Conditional state generation in a dispersive atom-cavity field interaction with a continuous external pump field

Christopher C. Gerry
Department of Physics and Astronomy, Lehman College, The City University of New York, 250 Bedford Park Boulevard West, Bronx, New York 10468-1589

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The interaction of an atom with both a quantized cavity field and an external classical driving field, the fields being degenerate in frequency, is studied in the regime where the atom and fields are highly detuned. The atom interacts dispersively with the quantized field but the classical driving field gives rise to the creation or destruction of photons conditional on the state of the atom. We show how this interaction can be used to generate coherent states of the cavity field and various forms of superpositions of macroscopically distinct states. This method is in contrast to the usual method used in microwave cavity QED of injecting a coherent state into a cavity via a waveguide attached to a klystron and where subsequently Schrödinger cat states may be generated by manipulating the field with injected atoms. The method proposed here could possibly be used in the case of an optical cavity. Further, we show that coherent states may be generated in the steady state from the competition between the driven dispersion interaction and dissipative single-photon losses, a form of optical balance.

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I. INTRODUCTION

The Jaynes-Cummings model (JCM) is a description of a two-level atom interacting with a single-mode quantized cavity electromagnetic field within the rotating wave approximation [1]. It is the simplest model of field-matter interactions relevant to quantum optics, having exactly integrable nonperturbative solutions. If the cavity field has somehow been prepared in a coherent state, the model predicts [2], and experiments confirm [3], the collapse and revival of the Rabi oscillations (nutations), a direct consequence of the quantization of the field. Numerous multilevel and multimode extensions of the original JCM have been studied over the years [4]. Even within the original JCM, an important variation in the form of the interaction occurs in the limit of a large (but not too large) detuning between the cavity field and the relevant atomic transition frequency. In such situations the interaction is dispersive, an interaction that has proved of great importance in various proposals and experiments for the production of superpositions of macroscopically (or at least mesoscopically) distinguishable quantum states, the so-called Schrödinger cat states, within the context of cavity QED [5]. The Schrödinger cat states are superpositions of coherent states differing by some macroscopic rotation in phase space (typically 180°) resulting from the dispersive atom-field interaction. But again, one needs to provide the cavity field with an initial coherent state. In experiments using submillimeter microwave cavities supporting a single-mode field and where circular Rydberg atoms are used to manipulate the field, a coherent state can be injected into the cavity through a waveguide attached to a klystron, itself driven by a classical field [6]. It is also possible to directly drive a current on the surface of one of the mirrors on the cavity. For optical cavities it is generally assumed that it is possible to drive one of the semitransparent cavity mirrors with an external laser field [7]. In this paper we describe what we believe is a more elegant and versatile procedure wherein a strong external coherent field (microwave or laser, depending on the type of cavity QED experiment) resonant with a cavity mode, interacts nonresonantly, i.e., dispersively, with an appropriately prepared atom passing through the cavity. The resulting interaction is conditional on the state of the atom and essentially converts the classical external field into a quantized cavity field of identical frequency (but possibly of different polarization). For the atom prepared in one of the bare atomic states, coherent states will be produced. Furthermore, we show that by preparing the injected atom in a superposition of the bare atomic states, various types of Schrödinger cat states may be generated, although manipulation and state reduction on the atom is required after it exists the cavity. The procedure described here should work for both microwave and optical cavities as long as dissipative effects can be ignored over the time scales of the interaction. We then show that when dissipative effects *are* taken into account, it is still possible to generate a coherent state as the result of the optical balance between the dispersive interaction and the dissipative interaction in the steady-state regime, a result that could be of particular importance for generating coherent states in optical cavity experiments involving single or few atoms.

The driven Jaynes-Cummings model for cases where the cavity and external driving field are close to or on resonance with the atom, has been studied by several authors. Alsing, Gou, and Carmichael [8] studied the Stark splittings in the quasienergies of the dressed states resulting from the presence of the driving field in the case where both fields are resonant with the atom. Jyotsna and Agarwal [9] studied the effect of the external field on the Rabi oscillations in the case where the cavity field is resonant with the atom and where the external field is both resonant and nonresonant. Dutra, Knight, and Moya-Cessa [10] studied a similar model but where the external field was taken to be quantized. Chough and Carmichael [11] have studied the JCM with an external

