Multiple transitions in atom optics: Intensity- and density-dependent effects

K. V. Krutitsky,* K.-P. Marzlin,† and J. Audretsch‡
Fakultät für Physik, Universität Konstanz, Fach M 674, D-78457 Konstanz, Germany
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Starting from the first principles of nonrelativistic QED we have derived the system of Maxwell-Schrödinger equations which can be used for theoretical description of atom optical phenomena at high densities of atoms and high intensities of the laser radiation. The role of multiple atomic transitions between ground and excited states in atom optics has been investigated. Nonlinear optical properties of interacting Bose gases are studied and an equation for the refractive index has been derived. We have investigated the role of light-induced and collisional nonlinearities in the diffraction of an ultracold atomic beam from an intense standing laser wave in the Raman-Nath and in the Bragg regimes.

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I. INTRODUCTION

In last decades the problem of interaction of the laser radiation with ultracold atomic gases has attracted a lot of attention. With the aid of the laser radiation one can manipulate the center-of-mass motion of ultracold atoms and one can observe different wave phenomena of ultracold atomic beams. Using various configurations of optical fields one can create diffraction gratings [1], beam splitters [2], lenses [3], waveguides [4], mirrors [5] for ultracold atomic beams, and one can collimate them [6].

After the experimental realization of Bose-Einstein condensation [7], which allows one to create rather dense atomic systems, the problem of interaction of photons with ultracold atoms has reached a new stage of development. The condensate behavior even in the absence of any external potentials is described by nonlinear equations, where the nonlinearities are caused by atomic collisions. Acting with the laser on a dense atomic sample one can induce nonlinearities of another kind in the behavior of the matter field caused by dynamical dipole-dipole interactions. This provides a possibility to create atomic solitons of different kinds [8,9], to change dramatically the effective scattering length of the condensate [10,11], to create nonlinear beam splitters [12], vortices [13], photonic band gaps, and defect states in a condensate [14].

In recent years, different approaches to the description of interaction of ultracold atoms in the field of optical radiation have been suggested and different aspects of the phenomenon have been considered. The properties of the laser radiation modified by atomic dipole-dipole interactions were investigated [15–17] and it was shown that they can be described by the refractive index, which is governed in the linear case—when the light intensity is low enough—by the Clausius-Mossotti relation known from classical optics if quantum correlations are neglected. If the quantum statistical correlations are taken into account, the formula for the refractive index contains additional terms defined by a position-dependent correlation function [15–17]. Nonlinear optical properties of a noninteracting Bose gas were studied as well [18].

The modification of the properties of the laser radiation should have a back influence on the behavior of an ultracold atomic ensemble (for example, on the motion of atomic beam). An attempt to consider this back influence was undertaken by several authors [8,12,16,19–21]. In the first works on the subject, the two-body interactions were modeled by the phenomenological contact potential [19]. Later on dynamical dipole-dipole interactions were taken into account within the framework of the nonrelativistic electrodynamics [8,12,16,20–24]. However, in papers by Zhang and Walls [12] and Lenz *et al.* [8] the averaged polarization of an ultracold atomic ensemble was computed as a function of the incident laser field, whereas it should be a function of the macroscopic or the local field, which are different from the external laser field due to dynamical dipole-dipole interactions. Wallis [21] used the correct form of the equations for the electromagnetic field, although his result for the matter-field equation seems to be inconsistent with the equation for the electromagnetic field [22]. This problem was considered also by Castin and Mølmer [20] and Ruostekoski and Javanainen [16]. But the equations used in those papers are very complicated because these are written in terms of the local-field and dipole-dipole interactions are presented explicitly in the form of the sum over dipole fields. This makes the analysis very difficult and an analytical study is practically impossible to perform.

In Refs. [22–24] a self-consistent quantum theory of atom optical processes has been developed. Making use of the Lorentz-Lorenz relation, which allows one to simplify the analysis, we obtained the general system of Maxwell-Schrödinger equations for atomic creation and annihilation operators and the propagation equation for the laser field, which can be used for the description of linear and nonlinear phenomena in atom optics of single-species and multispecies condensates at high densities of the atomic system. However, the treatment in [22–24] was restricted to low light intensities.

*Permanent address: Ulyanovsk Branch of Moscow Institute of Radio Engineering and Electronics of Russian Academy of Sciences, P.B. 9868, 48, Goncharov Str., Ulyanovsk 432011, Russia. Email address: kostya@spock.physik.uni-konstanz.de, ufire@mv.ru
†Email address: Peter.Marzlin@uni-konstanz.de
‡Email address: Juergen.Audretsch@uni-konstanz.de

In the present paper we shall continue the investigations started in [22,23]. Having in mind mainly atom optical applications, we shall derive the system of Maxwell-Schrödinger equations for the case of high density of the atoms and high light intensity. The nonlinear optical properties of the interacting Bose gas will also be discussed. The paper is organized in the following manner. In Sec. II we discuss basic equations, which govern the time evolution of the matter-field operators under the influence of the photons. In Sec. III a general procedure for the elimination of the excited state is developed and a compact analytical expression for the excited-state matter-field operator is derived. In order to accomplish the elimination procedure for the interacting gas, it is helpful to employ the Lorentz-Lorenz relation, which is briefly discussed in Sec. IV. In Sec. V we discuss nonlinear matter equations for the cases of low and high light intensity. Section VI is devoted to the investigation of linear and nonlinear optical properties of the interacting Bose gas. In Sec. VII we study the diffraction of an ultracold dense atomic beam from an intense standing light wave in the Raman-Nath and Bragg regimes. The conclusions are summarized in Sec. VIII.

II. HEISENBERG EQUATIONS OF MOTION FOR THE ATOMIC OPERATORS

We consider a system of bosonic ultracold two-level atoms with mass M, transition frequency ω_a, and transition dipole moment $\mathbf{d}$. We shall describe such a system in terms of matter-field operators. Let $|g\rangle$ and $|e\rangle$ be the vectors of the ground and excited states of the quantized atomic fields. Then the corresponding annihilation operators of the atoms in these internal states at the position $\mathbf{r}$ are $\hat{\phi}_g(\mathbf{r},t)$ and $\hat{\phi}_e(\mathbf{r},t)$. The matter-field operators are assumed to satisfy the bosonic equal time commutation relations.

