Im($\tilde{\rho}_{31}^{(1)}$) against the detuning of the probe Δ_p. It is found that, when $\Phi = \pi/2$, the amplitude of the gain peak becomes much smaller, but the corresponding linewidth becomes larger. It should be noticed that, when $\Phi = 3\pi/2$, al-

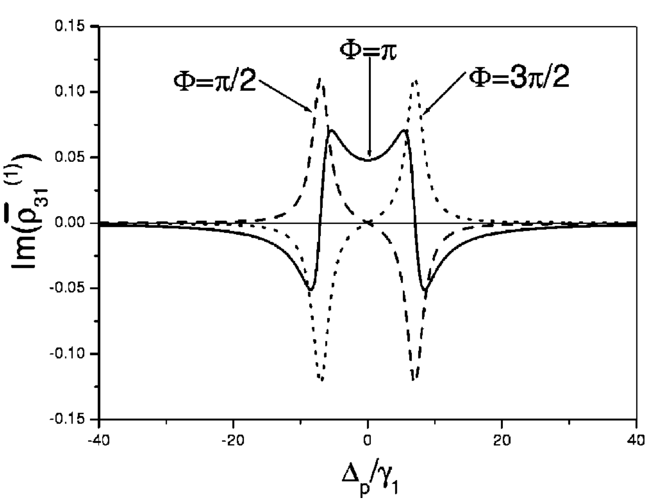

FIG. 3. Dependence of the probe gain Im($\tilde{\rho}_{31}^{(1)}$) upon the probe detuning Δ_p/γ_1 with $\eta = 1$ and different Φ. Other parameters are the same as those in Fig. 2.

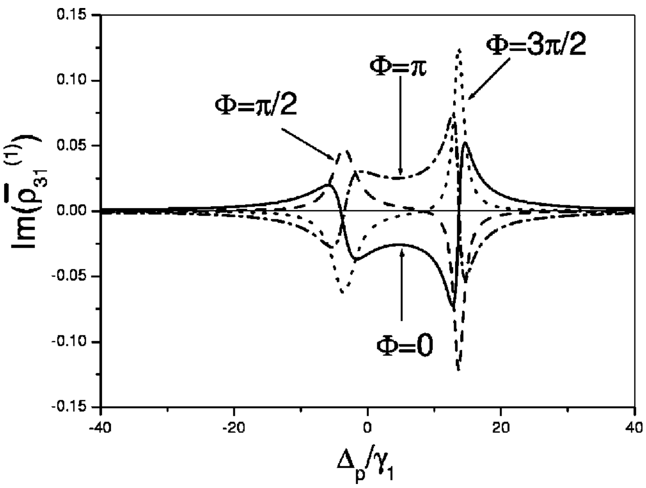

FIG. 4. Dependence of the probe gain $\mathrm{Im}(\tilde{\rho}_{31}^{(1)})$ upon the probe detuning Δ_p/γ_1 with $\eta=1$, $\Delta_c=10$, and different Φ. Other parameters are the same as those in Fig. 2.

though the amplitude of the gain peak does not change notably, the corresponding linewidth becomes smaller. When $\Phi=\pi$, much wider gain spectrum can be got, for in this case, $\delta\omega_p=\sqrt{\Delta_c^2+4g_c^2}$.

In order to get a deeper insight into the modulation effect of the relative phase Φ on the probe gain without inversion, in Fig. 5, we depict the probe gain $\mathrm{Im}(\tilde{\rho}_{31}^{(1)})$ at $\Delta_p=0,\pm 7$ against the relative phase Φ. Other parameters are the same as those in Fig. 2. It is shown that, at different detunings, we get different modulation amplitudes of the probe gain, and with different values of Φ, the largest probe gain corresponds to different detunings. The modulation period of the probe gain is always 2π.

Now, we give out a few physical interpretations of spectral features found in above figures. Clearly it is the SGC effect that leads to the dependence of the probe gain [i.e., $\mathrm{Im}(\tilde{\rho}_{31}^{(1)})$] on $\tilde{\rho}_{11}^{(1)}$, and then on the relative phase Φ. When the incoherent pumping rate Λ is not much smaller than γ_1 and γ_2, from Eq. (3), we find that $\tilde{\rho}_{11}^{(0)}$ and $\tilde{\rho}_{33}^{(0)}$ are in the order of $g_c\tilde{\rho}_{12}^{(0)}/(\Delta_c+i\gamma_1+i\gamma_2)$. Furthermore, in the limit of a weak probe and a strong coherent field, i.e., $g_c \gg \gamma_1, \gamma_2, \Lambda \gg g_p$, from Eq. (3) and Eq. (4) we find that $\tilde{\rho}_{11}^{(1)} \cong \tilde{\rho}_{11}^{(0)}$. Then in the same limit, according to Eq. (5), the contribution of spontaneously generated coherence ($\tilde{\rho}_{11}^{(1)}$) to the inversionless gain is much larger than those of population difference ($\tilde{\rho}_{11}^{(0)}-\tilde{\rho}_{33}^{(0)}$) and dynamically induced coherence ($\tilde{\rho}_{12}^{(0)}$) as shown by Fig. 2, so the behavior of the probe gain (or absorption) mainly depends on $\tilde{\rho}_{11}^{(1)}$. That is to say, in the limit of a strong coherent field and a weak probe field, Eq. (5) can be simplified into