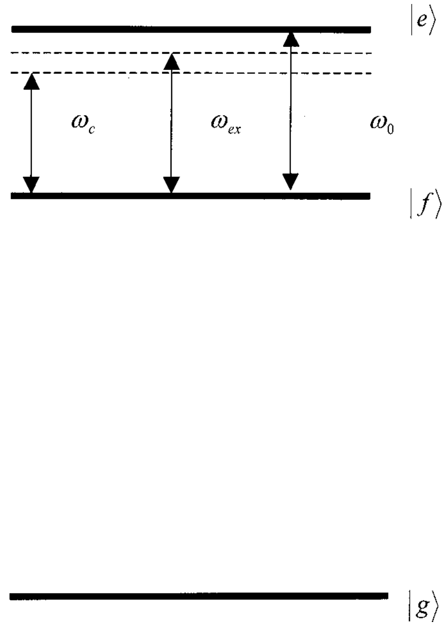

FIG. 1. The energy-level configuration of an atom interacting with both cavity and external driving fields. The transition frequency between levels e and f is ω_0 while ω_c and ω_{ex} are the cavity and external field frequencies, respectively. The cavity and external fields are close, but not too close, to resonance with the atomic transition frequency ω_0 such that the interaction is dispersive. We assume the condition $\omega_c=\omega_{ex}$. The level g is far out of resonance with any of the frequencies involved.

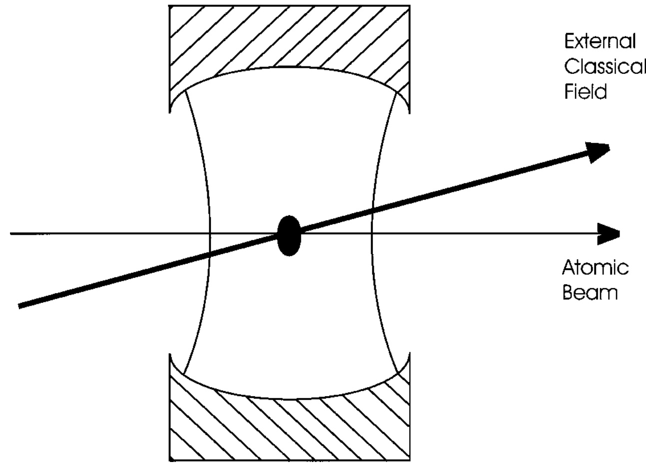

FIG. 2. An atom passes through a cavity and interacts dispersively with a cavity field mode. At the same time it interacts with an external classical field tuned into resonance with the cavity field. The atom enters the cavity prepared in either one, or a superposition of two, of the bare states indicated in Fig. 1. After the atom exits the cavity it may be subjected to classical fields implementing $\pi/2$ pulses and then selectively ionized in order to produce various types of Schrödinger cat states.

resonant driving field and have shown that the collapses and revivals of the mean photon number occur over a much longer time scale than the revival time of the Rabi oscillations for the atomic inversion. Joshi [12], in a similar vein, studied the driven two-photon JCM Nha, Chough, and An [13] studied the preparation of a temporally stable single-photon state an atom-cavity field system with a driving classical field. As far as the author is aware, the dispersive interaction with an external driving field has not previously been considered though it is a logical extension of previous work in this area.

This paper is organized as follows. In Sec. II, we discuss the driven Jaynes-Cummings model in the regime where the atom is equally detuned with both the quantized cavity field and driving external classical field such that the atom-cavity field coupling is dispersive. We discuss the generation of coherent states and superpositions of coherent states. In Sec. III, we consider the inclusion of dissipative effects, showing yet another mechanism to generate coherent states as the result of optical balance in the steady state. The paper concludes in Sec. IV with some brief remarks.

II. DRIVEN JAYNES-CUMMINGS MODEL IN THE DISPERSIVE REGIME

We consider an atom with three levels $|e\rangle$, $|f\rangle$, and $|g\rangle$ configured as in Fig. 1. We assume that only dipole transitions can occur consecutively as follows: $|e\rangle\leftrightarrow|f\rangle\leftrightarrow|g\rangle$. We let ω_0 be the $|e\rangle\leftrightarrow|f\rangle$ transition frequency and assume it near resonance with a single-mode cavity field of frequency ω_c. But the $|f\rangle\leftrightarrow|g\rangle$ transition we assume is far out of resonance with the cavity mode of interest (or any other cavity mode). A strong, classical, prescribed field, of frequency ω_{ex}, possibly of different polarization than the cavity field, interacts directly with an atom passing through the cavity, as pictured in Fig. 2. The Hamiltonian for the atom-cavity system is given by

$$\hat{H}=\tfrac{1}{2}\hbar\omega_0\sigma_3+\hbar\omega_c\hat{a}^\dagger\hat{a}+\hbar g(\hat{\sigma}_+\hat{a}+\hat{a}^\dagger\hat{\sigma}_-)+\hbar(Ee^{-i\omega_{ex}t}\hat{\sigma}_++E^*e^{i\omega_{ex}t}\hat{\sigma}_-). \quad (2.1)$$