The Heisenberg equations of motion for the atomic operators are easily derived from the Hamiltonian of the second-quantized atomic field interacting with the photons. We assume the incident laser field $\mathbf{E}_{in}(\mathbf{r},t)$ to be a monochromatic wave with the frequency $\omega_L = ck_L$, which is close to the frequency ω_a of the electric-dipole transition. In the reference frame rotating with the frequency ω_L and in making use of the electric-dipole approximation and the rotating-wave approximation, we obtain the following dynamical equations for the matter-field operators [22,23]:

$$i\hbar \frac{\partial \hat{\phi}_g}{\partial t} = \hat{H}_{cm}\hat{\phi}_g + \hat{H}_{ge}\hat{\phi}_e, \tag{1}$$

$$i\hbar \frac{\partial \hat{\phi}_e}{\partial t} = \hat{H}_{cm}\hat{\phi}_e - \hbar(\Delta + i\Gamma)\hat{\phi}_e + \hat{H}_{eg}\hat{\phi}_g, \tag{2}$$

where $\hat{H}_{cm} = -\hbar^2\nabla^2/(2M)$, $\Delta = \omega_L - \omega_a - \delta$ is the detuning of the frequency of the laser wave from the frequency of the atomic transition, δ and Γ are the Lamb shift and one-half of the spontaneous emission rate of a single atom in free space, respectively. Here we have introduced the operators $\hat{H}_{eg} = -\mathbf{d}\cdot\hat{\mathbf{E}}^{+}_{loc}$, and $\hat{H}_{ge} = -\mathbf{d}\cdot\hat{\mathbf{E}}^{-}_{loc}$ which are responsible for the transitions $|g\rangle \to |e\rangle$ and $|e\rangle \to |g\rangle$, respectively. $\hat{H}_{eg}$ and $\hat{H}_{ge}$ are related to the operator of the local electric field $\hat{\mathbf{E}}^{\pm}_{loc}(\mathbf{r},t)$. The positive-frequency part of this operator has the form

$$\begin{aligned}\hat{\mathbf{E}}^{+}_{loc}(\mathbf{r},t) = {} & \mathbf{E}^{+}_{in}(\mathbf{r}) + i\sum_{\mathbf{k}\lambda}\sqrt{\frac{2\pi\hbar\omega_k}{V}}\mathbf{e}_\lambda \hat{c}_{\mathbf{k}\lambda}(0) \\ & \times \exp[i\mathbf{k}\cdot\mathbf{r} - i(\omega_k - \omega_L)t] \\ & + \int d\mathbf{r}'\,\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times \frac{\hat{\mathbf{P}}^{+}(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|} e^{ik_L|\mathbf{r}-\mathbf{r}'|},\end{aligned} \tag{3}$$

where $\boldsymbol{\nabla}\times$ refers to the point $\mathbf{r}$ and the operator $\hat{c}_{\mathbf{k}\lambda}(0)$ corresponds to the free-space photon field. The polarization operator is $\hat{\mathbf{P}}^{+} = \mathbf{d}\hat{\phi}_g^{\dagger}\hat{\phi}_e$. Note that in Eq. (3) a small volume around the observation point $\mathbf{r}$ is excluded from the integration. The last term in Eq. (3) describes dipole-dipole interaction. Due to the structure of the local field given by Eq. (3) one can distinguish two kinds of the photons: primary laser photons and secondary photons, reradiated by the atoms.

Usually in atom optics of a Bose-Einstein condensate (BEC) one deals with equations for the ground-state matter-field operator $\hat{\phi}_g$. Therefore, one has to eliminate the excited-state matter-field operator $\hat{\phi}_e$ from the system of Eqs. (1) and (2). In the following section we shall develop a general procedure for the elimination of the excited state, which is in contrast to previous work (see, for instance, [12,22–24]) valid for relatively high atomic densities and relatively high laser radiation intensities.

We have to point out that the system of Eqs. (1) and (2) does not take into account ground state collisions of the atoms, which play an important role in a condensate. The corresponding terms can be included into the equation for the ground-state operator $\hat{\phi}_g$ after the elimination of the excited state in the same manner as it has been done in Ref. [10].

III. ELIMINATION OF THE EXCITED STATE

Let us assume that initially there are no atoms in the excited state. Then we can rewrite Eq. (2) in the form

$$\hat{\phi}_e(t) = -i\int_0^t e^{i[(\hat{H}_{cm}/\hbar) - \tilde{\Delta}]t'}\hat{H}_{eg}(t')e^{-i(\hat{H}_{cm}/\hbar)t'}\hat{\phi}_g(t')dt', \tag{4}$$

where $\tilde{\Delta} = \Delta + i\Gamma$. Using the identity

$$\int e^{ax}F(x)dx = \frac{e^{ax}}{a}\sum_{k=0}^{\infty}\frac{(-1)^k}{a^k}\frac{\partial^k}{\partial x^k}F(x), \tag{5}$$

we get

$$\begin{aligned}\hat{\phi}_e(\mathbf{r},t) = {} & \frac{e^{-i(\hat{H}_{cm}/\hbar)t}}{\hbar\tilde{\Delta}}\sum_{k=0}^{\infty}\frac{(-i)^k}{\tilde{\Delta}^k}\frac{\partial^k}{\partial t^k} \\ & \times [e^{i(\hat{H}_{cm}/\hbar)t}\hat{H}_{eg}(\mathbf{r},t)\hat{\phi}_g(\mathbf{r},t)].\end{aligned} \tag{6}$$

Since we are dealing with ultracold atoms, one can assume that their center-of-mass motion does not have any influence on the electromagnetic field propagation. If in addition the laser frequency in the medium remains unaltered, we may neglect the time dependence of $\hat{H}_{eg}$. In the case of a slow atomic motion we can neglect the center-of-mass motion $\hat{H}_{cm}$ in Eq. (6). Then we have

$$\hat{\phi}_e(\mathbf{r},t)=\frac{\hat{H}_{eg}(\mathbf{r})}{\hbar\tilde{\Delta}}\sum_{k=0}^{\infty}\frac{(-i)^k}{\tilde{\Delta}^k}\frac{\partial^k}{\partial t^k}\hat{\phi}_g(\mathbf{r},t). \tag{7}$$

From the system of Eqs. (1) and (2), with the center-of-mass motion neglected, one can derive the relation

$$\frac{\partial^n}{\partial t^n}\hat{\phi}_g(\mathbf{r},t)=-i^n\frac{\hat{H}_{ge}(\mathbf{r})\hat{H}_{eg}(\mathbf{r})}{\hbar^2\tilde{\Delta}^{2-n}}\times\sum_{k=n-1}^{\infty}\frac{(-i)^k}{\tilde{\Delta}^k}\frac{\partial^k}{\partial t^k}\hat{\phi}_g(\mathbf{r},t),\qquad n=1,2,\dots. \tag{8}$$