$$\tilde{\rho}_{31}^{(1)}=\frac{-ip^*g_c\tilde{\rho}_{11}^{(1)}}{(\gamma_1+\gamma_2+\Lambda-i\Delta_p)(\Lambda+i\Delta_c-i\Delta_p)+g_c^2}. \qquad (7)$$

From Eq. (7), it is easy to find the main spectral features of the probe gain. When $\Delta_p=\pm g_c$ and $\Delta_c=0$, we get

$$\mathrm{Im}(\tilde{\rho}_{31}^{(1)})\approx\frac{\mp 2\sqrt{\gamma_1\gamma_2}\cos\theta\sin\Phi\tilde{\rho}_{11}^{(1)}}{(2\Lambda+\gamma_1+\gamma_2)}, \qquad (8)$$

while when $\Delta_p=\Delta_c=0$, we get

$$\mathrm{Im}(\tilde{\rho}_{31}^{(1)})\approx\frac{-2\sqrt{\gamma_1\gamma_2}\cos\theta\cos\Phi\tilde{\rho}_{11}^{(1)}}{g_c}. \qquad (9)$$

According to Eq. (8), we cannot get the probe gain at $\Delta_p=\pm g_c$ simultaneously, which means that if the probe is amplified at $\Delta_p=g_c$, then it is surely absorbed at $\Delta_p=-g_c$, just as shown by Figs. 3 and 5. Furthermore, the behavior of the probe gain around $\Delta_p=g_c$ is also opposite to that around $\Delta_p=-g_c$ (see, Figs. 2 and 3). By comparing Eq. (8) with Eq. (9), we find that the inversionless gain at $\Delta_p=g_c$ or $\Delta_p=-g_c$ can be much larger than that at $\Delta_p=0$ (see, Figs. 3 and 5) as long as $g_c \gg \Lambda, \gamma_1, \gamma_2$. From Eqs. (8) and (9), it is also easy to understand why the maximal probe gain at $\Delta_p=0$ corresponds to $\Phi=\pi$, while the maximal probe gain at $\Delta_p=g_c(\Delta_p=-g_c)$ corresponds to $\Phi=3\pi/2$ ($\Phi=\pi/2$). In a word, when $g_c \gg \gamma_1, \gamma_2, \Lambda \gg g_p$, the inversionless gain manily stems from the spontaneously generated coherence, and it is sensitive to the relative phase Φ. Similar results can be obtained for unequal $\gamma's$.

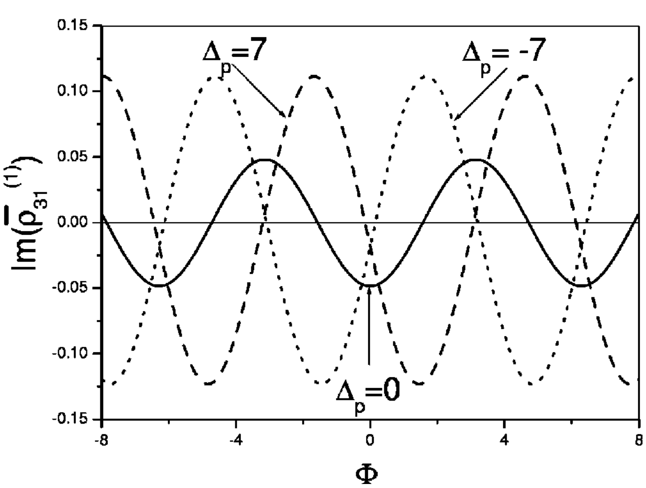

FIG. 5. Dependence of the probe gain $\mathrm{Im}(\tilde{\rho}_{31}^{(1)})$ upon the relative phase Φ with $\eta=1$ and different Δ_p. Other parameters are the same as those in Fig. 2.

V. CONCLUSIONS

In summary, we have shown that, in a Λ system with near-degenerate lower levels, with an incoherent pump, the SGC effect can be reserved even in the case of a weak probe. The inversionless gain can be achieved due to the spontaneously generated coherence as well as the dynamically induced coherence. In particular, the inversionless gain becomes quite sensitive to the relative phase between the probe and the coherent field. We can modulate the gain profile and the gain amplitude simultaneously just by changing the relative phase, which cannot be realized in a conventional Λ system. These results could be experimentally observed provided that the dipole elements for the two optical transition in the Λ system are nonorthogonal. The nonorthogonality can be obtained from the mixing of the levels arising from internal fields or external microwave fields [8,21].

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