Here

$$\hat{\sigma}_3=|e\rangle\langle e|-|f\rangle\langle f|,\quad \hat{\sigma}_+=|e\rangle\langle f|,\quad \hat{\sigma}_-=|f\rangle\langle e|, \quad (2.2)$$

g is the coupling constant between the atom and the cavity field mode, E is proportional to the coupling constant between the atom and the external classical field of frequency ω_{ex} and the amplitude of that field. We have assumed for the moment that ω_0, ω_c, and ω_{ex} are different. To remove the time dependence in $\hat{H}$, we use the operator $\hat{R}=\exp[-i\omega_{ex}t(\hat{\sigma}_3+\hat{a}^\dagger\hat{a})]$ to transform to a frame rotating at the frequency ω_{ex}. The Hamiltonian in the rotating frame (essentially the interaction picture) is then

$$\hat{H}_R=\tfrac{1}{2}\hbar(\omega_0-\omega_{ex})\hat{\sigma}_3+\hbar(\omega_c-\omega_{ex})\hat{a}^\dagger\hat{a}+\hbar g[\hat{\sigma}_+(\hat{a}+\lambda)+(\hat{a}^\dagger+\lambda^*)\hat{\sigma}_-], \quad (2.3)$$

where $\lambda=E/g$. We now assume the resonance condition between the external and cavity fields, $\omega_c=\omega_{ex}$, and obtain

$$\hat{H}_R=\tfrac{1}{2}\hbar\Delta\hat{\sigma}_3+\hbar g[\hat{\sigma}_+(\hat{a}+\lambda)+(\hat{a}^\dagger+\lambda^*)\hat{\sigma}_-], \quad (2.4)$$

where we have set $\Delta=\omega_0-\omega_c$. At this point we may introduce auxiliary Bose operators $\hat{b}=\hat{a}+\lambda$ and $\hat{b}^\dagger=\hat{a}^\dagger+\lambda^*$ satisfying $[\hat{b},\hat{b}^\dagger]=1$ so that we may write

$$\hat{H}_R=\tfrac{1}{2}\hbar\Delta\hat{\sigma}_3+\hbar g[\hat{\sigma}_+\hat{b}+\hat{b}^\dagger\hat{\sigma}_-], \tag{2.5}$$

which superficially has the appearance of the interaction picture Hamiltonian of the usual detuned JCM. Alternatively, introducing the displacement operator $\hat{D}(\lambda)=\exp(\lambda\hat{a}^\dagger-\lambda^*\hat{a})$ we may write

$$\hat{H}_R=\tfrac{1}{2}\hbar\Delta\hat{\sigma}_3+\hbar g\hat{D}^\dagger(\lambda)[\hat{\sigma}_+\hat{a}+\hat{a}^\dagger\hat{\sigma}_-]\hat{D}(\lambda), \tag{2.6}$$

where we have made use of the relations

$$\hat{D}^\dagger(\lambda)\begin{Bmatrix}\hat{a}\\ \hat{a}^\dagger\end{Bmatrix}\hat{D}(\lambda)=\begin{Bmatrix}\hat{a}+\lambda\\ \hat{a}^\dagger+\lambda^*\end{Bmatrix}. \tag{2.7}$$

In the limit of moderately large detuning between the atom and the fields, one can use the standard techniques [14] to obtain the effective atom-field interaction Hamiltonian

$$\begin{aligned}\hat{H}_{\rm eff}&=\hbar\chi[\hat{\sigma}_+\hat{\sigma}_-+\hat{b}^\dagger\hat{b}\hat{\sigma}_3]\\ &=\hbar\chi[\hat{\sigma}_+\hat{\sigma}_-+\hat{D}^\dagger(\lambda)\hat{a}^\dagger\hat{a}\hat{D}(\lambda)\hat{\sigma}_3]\\ &=\hbar\chi[\hat{\sigma}_+\hat{\sigma}_-+(\hat{a}^\dagger\hat{a}+\lambda\hat{a}^\dagger+\lambda^*\hat{a}+|\lambda|^2)\hat{\sigma}_3],\end{aligned} \tag{2.8}$$

where $\chi=g^2/\Delta$. Obviously, in the limit $\lambda\to 0$ (no external driving field) we recover the usual dispersive interaction Hamiltonian $\hat{H}_{\rm eff}=\hbar\chi[\hat{\sigma}_+\hat{\sigma}_-+\hat{a}^\dagger\hat{a}\hat{\sigma}_3]$. But with $\lambda\neq 0$, the interaction is no longer purely dispersive as it contains terms of the form $(\lambda\hat{a}^\dagger+\lambda^*\hat{a})\hat{\sigma}_3$ that creates or destroys photons in the cavity conditional on the state of the atom. If the atom is prepared in the far off-resonance state $|g\rangle$, the cavity field is unaffected. But with the atom prepared in either states $|e\rangle$ or $|f\rangle$, and if the cavity field is initially in a vacuum state $|0\rangle$, the external classical driving field will generate a coherent state of the quantized field.