Substituting this relation iteratively into Eq. (7), we obtain

$$\hat{\phi}_e(\mathbf{r},t)=\frac{\hat{H}_{eg}(\mathbf{r})}{\hbar\tilde{\Delta}}\sum_{m=0}^{\infty}(-1)^m a_m\left[\frac{\hat{H}_{ge}(\mathbf{r})\hat{H}_{eg}(\mathbf{r})}{\hbar^2\tilde{\Delta}^2}\right]^m\hat{\phi}_g(\mathbf{r},t), \tag{9}$$

$$a_0=1,$$

$$a_{m+1}=\sum_{k_1=0}^{1}\sum_{k_2=0}^{k_1+1}\sum_{k_3=0}^{k_2+1}\cdots\sum_{k_m=0}^{k_{m-1}+1}1=2\frac{(2m+1)!}{m!(m+2)!},$$

$$m=0,1,\dots\ . \tag{10}$$

The zeroth-order term ($m=0$) in Eq. (9) corresponds to the transition $|g\rangle\rightarrow|e\rangle$. The next term ($m=1$) corresponds to the transition $|g\rangle\rightarrow|e\rangle\rightarrow|g\rangle\rightarrow|e\rangle$ and so on. This amounts to the higher-order adiabatic elimination. Thereby different terms in Eq. (9) describe multiple transitions of the atoms between ground and excited states, caused by the influence of the photons. In a typical atom optical situation when the electromagnetic field has a form of a standing wave, these terms correspond to the processes with the momentum transfer from the laser beam to the atoms $\pm 2\hbar\mathbf{K}(m+1)$, where $\mathbf{K}$ is a wave vector of the laser wave in a medium.

We assume that $\hat{\epsilon}(\mathbf{r})=\hat{H}_{ge}(\mathbf{r})\hat{H}_{eg}(\mathbf{r})/(\hbar^2\tilde{\Delta}^2)$ acts only on states $|\psi\rangle$ for which the series in Eq. (9) converges, i.e., $|\psi\rangle$ can be decomposed into eigenstates of $\hat{\epsilon}$ whose eigenvalues satisfy the condition $|\epsilon_n|<1/4$. Then the result of the summation is given by [25]

$$\hat{\phi}_e(\mathbf{r},t)=\frac{\hat{H}_{eg}(\mathbf{r})}{\hbar\tilde{\Delta}}\frac{\sqrt{1+4\hat{\epsilon}(\mathbf{r})}-1}{2\hat{\epsilon}(\mathbf{r})}\hat{\phi}_g(\mathbf{r},t), \tag{11}$$

and the equation for the ground state (1) takes the form

$$i\hbar\frac{\partial}{\partial t}\hat{\phi}_g(\mathbf{r},t)=\left[\hat{H}_{cm}+\hat{G}(\hat{\phi}_g(\mathbf{r},t))+\frac{\hbar\tilde{\Delta}}{2}[\sqrt{1+4\hat{\epsilon}(\mathbf{r})}-1]\right]\times\hat{\phi}_g(\mathbf{r},t), \tag{12}$$

where $\hat{G}(\hat{\phi}_g)$ is an operator which takes into account ground-state collisions. The general form of this operator is not known and one has the corresponding explicit expressions only in the low-density approximations.

Although we do not see $\hat{\phi}_e$ explicitly on the right-hand-side of Eqs. (11) and (12), it is still there, because $\hat{\mathbf{E}}^{\pm}_{loc}$ depends on $\hat{\phi}_e$ and this dependence, which is given by Eq. (3), is very complicated. Formally we could iteratively substitute Eqs. (3) and (11) into Eq. (12). However, by performing such a procedure we would get an equation for the ground-state operator in a form that would be impossible to use. Therefore, the algorithm described in this section does really eliminate the excited state only in the case when the dipole-dipole interactions are small, i.e., when $\hat{\mathbf{E}}^{\pm}_{loc}\approx\mathbf{E}^{\pm}_{in}$. In this case Eqs. (11) and (12) coincide with Eqs. (12) and (14) in Ref. [26], where they have been obtained in a different manner. In order to eliminate the excited state in the case when the dipole-dipole interactions play an important role, it is useful to employ in addition the Lorentz-Lorenz relation, which will be briefly discussed in the following section.

IV. LOCAL-FIELD CORRECTION

As was mentioned in the preceding section, the solution of Eqs. (3), (11), and (12) is a rather complicated mathematical problem because these equations contain explicitly the dipole-dipole interactions. In many particular situations such a detailed microscopic description of matter is not necessary and it is more convenient to consider optical properties of the medium on a macroscopic level. This can be done by introducing the macroscopic field $\hat{\mathbf{E}}_{mac}(\mathbf{r},t)$, instead of the local field $\hat{\mathbf{E}}_{loc}(\mathbf{r},t)$ in the equations for the matter fields.

As in Ref. [27] we can introduce the macroscopic field by imposing the requirement that it is a solution of the macroscopic Maxwell equations for a charge-free and current-free polarization medium. Taking into account that in most of the practical situations the electromagnetic processes are much faster than the center-of-mass motion of the atoms, we can neglect the time dependence of the local field $\hat{\mathbf{E}}^{\pm}_{loc}$ and the polarization $\hat{\mathbf{P}}^{\pm}$. Then the system of Maxwell equations can be written in the form of the wave equation

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r})=k_L^2[\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r})+4\pi\hat{\mathbf{P}}^{\pm}(\mathbf{r})]. \tag{13}$$

Using Eq. (13) and the definition of the local field (3), we get the following relation:

$$\hat{\mathbf{E}}^{\pm}_{loc}(\mathbf{r})=\hat{\mathbf{E}}^{\pm}_{mac}(\mathbf{r})+\frac{4\pi}{3}\hat{\mathbf{P}}^{\pm}(\mathbf{r}). \tag{14}$$

This equation is often called in the literature the Lorentz-Lorenz relation. It constitutes the basis of the local-field effects in classical [28], quantum [29] and nonlinear optics (see [30–32] and references therein).

V. NONLINEAR MATTER EQUATION

As mentioned above, the elimination of the excited state is, in general, a complicated mathematical problem. Let us consider some important special cases, when one can obtain rather simple analytical solutions.