As a specific example, suppose an atom prepared in state $|f\rangle$ is injected through a cavity in the vacuum state. Then, while the atom is inside the cavity, the atom-field system evolves according to

$$\begin{aligned}|\psi(t)\rangle&=\exp[-i\hat{H}_{\rm eff}t/\hbar]|0\rangle|f\rangle\\ &=\exp[i\chi t(\hat{a}^\dagger\hat{a}+\lambda\hat{a}^\dagger+\lambda^*\hat{a}+|\lambda|^2)]|0\rangle|f\rangle\\ &=\exp[i|\lambda|^2\sin(\chi t)]|-\lambda(1-e^{i\chi t})\rangle|f\rangle,\end{aligned} \tag{2.9}$$

where $|-\lambda(1-e^{i\chi t})\rangle$ is a coherent state of the cavity field. On the other hand, if the atom is initially in the state $|e\rangle$, it is easy to see that we obtain

$$\begin{aligned}|\psi(t)\rangle&=\exp[-i\hat{H}_{\rm eff}t/\hbar]|0\rangle|e\rangle\\ &=\exp[-i\chi t-i\chi t(\hat{a}^\dagger\hat{a}+\lambda\hat{a}^\dagger+\lambda^*\hat{a}+|\lambda|^2)]|0\rangle|e\rangle\\ &=e^{-i\chi t}\exp[-i|\lambda|^2\sin(\chi t)]|-\lambda(1-e^{-i\chi t})\rangle|e\rangle.\end{aligned} \tag{2.10}$$

The coherent states $|-\lambda(1-e^{\pm i\chi t})\rangle$ are obviously not simple inversions (i.e., not separated by a rotation 180°) of each other in the phase space. But if we write $\lambda=|\lambda|e^{i\vartheta}$ it is easy to see that they are really reflections of each other about a line along $\vartheta+\pi$. The two coherent states evolve from the origin (the vacuum state) at $t=0$ in opposite directions about a circle centered on $-\lambda$. For $t<\pi/2\chi$ the coherent states can have a large angular separation in phase space, in fact, greater than 90°, the angular separation *becoming* 90° for $t=\pi/2\chi$. Of course, the separations in phase space will also depend on $|\lambda|$ and thus may be quite large. For $t=\pi/\chi$ the two states become identical: $|-2\lambda\rangle$. For the simplification of notation in future applications we set

$$|\psi_f(t)\rangle=\exp[i|\lambda|^2\sin(\chi t)]|-\lambda(1-e^{i\chi t})\rangle, \tag{2.11a}$$

$$|\psi_e(t)\rangle=e^{-i\chi t}\exp[-i|\lambda|^2\sin(\chi t)]|-\lambda(1-e^{i\chi t})\rangle, \tag{2.11b}$$

Suppose we prepare the atom in the general superposition state $\sin\theta|e\rangle+e^{i\phi}\cos\theta|f\rangle$ and the cavity field in the vacuum state. Then at time $t\geqslant 0$ we obtain the entangled state

$$|\psi(t)\rangle=\sin\theta|\psi_e(t)\rangle e\rangle+e^{i\phi}\cos\theta|\psi_f(t)\rangle|f\rangle. \tag{2.12}$$

For the simplest case where $\theta=\pi/4$ and $\phi=0$ we have

$$|\psi(t)\rangle=\frac{1}{\sqrt{2}}(|\psi_e(t)\rangle|e\rangle+|\psi_f(t)\rangle|f\rangle). \tag{2.13}$$

If the atom leaves the cavity after time t (and henceforth t should be understood as the time the atom and the fields have interacted) and is then manipulated by resonant classical fields effecting $\pi/2$ pulses causing the transformations $|e\rangle\to(|e\rangle+|f\rangle)/\sqrt{2}$ and $|f\rangle\to(|f\rangle-|e\rangle)/\sqrt{2}$, we obtain

$$|\psi(t)\rangle\to\tfrac{1}{2}[|e\rangle(|\psi_e(t)\rangle-\psi_f(t)\rangle)+|f\rangle(|\psi_e(t)\rangle+|\psi_f(t)\rangle)]. \tag{2.14}$$