A. Low light intensity

We consider first the special case of a low light intensity. In this case we can keep only the first term $(m=0)$ in Eq. (9), which is linear with respect to the electromagnetic field strength:

$$\hat{\phi}_e(\mathbf{r},t) = -\frac{\mathbf{d}\cdot\hat{\mathbf{E}}^+_{loc}(\mathbf{r})}{\hbar\tilde{\Delta}}\hat{\phi}_g(\mathbf{r},t). \tag{15}$$

If we substitute Eq. (14) into Eq. (15) and take into account Eq. (15), we obtain

$$\hat{\phi}_e(\mathbf{r},t) = -\left[\frac{\hat{\Omega}^+(\mathbf{r})}{2\tilde{\Delta}} + \frac{4\pi}{3}\alpha|\hat{\phi}_g(\mathbf{r},t)|^2\frac{\mathbf{d}\cdot\hat{\mathbf{E}}^+_{loc}(\mathbf{r})}{\hbar\tilde{\Delta}}\right]\hat{\phi}_g(\mathbf{r},t), \tag{16}$$

where the position-dependent Rabi frequency $\hat{\Omega}^+(\mathbf{r}) = 2\mathbf{d}\cdot\hat{\mathbf{E}}^+_{mac}(\mathbf{r})/\hbar$ is related to the macroscopic electric field, $\alpha = -d^2/(\hbar\tilde{\Delta})$ is the atomic polarizability, and $|\hat{\phi}_g|^2 = \hat{\phi}_g^\dagger\hat{\phi}_g$. Repeating the same procedure infinite number of times we come to the result

$$\hat{\phi}_e(\mathbf{r},t) = -\frac{\hat{\Omega}^+(\mathbf{r})}{2\tilde{\Delta}}\sum_{n=0}^{\infty}\left(\frac{4\pi}{3}\alpha|\hat{\phi}_g(\mathbf{r},t)|^2\right)^n\hat{\phi}_g(\mathbf{r},t). \tag{17}$$

The series in Eq. (17) converges, provided that the eigenvalues of the operator $4\pi/3|\alpha||\hat{\phi}_g|^2$ are less than 1. In this case we get

$$\hat{\phi}_e(\mathbf{r},t) = -\frac{\hat{\Omega}^+(\mathbf{r})}{2\tilde{\Delta}\left[1-\frac{4\pi}{3}\alpha|\hat{\phi}_g(\mathbf{r},t)|^2\right]}\hat{\phi}_g(\mathbf{r},t). \tag{18}$$

This adiabatic solution has been obtained in Ref. [22] using a slightly different technique. Note that a singularity occurs in Eq. (18) under the condition $4\pi/3|\alpha||\phi_g|^2=1$. However, as it follows from our derivation, we never encounter this singularity, because in the region $4\pi/3|\alpha||\phi_g|^2 \geqslant 1$ Eq. (18) is not valid. This was not clear from the derivation given in Ref. [22].

Then substituting Eq. (18) in Eq. (1), we obtain as a result an equation for the ground-state matter field $\hat{\phi}_g$,

$$i\hbar\frac{\partial\hat{\phi}_g(\mathbf{r},t)}{\partial t} = \left[\hat{H}_{cm} + \hat{G}(\hat{\phi}_g(\mathbf{r},t)) + \frac{\hbar|\hat{\Omega}^+(\mathbf{r})|^2}{4\tilde{\Delta}\left|1-\frac{4\pi}{3}\alpha|\hat{\phi}_g(\mathbf{r},t)|^2\right|^2}\right]\hat{\phi}_g(\mathbf{r},t), \tag{19}$$

where $|\hat{\Omega}^+|^2 = \hat{\Omega}^-\hat{\Omega}^+$.

B. High light intensity

If the light intensity is high, we have to take into account higher-order terms in the expansion (9). In the case of a dilute gas, i.e., when $4\pi/3|\alpha||\phi_g|^2 \ll 1$ it is enough to keep only linear terms with respect to the atomic density. Then we obtain

$$\hat{\phi}_e(\mathbf{r},t) = -\frac{\hat{\Omega}^+(\mathbf{r})}{2\tilde{\Delta}}\left[1+\frac{4\pi}{3}\frac{|\hat{\phi}_g(\mathbf{r},t)|^2}{\sqrt{1+4\hat{\varepsilon}_m}}\times\left(\alpha-\frac{\alpha^*}{4\hat{\varepsilon}_m^\dagger}\left|\sqrt{1+4\hat{\varepsilon}_m^\dagger}-1\right|^2\right)\right]\hat{\phi}_g(\mathbf{r},t), \tag{20}$$

where $\hat{\varepsilon}_m = |\hat{\Omega}^+|^2/(4\tilde{\Delta}^2)$ is a bounded operator with eigenvalues ε_m that obey the condition $|\varepsilon_m|<1/4$. The operator $\hat{G}$ of Eq. (12) in this case is given by $\hat{G} = 4\pi\hbar^2 a/M|\hat{\phi}_g(\mathbf{r},t)|^2$, where a is a scattering length of the condensate, and we get the following equation for the ground-state matter-field operator:

$$i\hbar\frac{\partial\hat{\phi}_g(\mathbf{r},t)}{\partial t} = \left[\hat{H}_{cm} + \frac{\hbar|\hat{\Omega}^+(\mathbf{r})|^2}{4\tilde{\Delta}}\frac{\sqrt{1+4\hat{\varepsilon}_m(\mathbf{r})}-1}{2\hat{\varepsilon}_m(\mathbf{r})} + \frac{4\pi\hbar^2\hat{a}_{eff}(\mathbf{r})}{M}|\hat{\phi}_g(\mathbf{r},t)|^2\right]\hat{\phi}_g(\mathbf{r},t), \tag{21}$$

where an effective scattering length

$$\hat{a}_{eff}(\mathbf{r}) = a + \frac{\hbar|\hat{\Omega}^+(\mathbf{r})|^2}{4\tilde{\Delta}\sqrt{1+4\hat{\varepsilon}_m}}\left(\alpha\frac{\sqrt{1+4\hat{\varepsilon}_m}-1}{2\hat{\varepsilon}_m} + \mathrm{H.c.}\right)\frac{M}{3\hbar^2} \tag{22}$$

contains a light-induced contribution, which vanishes if the light intensity tends to zero.

The dependence of the effective scattering length a_{eff} on the detuning Δ is depicted in Fig. 1. If the detuning Δ is comparable to Γ, one can obtain rather big negative corrections to the scattering length a. However, inelastic processes caused by the spontaneous emission and described by the imaginary part of a_{eff} give also a significant contribution in

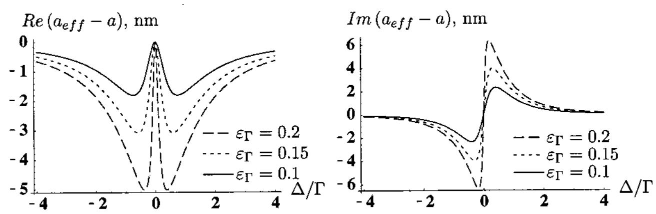

FIG. 1. Light-induced scattering length modification at different values of $\varepsilon_\Gamma = |\mathbf{d}\cdot\mathbf{E}^+_{mac}|^2/(\hbar\Gamma)^2$. The parameters are $d = 10^{-18}$ esu, $M = 10^{-23}$ g. The imaginary part of a_{eff} is determined by the spontaneous emission.

this region. In order to reduce spontaneous emission the detuning Δ must be of the order of at least few Γ. This results in a modification of the scattering length of the order of -1 nm, which is comparable to the typical values of the scattering length a for alkali atoms.