If by selective ionization the atom is found to be in state $|f\rangle$ ($|e\rangle$), we project the cavity field onto the states (apart from normalization)

$$|\psi_e(t)\rangle\pm|\psi_f(t)\rangle, \tag{2.15}$$

which constitute forms of Schrödinger cat states. For the time $t=\pi/2\chi$ we have

$$\begin{aligned}&[|\psi_e(t)\rangle_F\pm|\psi_f(t)\rangle_F]|_{t=\pi/2\chi}\\ &=e^{i|\lambda|^2}|-\lambda(1-i)\rangle\mp ie^{-i|\lambda|^2}|-\lambda(1+i)\rangle\\ &=e^{i|\lambda|^2}|-\lambda e^{-i\pi/4}\rangle\mp ie^{-i|\lambda|^2}|-\lambda e^{i\pi/4}\rangle,\end{aligned} \tag{2.16}$$

obviously superpositions of coherent states separated by 90° in phase space.

There exists another possibility. Suppose the atom is prepared in a superposition of the far off-resonance state $|g\rangle$ and either $|f\rangle$ or $|e\rangle$. As an example, we take the initial state as

$$|\psi(0)\rangle = |0\rangle \frac{1}{\sqrt{2}}(|f\rangle + |g\rangle). \tag{2.17}$$

This evolves into

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}[|\psi_f(t)\rangle|f\rangle + |0\rangle|g\rangle]. \tag{2.18}$$

If, when the atom exits the cavity, we subject it to a $\pi/2$ pulse from a classical resonant field causing the transformations $|f\rangle \to (|f\rangle + |g\rangle)/\sqrt{2}$ and $|g\rangle \to (|g\rangle - |f\rangle)/\sqrt{2}$, we obtain

$$\tfrac{1}{2}[|f\rangle(|\psi_f(t)\rangle - |0\rangle) + |g\rangle(|\psi_f(t)\rangle + |0\rangle]. \tag{2.19}$$

Selective state reduction measurements on the atom project the cavity field into either of the (unnormalized) states

$$|\psi_f(t)\rangle \pm |0\rangle, \tag{2.20}$$

i.e., superpositions of a coherent state with the vacuum. States of this sort have previously been discussed as an example of a quantum switch by Davidovich *et al.*, [15] though the mechanism of generation is a bit different than in our case. In the former, the external source of microwave radiation is a klystron, driven by a classical current, coupled to the cavity mode structure via a waveguide. Such arrangements have in fact been used to inject coherent states in a cavity field mode in certain experiments [6]. (It would also be possible to drive one of the cavity mirrors with a current resonant with the cavity mode.) In the case of the quantum switch of Davidovich *et al.* [15], a coherent state is generated in the cavity by a frequency pulling effect associated with an atom in a particular state. Effectively, the refractive index of the cavity is given a change large enough to tune the cavity into resonance with the klystron source. Such a coupling is represented by the replacements $\hat{\sigma}_+ \to \hat{a}^\dagger$ and $\hat{\sigma}_- \to \hat{a}$ in the last term of Eq. (2.1), whereas our proposal requires a classical external field interacting directly with an atom. Further, in our case both external and cavity fields are assumed from the outset at resonance with each other but not with the atom. Thus the generation of a coherent state in the cavity, or not, is conditional on the state of the atom. No frequency pulling effect is involved.

So far we have made no approximations. But now suppose that χ is sufficiently small, or that t is sufficiently short, so that we may take $\chi t \ll 1$. Then, from Eqs. (2.11) we will have

$$|\psi_e(t)\rangle \simeq \exp(i\chi|\lambda|^2 t)|i\lambda\chi t\rangle,$$

$$|\psi_f|(t)\rangle \simeq \exp(-i\chi t|\lambda|^2)|-i\lambda\chi t\rangle. \tag{2.21}$$

This approximation is independent of $|\lambda|$. But for a strong external classical field, $|\lambda|$ will be large and thus, apart from the factors $\exp(\pm i\chi|\lambda|^2 t)$, we generate coherent states of equal amplitude but separated in phase space by 180°. Under this approximation, the Schrödinger cat states in Eq. (2.13) take the form

$$e^{i\chi|\lambda|^2 t}|i\lambda\chi t\rangle \pm e^{-i\chi|\lambda|^2 t}|-i\lambda\chi t\rangle, \tag{2.22}$$

a superposition of maximally separated equal amplitude coherent states for which the condition $\chi t \ll 1$ holds. If $|\lambda|$ is large enough to attain the condition $\chi|\lambda|^2 t = \pi$, the Schrödinger cat states of Eq. (2.22) will become the even and odd coherent states. This procedure discussed here is an alternative to the usual cavity QED approach to generating Schrödinger cat states where one injects a coherent state $|\alpha\rangle$ directly into the cavity via a waveguide and then by injecting a sequence of suitably prepared atoms manipulates the field state into the form $|\alpha e^{i\varphi/2}\rangle \pm |\alpha e^{-i\varphi/2}\rangle$ where $\varphi = \chi t$. For $\varphi = \pi$, we obtain the maximally separated states $|i\alpha\rangle \pm |-i\alpha\rangle$, which happen to be the even or odd Schrödinger cat states. But the states in Eq. (2.19) are *always* separated by 180° as long as the condition $\chi t \ll 1$ holds. Thus for a strong enough external field, it may be possible to produce the Schrödinger cat states in a much shorter time scale than in the conventional approach.

There is another way to reach these approximate results. If we go back to the effective Hamiltonian of Eq. (2.8) and, under the assumption that $|\lambda|$ is large enough so that $\langle \hat{a}^\dagger \hat{a}\rangle \ll |\lambda|\langle \hat{a}\rangle$, we may simply drop the $\hat{a}^\dagger \hat{a}\hat{\sigma}_3$ term to obtain the approximate effective Hamiltonian

$$\hat{H}_{\text{eff}} = \hbar\chi\hat{\sigma}_3(\lambda\hat{a}^\dagger + \lambda^*\hat{a} + |\lambda|^2), \tag{2.23}$$

where we have also dropped the term $\hat{\sigma}_+\hat{\sigma}_-$, which is at most of order unity. For the initial state $|0\rangle|f\rangle$, we have

$$\exp[-i\hat{H}_{\text{eff}}t/\hbar]|0\rangle|f\rangle = e^{i\chi|\lambda|^2 t}\hat{D}(i\chi\lambda t)|0\rangle|f\rangle$$

$$= e^{i\chi|\lambda|^2 t}|i\chi\lambda t\rangle|f\rangle. \tag{2.24}$$

Similarly, for the initial state $|0\rangle|e\rangle$ we obtain

$$e^{-i\chi|\lambda|^2 t}|-i\chi\lambda t\rangle|e\rangle.$$

These results are in agreement with Eqs. (2.21).

In the preceding, section we have assumed that only one atom passes through the cavity. If the atom is prepared in the state $|f\rangle$ then according to Eq. (2.9), the cavity field, if initially in the vacuum, becomes $|-\lambda(1-e^{i\chi t})\rangle$. This coherent state reaches its maximum amplitude when the interaction time t satisfies the condition $\chi t = \pi$ for which we obtain $|-2\lambda\rangle$. The coherent state can be amplified by sending in a sequence of identically prepared atoms. In the ideal case of a sequence of N atoms, each prepared in the state $|f\rangle$, all having the same speed and thus the same interaction time $t = \pi/\chi$, the coherent state $|-2N\lambda\rangle$ would be generated, assuming negligible dissipation during the process. In this way a large amplitude coherent state could be built up in the cavity. More generally, if t_i is the interaction time of the ith atom, then, apart from an irrelevant overall phase factor, the cavity field would be in the coherent state $|\alpha\rangle$, where $\alpha = -\lambda\Sigma_{i=1}^N(1 - e^{i\chi t_i})$. If the $t_i \approx \pi/\chi$ it would still be possible to obtain a large amplitude coherent state although it may not be possible to determine α precisely if the interaction times are stochastic.

III. INCLUSION OF DISSIPATIVE INTERACTIONS AND THE GENERATION OF COHERENT STATES BY OPTICAL BALANCE

So far, we have not considered the dissipative effects of losses through the walls of the cavity. This can be justified if the time scales involved in any of the above processes are short compared to the characteristic time scales of the decay process. But here we include the effects of losses and show that even in this case it is still possible to generate coherent states in the cavity and not just at short times but also in the long-time steady-state regime. The latter results from the optical balance achieved between the competition from the action of the external driving field, which tends to create photons, and the loss mechanism [16].