In contrast to the photoassociation mechanism of the scattering length modification [10,11], our mechanism does not provide big corrections and does not allow one to reverse the sign of the scattering length for the condensates with attractive interactions. Nevertheless, as it will be shown in Sec. VII it has to be taken into account in certain atom optical processes.

C. Decomposition up to the order $1/\tilde{\Delta}^3$

In the limit $\Delta \gg \Gamma$, which is the most interesting one for atom optical applications, strong-field effects do not give a big contribution. In addition, one has to take into account that the densities of the condensates currently achievable in the experiments are not higher than 5×10^{15} cm^{-3}. Therefore, in order to provide a satisfactory description of atom optical processes it is enough to keep the terms up to the order $1/\tilde{\Delta}^3$ in the expansion (9). Employing the Lorentz-Lorenz relation (14) we get the following expression for the excited-state operator:

$$\hat{\phi}_e = -\frac{\hat{\Omega}^+}{2\tilde{\Delta}}\left[1+\frac{4\pi}{3}\alpha|\hat{\phi}_g|^2+\left(\frac{4\pi}{3}\alpha|\hat{\phi}_g|^2\right)^2-\frac{|\hat{\Omega}^+|^2}{4\tilde{\Delta}^2}\right]\hat{\phi}_g. \tag{23}$$

The operator $\hat{G}$ in this case is given by

$$\hat{G}(\hat{\psi}_g) = \frac{4\pi\hbar^2 a}{M}\left[1+\frac{32}{3}\sqrt{\frac{a^3}{\pi}}|\hat{\phi}_g|\right]|\hat{\phi}_g|^2, \tag{24}$$

where the second term takes into account the effects of quantum fluctuations [33]. Then the equation for the ground-state operator takes the form

$$\begin{aligned} i\hbar\frac{\partial\hat{\phi}_g}{\partial t} &= \hat{H}_{cm}\hat{\phi}_g + \frac{4\pi\hbar^2 a}{M}\left[1+\frac{32}{3}\sqrt{\frac{a^3}{\pi}}|\hat{\phi}_g|\right]|\hat{\phi}_g|^2\hat{\phi}_g \\ &\quad + \frac{\hbar|\hat{\Omega}^+|^2}{4\tilde{\Delta}}\Bigg[1+\frac{4\pi}{3}(\alpha+\alpha^*)|\hat{\phi}_g|^2 \\ &\quad + \left(\frac{4\pi}{3}|\hat{\phi}_g|^2\right)^2[\alpha^2+|\alpha|^2+(\alpha^*)^2]-\frac{|\hat{\Omega}^+|^2}{4\tilde{\Delta}^2}\Bigg]\hat{\phi}_g. \end{aligned} \tag{25}$$

Neglecting the summands, which contain $|\hat{\phi}_g|^4$, $|\hat{\Omega}^+|^4$, and the scattering length a, we get the same equation as in Ref. [12].

Let us compare the light-induced and collisional nonlinear terms in Eq. (25) in the limit $\Delta \gg \Gamma$. The leading nonlinear terms in Eq. (25) are proportional to the density $|\hat{\phi}_g|^2$. Their relative contributions are defined by the quantities

$$U_E = -\frac{|\Omega^+|^2}{\Delta^2}\frac{2\pi}{3}d^2, \qquad U_C = \frac{4\pi\hbar^2 a}{M},$$

respectively. The typical orders of magnitude of the parameters for alkali-metal atoms are (in CGS system of units) $d \sim 10^{-18}$ esu, $|a| \sim 10^{-7}$ cm, $M \sim 10^{-23}$ g, and we get the estimates

$$|U_E| \sim \frac{|\Omega^+|^2}{\Delta^2}10^{-36}\ \text{erg cm}^{-3}, \qquad |U_C| \sim 10^{-37}\ \text{erg cm}^{-3}.$$

Therefore, we see that at $|\Omega^+|^2/\Delta^2 \sim 0.1$ U_E and U_C are of the same order of magnitude.

The higher-order light-induced and collisional nonlinear terms in Eq. (25) have different dependence on the density. Their relative contributions are defined by

$$V_E = \frac{3\hbar|\Omega^+|^2}{4\Delta}\left(\frac{4\pi}{3}\frac{d^2}{\hbar\Delta}\right)^2|\phi_g|^2,$$

$$V_C = \frac{128\pi\hbar^2 a}{3M}\sqrt{\frac{a^3}{\pi}}|\phi_g|,$$

respectively. For the densities $|\phi_g|^2 \sim 10^{14}$ cm^{-3} and detunings $|\Delta| \sim 10^8$ Hz, we have

$$|V_E| \sim \frac{|\Omega^+|^2}{\Delta^2}10^{-38}\ \text{erg cm}^{-3}, \qquad |V_C| \sim 10^{-39}\ \text{erg cm}^{-3}.$$

Thus, we see that the light-induced and collisional nonlinear terms in Eq. (25) have about the same order of magnitude.

VI. OPTICAL PROPERTIES OF THE ULTRACOLD GAS

We substitute Eq. (11) into the definition of the polarization field. This gives us a general nonlinear relation between the polarization and the local field,

$$\hat{\mathbf{P}}^{+}=\alpha\frac{\sqrt{1+4\hat{\epsilon}}-1}{2\hat{\epsilon}}|\hat{\phi}_g|^2\hat{\mathbf{E}}^{+}_{loc}. \tag{26}$$

The optical properties of a gas are determined by the refractive index. In order to work out the refractive index one has to relate the polarization to the macroscopic electric field, which can be done by means of the Lorentz-Lorenz formula (14). This relation has the form

$$\hat{\mathbf{P}}^{+}=\hat{\chi}\hat{\mathbf{E}}^{+}_{mac}, \tag{27}$$

where the dielectric susceptibility depends, in general, on the atomic density $|\hat{\phi}_g|^2$ and the macroscopic electric-field strength $\hat{\mathbf{E}}_{mac}$. From Eq. (13) we get the following equation for the macroscopic electric field:

$$\nabla\times\nabla\times\hat{\mathbf{E}}^{+}_{mac}-k_L^2\hat{n}^2\hat{\mathbf{E}}^{+}_{mac}=0, \tag{28}$$

with the refractive index $\hat{n}$ given by $\hat{n}=(1+4\pi\hat{\chi})^{1/2}$. We would like to stress that the refractive index is a rather important parameter. On the one hand, it describes the propagation of the laser radiation in a medium. On the other hand, the refractive index of the gas plays an important role in atom optical processes, because as one can see from Eq. (28) it modifies the laser wave vector and, therefore, the momentum transfer from the laser to the atoms.

A. Low light intensity

In the special case of the low light intensity we have obtained the adiabatic solution (18). Then the expression for the refractive index takes the form

$$\hat{n}^2=\frac{1+\dfrac{8\pi}{3}\alpha|\hat{\phi}_g|^2}{1-\dfrac{4\pi}{3}\alpha|\hat{\phi}_g|^2}. \tag{29}$$

As it follows from our physical interpretation of the expansion (9), the Clausius-Mossotti formula (29) corresponds to the quantum transition of the type $|g\rangle\to|e\rangle$. Therefore, it takes into account pair interactions between the atoms when one atom emits a photon, which is absorbed by another atom and so on.

B. High light intensity

In the case of a high light intensity and low atomic density the expression for the excited-state operator $\hat{\phi}_e$ is given by Eq. (20), which leads to the following result for the refractive index:

$$\hat{n}=1+2\pi\alpha|\hat{\phi}_g|^2\frac{\sqrt{1+4\hat{\varepsilon}_m}-1}{2\hat{\varepsilon}_m}. \tag{30}$$

The dependence of the refractive index n on the detuning Δ is shown in Fig. 2. The refractive index strongly depends on the laser intensity when the laser frequency ω_L is close to the atomic transition frequency ω_a. On the other hand, strong modifications of the real part of the refractive index are inevitably accompanied by the increase of absorption, which is determined by the imaginary part of the refractive index.

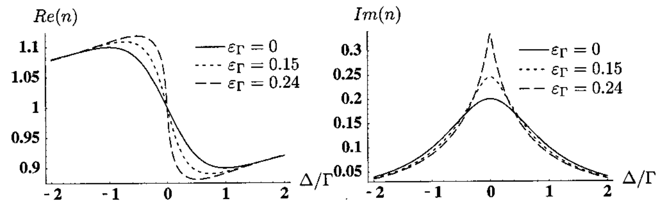

FIG. 2. Refractive index at different values of $\varepsilon_\Gamma=|\mathbf{d}\cdot\mathbf{E}^{+}_{mac}|^2/(\hbar\Gamma)^2$. The parameter $2\pi d^2\rho_g/(\hbar\Gamma)=0.2$.

C. Decomposition up to the order $1/\tilde{\Delta}^3$

If we keep the terms up to the order $1/\tilde{\Delta}^3$, we have to use Eq. (23) for the excited state operator. Then the refractive index takes the form

$$\hat{n}=1+2\pi\alpha|\hat{\phi}_g|^2\left[1+\frac{\pi}{3}\alpha|\hat{\phi}_g|^2+\frac{5}{8}\left(\frac{4\pi}{3}\alpha|\hat{\phi}_g|^2\right)^2-\frac{|\mathbf{d}\cdot\hat{\mathbf{E}}^{+}_{mac}|^2}{\hbar^2\tilde{\Delta}^2}\right], \tag{31}$$

which corresponds to the Kerr-type optical nonlinearity.

Equations (25), (28), and (31) can be considered as an atom optical analog of the system of Maxwell-Schrödinger equations used in quantum and nonlinear optics. In general, they have to be solved in a self-consistent way and in most of the situations solutions can be obtained only by doing numerical calculations. In the following section we shall consider some particular examples.

VII. DIFFRACTION OF AN ULTRACOLD ATOMIC BEAM FROM AN INTENSE STANDING LIGHT WAVE

In this section we will be dealing with the system of Eqs. (25), (28), and (31). Assuming that $\Delta\gg\Gamma$, we can neglect spontaneous emission rate Γ. Alternatively, we can consider short interaction time τ, such that $W\Gamma\tau<1$, where W is a population of the excited atomic state [1]. This condition allows one also to neglect Γ. In addition we shall restrict ourselves by the situations when the number of photons and atoms is high and they are in coherent states, i.e., we will consider atomic BEC interacting with a laser field. This allows us to replace all the operators by macroscopic functions.

We consider a typical scheme for the observation of diffraction in atom optics: An incident atomic beam moves in the plane (y,z) and crosses under a certain angle two laser waves counterpropagating along the y axis with wave vectors $+\mathbf{k}_L$ and $-\mathbf{k}_L$, respectively, and with Gaussian envelope. From the uncertainty relation it follows that in order to get a distinct diffraction pattern, the width of the atomic wave packet w_y should be sufficiently large compared to the wavelengh of the laser radiation in a medium. In this case the atoms can be described as a homogeneous medium with constant refractive index. If the spontaneous emission does not make any contribution, the effect of the atoms on the laser beam is purely dispersive and only the wavelength will be shifted. This means that in a medium we shall have a standing wave that is formed by counter propagating laser beams with the wave vectors $+n\mathbf{k}_L$ and $-n\mathbf{k}_L$, respectively. In this approximation the solution of Eq. (28) is given by

$$|\Omega^+|^2=|\Omega_0|^2\exp(-z^2/w_L^2)\cos^2 nk_Ly. \qquad (32)$$

We assume that the longitudinal kinetic energy of the atomic beam, associated with the center-of-mass motion in z direction, is large compared to the nonlinear potential in Eq. (25). Then the z component of the atomic velocity will not change much and, therefore, the motion of atoms in z direction during the whole evolution can be treated classically. Only the motion in y direction should be treated quantum mechanically. In such a situation the coordinate z plays the role of time and we can change the variable $t=z/v_g$ in Eq. (25) with v_g being the group velocity.

A. Raman-Nath regime

We assume that we are in the Raman-Nath regime and we can neglect the transverse kinetic energy during the interaction of the atoms with the electromagnetic field. This approximation is valid for heavy atoms or if the interaction is so strong that it can have a considerable effect on the atoms without leading to large spatial variations during the interaction time [34]. In this case the density of atoms remains unaltered, but their phase changes. Making use of all the assumptions stated above, we can write down the solution of Eq. (25) for $z\gg w_L$ (in the far zone) in the following form:

$$\begin{aligned}\phi_g(y,\infty)=&\phi_g(y,-\infty)\exp(-i\beta)\\&\times\exp\left\{\int_{-\infty}^{\infty}\frac{-i|\Omega^+(y,z)|^2}{4\Delta v_g}\left[1+\frac{8\pi}{3}\alpha\rho_g\right.\right.\\&\left.\left.+3\left(\frac{4\pi}{3}\rho_g\right)^2-\frac{|\Omega^+(y,z)|^2}{4\Delta^2}\right]dz\right\}, \qquad (33)\end{aligned}$$

where $\rho_g=|\phi_g|^2$ is the density of atoms in the ground state and

$$\beta\approx\frac{4\pi\hbar a w_L}{Mv_g}\sqrt{\pi}\left(1+\frac{32}{3}\sqrt{\frac{a^3\rho_g}{\pi}}\right)\rho_g. \qquad (34)$$