In the absence of interactions other than with the walls of the cavity and with the walls at zero temperature, the decay of any field established in the cavity is described by the master equation

$$\frac{\partial\hat{\rho}_F}{\partial t}=-\kappa(\hat{a}^\dagger\hat{a}\hat{\rho}_F-2\hat{a}\hat{\rho}_F\hat{a}^\dagger+\hat{\rho}_F\hat{a}^\dagger\hat{a}), \tag{3.1}$$

where κ is the rate of single-photon losses and $\hat{\rho}_F$ is the density operator of the field. The decay time of the field is $t_{\text{decay}}=1/\kappa$. In most cases, workers have assumed that the dissipative interaction can be ignored during the formation of any particular state as long as the time of formation is short compared to any of the other relevant time scales involved, such as the decay times of atoms and the decay time of the cavity. If the field is initially in a coherent state $|\alpha\rangle$, it remains in a (pure) coherent state but with a decaying amplitude: $\hat{\rho}_F(t)=|\alpha e^{-\kappa t/2}\rangle\langle\alpha e^{-\kappa t/2}|$. On the other hand, if the field is initially in a superposition of the form $|\alpha\rangle\pm|-\alpha\rangle$, this initially pure state decoheres into a statistical mixture such that for times $t\gg t_{\text{decoh}}=t_{\text{decay}}/|\alpha|^2$ (t_{decoh} being the decoherence time),

$$\hat{\rho}_F(t\gg t_{\text{decoh}})\approx\tfrac{1}{2}[|\alpha e^{-\kappa t/2}\rangle\langle\alpha e^{-\kappa t/2}| +|-\alpha e^{-\kappa t/2}\rangle\langle-\alpha e^{-\kappa t/2}|]. \tag{3.2}$$

But here we are interested in the long-time dynamics of our model interaction, including the effects of dissipation, in the case where the external driving field is maintained. Thus we must modify our master equation to

$$\frac{\partial\hat{\rho}}{\partial t}=-\frac{i}{\hbar}[\hat{H}_{\text{eff}},\hat{\rho}]-\kappa(\hat{a}^\dagger\hat{a}\hat{\rho}-2\hat{a}\hat{\rho}\hat{a}^\dagger+\hat{\rho}\hat{a}^\dagger\hat{a}), \tag{3.3}$$

where $\hat{H}_{\text{eff}}$ is given by Eq. (2.8) and where $\hat{\rho}$ is now the density operator of the atom-field system. As a definite example, we suppose the initial density operator to be $\hat{\rho}(0)=\hat{\rho}_F(0)\otimes\hat{\rho}_A(0)$, where the initial field density operator is $\hat{\rho}_F(0)=|0\rangle\langle 0|$ and where $\hat{\rho}_A=|f\rangle\langle f|$ is the initial atomic density operator. As long as the atom is initially in one of its bare states, evolution under the dispersive interaction will not create entanglement between the atom and field and so we may write $\hat{\rho}(t)=\hat{\rho}_F(t)\otimes\hat{\rho}_A$, such that our master equation for the atom-field system reduces to a master equation for the field only,

$$\frac{\partial\hat{\rho}_F}{\partial t}=i\chi[(\hat{a}^\dagger\hat{a}+\lambda\hat{a}^\dagger+\lambda^*\hat{a}),\hat{\rho}_F] -\kappa(\hat{a}^\dagger\hat{a}\hat{\rho}_F-2\hat{a}\hat{\rho}_F\hat{a}^\dagger+\hat{\rho}_F\hat{a}^\dagger\hat{a}). \tag{3.4}$$

At times short enough to ignore the dissipative interaction, the field evolves into the coherent state given by Eq. (2.9a). But we are interested in the long-time steady-state solution and to that end we now make the transformation to a second rotating frame,

$$\hat{\rho}_F=\exp(-i\chi\hat{a}^\dagger\hat{a}t)\hat{\tilde{\rho}}_F\exp(i\chi\hat{a}^\dagger\hat{a}t). \tag{3.5}$$

Equation (3.4) now takes the form

$$\frac{\partial\hat{\tilde{\rho}}_F}{\partial t}=i\chi[(\lambda\hat{c}^\dagger+\lambda^*\hat{c}),\hat{\tilde{\rho}}_F]-\kappa(\hat{c}^\dagger\hat{c}\hat{\tilde{\rho}}_F-2\hat{c}\hat{\tilde{\rho}}_F\hat{c}^\dagger+\hat{\tilde{\rho}}_F\hat{c}^\dagger\hat{c}), \tag{3.6}$$

where

$$\hat{c}=e^{i\chi\hat{a}^\dagger\hat{a}t}\hat{a}e^{i\chi\hat{a}^\dagger\hat{a}t}=\hat{a}e^{-i\chi t}. \tag{3.7}$$