We represent ρ_g as a Gaussian wave packet with width w_y,

$$\rho_g=\rho\exp(-y^2/w_y^2). \qquad (35)$$

Then we substitute Eqs. (32) and (35) into Eq. (33) and take into account that the width of the atomic wave packet must be much larger than the wavelength of the laser radiation, i.e., $w_y\gg 2\pi/(nk_L)$. After integration, we get the following result:

$$\phi_g(y,\infty)=\phi_g(y,-\infty)e^{-i(\varphi+\beta)}\sum_{q=-\infty}^{\infty}e^{i2qnk_Ly}(-i)^qC_q, \qquad (36)$$

which is represented here in the form of a Fourier series expansion. We use the notations

$$\begin{aligned}C_q=&J_q(\varphi)+A[J_{q+1}(\varphi)-J_{q-1}(\varphi)]\\&+iA\left[\frac{3}{2}J_q(\varphi)-\frac{J_{q+2}(\varphi)+J_{q-2}(\varphi)}{4}\right], \qquad (37)\end{aligned}$$

$$\varphi=\frac{|\Omega_0|^2w_L\sqrt{\pi}}{8\Delta v_g}\left[1+\frac{8\pi}{3}\alpha\rho+3\left(\frac{4\pi}{3}\alpha\rho\right)^2\right],$$

$$A=\frac{|\Omega_0|^4}{64\sqrt{2}\Delta^3}\frac{w_L}{v_g}\sqrt{\pi}. \qquad (38)$$

J_q is the qth-order Bessel function.

From the solution (36) it follows that the momentum transferred from the laser beam to the atomic beam is determined by the wave number of the incident laser radiation k_L and the refractive index of the gas n. The probability to find the beam in a momentum state $2qnk_L$ is given by

$$P_q=|C_q|^2=J_q^2(\varphi)-2A\frac{\partial}{\partial\varphi}J_q^2(\varphi), \qquad q=0,\pm1,\pm2,\ldots, \qquad (39)$$

with P_0 being the probability to find the atomic beam in the same momentum state as for the incident atomic beam. The angle of diffraction α_q for a particular momentum state q is thereby given by

$$\tan\alpha_q=\frac{2qn\hbar k_L}{mv_g}. \qquad (40)$$

Therefore the diffraction pattern, as it follows from Eqs. (36), (39), and (40), depends on the density of the atomic beam and the intensity of the laser radiation. Since the parameter φ, which is proportional to the intensity of the laser wave, corresponds to $m=0$ in Eq. (9), it describes the processes with the momentum transfer $\pm 2n\hbar\mathbf{k}_L$.

The parameter A, which is proportional to the intensity squared, describes the contribution of the intensity-dependent processes with the momentum transfer $\pm4n\hbar\mathbf{k}_L$ [$m=1$ in Eq. (9)]. For estimations we choose the value of the parameter $|\Omega_0|^2w_L\sqrt{\pi}/(8\Delta v_g)$ within the range 3–5 [1] and assume that $|\Omega_0|^2/\Delta^2=0.4$, which is quite realistic and at the same time consistent with our derivations. Then we get that the contribution of the processes with the momentum transfer $\pm4n\hbar\mathbf{k}_L$ is of the order of 10%. If one assumes a laser beam width $w_L\sim10$ μm, an atomic beam propagating with the group velocity $v_g>10$ m/s interacts with the laser for a time $\tau<1$ μs. Spontaneous emission is then sufficiently supressed.

The phase shift β in Eq. (36), which is produced by the contact interaction of the atoms in the beam, can be rather big compared to π. However, it does not play any important physical role. In the following section we shall consider another situation, when the contact interaction is really important.

B. Bragg regime

In the Raman-Nath approximation only the momentum conservation in the process of atom-photon interaction is taken into account. If we take into account additionally the energy conservation, we come to the Bragg regime.

Bragg diffraction of a dense atomic beam has been considered in Ref. [12], where the authors have solved a matter wave equation similar to Eq. (25), but without the terms proportional to $1/\Delta^3$ and without the contact interaction. Here we shall consider this more general case.

We can look for a solution of Eq. (25) with the Rabi frequency (32) in the form

$$\phi_g(y,z,t)=\sum_{q=-\infty}^{\infty}\phi_q(y,z)e^{i(K_{0y}+2qnk_L)y}e^{i[K_{0z}z-(E/\hbar)t]}. \tag{41}$$

Since the atomic wave packet width w_y is assumed to be large compared to the laser wave length, one can neglect its spatial dispersion and use the slowly varying amplitude approximation. We substitute Eq. (41) into Eq. (25) and neglect the second-order derivatives of ϕ_q with respect to y and z. Then for the first-order Bragg diffraction only the terms with $q=0,1$ in Eq. (41) have to be considered and we obtain the following system of equations:

$$\begin{aligned}
i\left(v_g\frac{\partial\phi_0}{\partial z}-v_R\frac{\partial\phi_0}{\partial y}\right)=&[\omega_R+f_1(z)]\phi_0+f_2(z)\phi_1+\frac{8\pi}{3}\alpha g(z)[2(|\phi_0|^2+2|\phi_1|^2)\phi_0+(2|\phi_0|^2+|\phi_1|^2)\phi_1+\phi_1^*\phi_0\phi_0]\\
&+3\left(\frac{4\pi}{3}\alpha\right)^2 g(z)[2(|\phi_0|^4+6|\phi_0|^2|\phi_1|^2+3|\phi_1|^4)\phi_0+(3|\phi_0|^4+6|\phi_0|^2|\phi_1|^2+|\phi_1|^4)\phi_1\\
&+(2|\phi_0|^2+3|\phi_1|^2)\phi_1^*\phi_0\phi_0]+\frac{4\pi\hbar a}{M}\left[1+\frac{32}{3}\sqrt{\frac{a^3}{\pi}}|\phi_0+\phi_1\exp(2ink_Ly)|\right](|\phi_0|^2+2|\phi_1|^2)\phi_0,
\end{aligned} \tag{42}$$

$$\begin{aligned}
i\left(v_g\frac{\partial\phi_1}{\partial z}+v_R\frac{\partial\phi_1}{\partial y}\right)=&[\omega_R+f_1(z)]\phi_1+f_2(z)\phi_0+\frac{8\pi}{3}\alpha g(z)[2(|\phi_1|^2+2|\phi_0|^2)\phi_1+(2|\phi_1|^2+|\phi_0|^2)\phi_0+\phi_0^*\phi_1\phi_1]\\
&+3\left(\frac{4\pi}{3}\alpha\right)^2 g(z)[2(|\phi_1|^4+6|\phi_0|^2|\phi_1|^2+3|\phi_0|^4)\phi_1+(3|\phi_1|^4+6|\phi_0|^2|\phi_1|^2+|\phi_0|^4)\phi_0\\
&+(2|\phi_1|^2+3|\phi_0|^2)\phi_0^*\phi_1\phi_1]+\frac{4\pi\hbar a}{M}\left[1+\frac{32}{3}\sqrt{\frac{a^3}{\pi}}|\phi_0+\phi_1\exp(2ink_Ly)|\right](|\phi_1|^2+2|\phi_0|^2)\phi_1,
\end{aligned} \tag{43}$$

where $v_g=\hbar K_{0z}/M$ is a group velocity, $\omega_R=\hbar n^2k_L^2/(2M)$ and $v_R=\hbar nk_L/M$ are the photon recoil frequency and photon recoil velocity, respectively,

$$f_1(z)=2g(z)-6\frac{g^2(z)}{\Delta},\qquad f_2(z)=g(z)-4\frac{g^2(z)}{\Delta},$$

$$g(z)=\frac{|\Omega_0|^2}{16\Delta}e^{-z^2/w_L^2}.$$

For the incident atomic beam with a Gaussian density profile and with the wave vector in y direction matching the single-photon recoil momentum, we have the following initial conditions

$$\phi_0(y,-\infty)=\sqrt{\rho}\exp\left(-\frac{y^2}{2w_y^2}\right),\qquad \phi_1(y,-\infty)=0. \tag{44}$$

In the absence of the electromagnetic field $[g(z)=0]$, Eqs. (42) and (43) describe the free propagation of the incident atomic beam without changing its shape. If the electromagnetic field is switched on there is a coupling between the two modes ϕ_0 and ϕ_1. If the atomic density is negligible one can neglect all the nonlinear terms in Eqs. (42) and (43). Neglecting also the propagation of the atomic beam in y direction, we obtain the following solution in the far zone ($z\gg w_L$):

$$\begin{aligned}
|\phi_0(y,\infty)|^2&=|\phi_0(y,-\infty)|^2\cos^2\Theta,\\
|\phi_1(y,\infty)|^2&=|\phi_0(y,-\infty)|^2\sin^2\Theta,
\end{aligned} \tag{45}$$

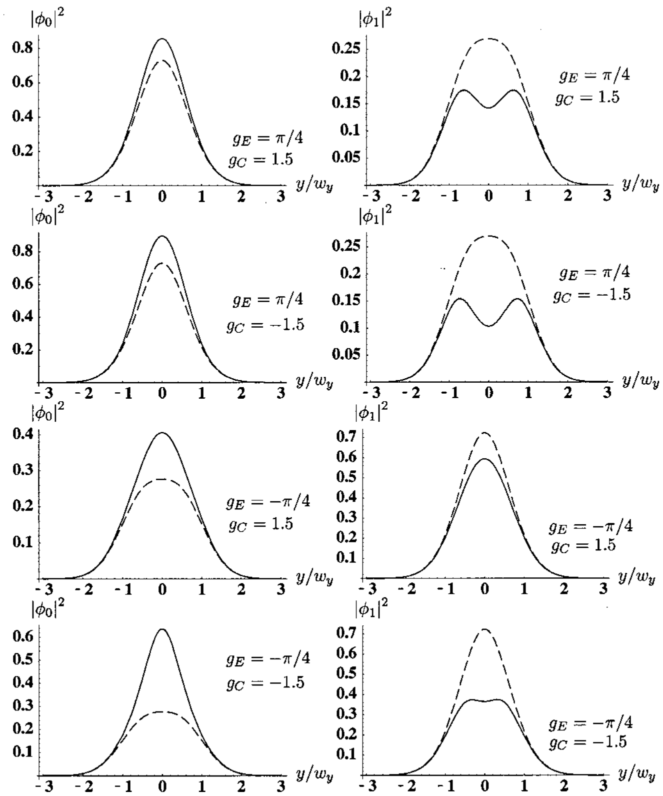

FIG. 3. Diffraction pattern in the wave zone at different values of the parameters $g_E = |\Omega_0|^2 w_L / 16\Delta v_g \sqrt{\pi}$ and $g_C = 4\pi\hbar a w_L / M v_g$, $8\pi/3|\alpha|\rho = 0.3$. Solid lines, all nonlinearities are taken into account; dashed lines, only ϕ^2 light-induced nonlinearities are taken into account. The origin of the reference frame is shifted to the center of the corresponding mode.

where

$$\Theta(y,\infty) = \frac{|\Omega_0|^2}{16\sqrt{2}\Delta} \frac{w_L}{v_g} \sqrt{\pi} \left(1 - \frac{1}{4\sqrt{2}} \frac{|\Omega_0|^2}{\Delta^2} \right). \qquad (46)$$

The second term in the braces describes the contribution of the processes with the momentum transfer $\pm 4n\hbar \mathbf{k}_L$.

If the condensate density is high enough, the coupling between the modes ϕ_0 and ϕ_1 becomes essentially nonlinear. We have numerically solved the system of Eqs. (42) and (43) for that case with the initial conditions (44). The diffraction pattern in the far zone ($z \gg w_L$) is shown in Fig. 3, where the results obtained under the same assumptions as in Ref. [12] are also presented for comparison. The diffraction pattern is mainly determined by the linear terms and the nonlinear terms of the kind ϕ^2 in Eqs. (42) and (43), and one can clearly see an interplay of collisional and light-induced processes. The contribution of the higher-order terms is about few percent.

VIII. CONCLUSION

Starting from the microscopic model and making use of the multipolar formulation of QED and electric-dipole approximation, we have derived the general system of Maxwell-Schrödinger equations for atomic creation and annihilation operators and the propagation equation for the laser field. It describes the modification of the properties of the external off-resonant laser radiation in a medium due to dipole-dipole interactions and the influence of this modification on the center-of-mass motion of the ultracold atoms as a single dynamical process. The system can be used, for instance, for the self-consistent analysis of linear and nonlinear phenomena in the atom optics of Bose condensates at relatively high densities of the atomic system and high intensities of the laser radiation.

A general procedure for the elimination of the excited state has been developed. The annihilation and creation operators of the excited state for large detuning are represented in the form of a series expansion in powers of the inverse detuning, which corresponds to multiple transitions of atoms between the ground and excited electronic states. We have derived compact analytical expressions, which relate annihilation and creation operators of the excited state to the corresponding operators of the ground state.

The optical properties of an interacting ultracold Bose gas are investigated and an equation for the intensity-dependent refractive index is derived. The refractive index is shown to be an important parameter in atom optical processes, because it defines, on the one hand, the propagation of the laser radiation in a medium and, on the other, the momentum transfer from the laser beam to the atomic beam.

As a typical atom optical application we have considered the diffraction of an ultracold atomic beam from an intense standing laser wave in the Raman-Nath and in the Bragg regimes. It has been shown that in the Bragg regime the light-induced interaction as well as the contact interaction can make a significant contribution to the diffraction pattern, while in the Raman-Nath regime the diffraction pattern is determined by electromagnetic processes and the contact interaction gives only an unimportant phase shift.

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