Of course, we have $[\hat{c},\hat{c}^\dagger]=1$. We make one more transformation defining yet another Bose operator $\hat{d}=\hat{c}-i\chi\lambda/\kappa$ so that we may rewrite Eq. (3.6) as

$$\frac{\partial\hat{\tilde{\rho}}_F}{\partial t}=-\kappa(\hat{d}^\dagger\hat{d}\hat{\tilde{\rho}}_F-2\hat{d}\hat{\tilde{\rho}}_F\hat{d}^\dagger+\hat{\tilde{\rho}}_F\hat{d}^\dagger\hat{d}). \tag{3.8}$$

As $t\to\infty$ we approach the steady state in this second rotating frame where one has $\partial\hat{\tilde{\rho}}_F/\partial t=0$. Evidently the right-hand side of Eq. (3.8) vanishes for $\hat{d}\hat{\tilde{\rho}}_F(\infty)=0=\hat{\tilde{\rho}}_F(\infty)\hat{d}^\dagger$, which in turn means that we must have $\hat{\tilde{\rho}}_F(\infty)=|z\rangle_{2\,2}\langle z|$ where $|z\rangle_2$ is a coherent state satisfying the eigenvalue problem $\hat{c}|z\rangle_2=z|z\rangle_2$ and where $z=i\chi\lambda/\kappa$. The subscript 2 indicates that the coherent state is in the second rotating frame. From Eq. (3.5) we obtain the steady-state solution in the first rotating frame as

$$\hat{\rho}_F(t\to\infty)=|ze^{-i\chi t}\rangle\langle ze^{-i\chi t}|. \tag{3.9}$$

Note that this is not independent of time but does satisfy Eq. (3.4) as an identity provided that $\partial\hat{\tilde{\rho}}_F/\partial t=0$. Thus the steady-state solution of Eq. (3.3) is $\hat{\rho}(t\to\infty)=|ze^{-i\chi t}\rangle\langle ze^{-i\chi t}|\otimes|f\rangle\langle f|$. The coherent field state will be maintained in the cavity under the conditions of optical balance as long as the atom is present. Once the atom leaves the cavity, optical balance can no longer be maintained and the coherent state simply decays in amplitude as described above.

In the case of an optical cavity, we must add one further term to the master equation above in order to take into account the spontaneous emission from the atom out of the sides of the cavity. Equation (3.3) must be modified to

$$\frac{\partial\hat{\rho}}{\partial t}=-\frac{i}{\hbar}[\hat{H}_{\rm eff},\hat{\rho}]-\kappa(\hat{a}^{\dagger}\hat{a}\hat{\rho}-2\hat{a}\hat{\rho}\hat{a}^{\dagger}+\hat{\rho}\hat{a}^{\dagger}\hat{a})-\frac{\gamma}{2}(\hat{\sigma}_{+}\hat{\sigma}_{-}\hat{\rho}-2\hat{\sigma}_{-}\hat{\rho}\hat{\sigma}_{+}+\hat{\rho}\hat{\sigma}_{+}\hat{\sigma}_{-}), \tag{3.10}$$

where γ is the spontaneous emission rate. But we still obtain the same steady-state solutions at long time, namely, $\hat{\rho}(t\to\infty)=|ze^{-i\chi t}\rangle\langle ze^{-i\chi t}|\otimes|f\rangle\langle f|$. The last term in Eq. (3.10) vanishes identically owing to the atom being in the pure state $|f\rangle$. Note that in this case, the *initial* state of the atom need not be $|f\rangle$ but spontaneous emission certainly will bring it to that state in sufficiently long time.

IV. CONCLUSIONS

In this paper, we have studied a variant of the Jaynes-Cummings model with a continuous external pump field for which both the cavity field and the external field are detuned from the atomic transition frequency. The model has distinct features over just the usual dispersive interaction with no external driving field. It can be applied to generate coherent states in both microwave and optical cavities. In the case of microwave cavities it could replace the procedure described in Ref. [6] or those methods proposed in connection with micromaser experiments [17]. In the case of optical cavities, the methods proposed here could be applied to the "single atom laser" experiments of the type discussed by An *et al.* [18] The interaction can be used to generate various forms of Schrödinger cat states without the prior establishment of a coherent state. This could be an important feature in cases where the relevant decay times are short, as for optical cavities. Under certain conditions, the components of the Schrödinger cat state are *always* separated by 180° in phase space, this being impossible without the external driving field. With the inclusion of dissipative interactions, we have shown that coherent states may be maintained in the cavity field in the long-time steady-state regime, as long as the atom remains in the cavity. This steady-state behavior has no counterpart in the undriven dispersive model and is possible only in the presence of the external driving field. This feature of the interaction including dissipation may be of considerable importance in optical cavity QED experiments where a coherent state needs to be maintained over an extended period of time.

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