Teleportation improvement by conditional measurements on the two-mode squeezed vacuum

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We show that by making conditional measurements on the Einstein-Podolsky-Rosen (EPR) squeezed vacuum [T. Opatrný, G. Kurizki, and D.-G. Welsch, Phys. Rev. A **61**, 032302 (2000)], one can improve the efficacy of teleportation for both the position-difference, momentum-sum, and number-difference, phase-sum continuous variable teleportation protocols. We investigate the relative abilities of the standard and conditional EPR states, and show that by conditioning we can improve the fidelity of teleportation of coherent states from below to above the $\bar{F}=2/3$ boundary, thereby achieving unambiguously quantum teleportation.

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I. INTRODUCTION

Over recent times, teleportation has shown itself to be a fundamental building block in the business of quantum information processing [1–8]. In continuous variable teleportation the entanglement resource is usually the two-mode squeezed state, or the Einstein-Podolsky-Rosen (EPR) state [20]. The quality of teleportation depends upon how squeezed the EPR state can be made. High levels of squeezing are hard to achieve, so other techniques for improving teleportation need to be considered. Opatrný, Kurizki, and Welsch [9] showed that one can improve standard continuous variable teleportation, by conditioning off detection results from very weakly reflective beam splitters inserted into each arm of the entanglement resource. Making such conditional measurements selects a subensemble of more highly entangled states that can then be used to teleport more effectively. From this point of view it is similar to a distillation protocol. The conditioning procedure also gives information on when one should attempt to teleport the input state, thereby improving the efficiency of teleportation. Opatrný, Kurizki, and Welsch looked at the example of teleportation of *Schrödinger cat states*.

In this paper we concentrate on the more experimentally relevant situation of coherent states. We consider the relative merits of the conditioned and unconditioned EPR states for the original scheme [5] (generalized to include a variable gain and output state amplitude) and show that the conditional entanglement resource improves the efficacy of teleportation. We maximize the average fidelity over the gain to show a teleportation efficiency improvement from using the conditioning procedure. We show that the conditioning procedure can produce unambiguous quantum teleportation of coherent states from an entanglement resource initially unable to do so. The number-difference, phase-sum teleportation protocol of Milburn and Braunstein [10] is also analyzed. The conditioning procedure improves the output fidelity of the teleportation scheme; the conditional states being shown to be simultaneous eigenstates of number difference and phase sum for sufficiently high levels of squeezing.

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II. ENTANGLEMENT RESOURCE IMPROVEMENT

Following Opatrný, Kurizki, and Welsch [9], we introduce a very weakly reflective beam splitter into each EPR beam and look for coincidences occurring from only one photon being reflected. Such coincidences tell us when we have a ‘‘good’’ resource and, therefore, when to teleport, it merely being a matter of time to wait for such an occurrence. The photon-subtracted EPR state is an entanglement resource produced by these conditional measurements [9]. Consider the experimental schematic shown in Fig. 1. To obtain an expression for the photon-subtracted resource we calculate the effect of introducing a beam splitter into each mode of the EPR state,

$$|\psi\rangle_{EPR}=\sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle_{AB}, \qquad (1)$$

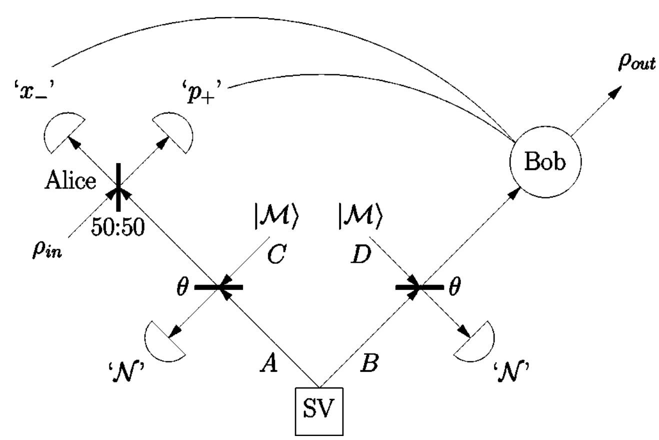

FIG. 1. Schematic of continuous variable teleportation. SV is the two-mode squeezed vacuum entanglement resource, one beam of which goes to Alice (labeled A), the other to Bob (labeled B). Alice mixes the unknown input state ρ_{in} on the 50:50 beam splitter and measures position difference x_- and momentum sum p_+. She sends this information via a classical channel to Bob who then makes the relevant local unitary operations on his beam, dependent upon the information from Alice, to recreate the input state at his location ρ_{out}. The conditional resource is made by inserting beam splitters of reflectivity θ in each arm of the teleporter, a Fock state $|\mathcal{M}\rangle$ at the spare port of the beam splitters, and detecting $\mathcal{N}$ at the detectors. The left-hand beam splitter's input mode is labeled C; the right-hand beam splitter's input mode is labeled D.

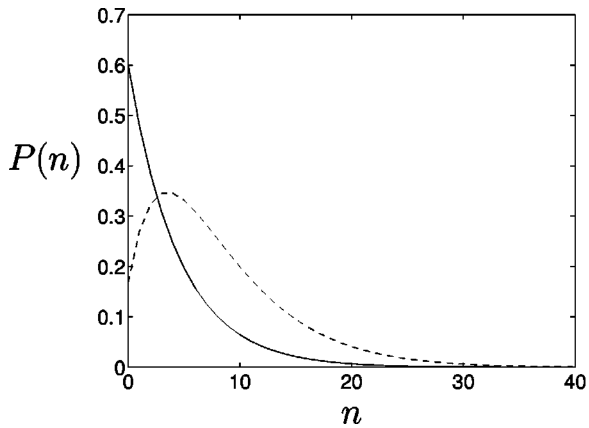

FIG. 2. Photon number distributions for the standard EPR state (solid curve) and the photon-subtracted EPR state (dashed curve) for a squeezing parameter of $\lambda = 0.8$.

where λ is the squeezing parameter, the labels A and B refer to the first and second modes, respectively, and we have made the definition $|n,n\rangle \equiv |n\rangle \otimes |n\rangle$. Modes A and B refer, respectively, to the left- and right-hand output of the parametric down converter [(SV) where SV is squeezed vacuum] in Fig. 1. Modes C and D being the modes at the spare port of the left- and right-hand beam splitters, respectively. The effect of the beam splitter is described by the unitary operator

$$U(\theta) = \exp[i\theta(a^\dagger c + c^\dagger a)], \tag{2}$$

where θ is the beam splitter reflectivity and a, $a^\dagger$, c, and $c^\dagger$ are the annihilation and creation operators for modes A and C, respectively. We expand to second order in the beam splitter reflectivity and condition on the result $\mathcal{N} = 1$ at each detector with the vacuum $|\mathcal{M}\rangle = |0\rangle$ at the spare port of each beam splitter. The photon-subtracted state in the Fock basis is

$$|\psi\rangle_{PS} = \sqrt{\frac{(1-\lambda^2)^3}{1+\lambda^2}} \sum_{n=0}^{\infty} (n+1)\lambda^n |n,n\rangle_{AB}, \tag{3}$$

where PS denotes that this is the photon-subtracted resource. The probability of obtaining this state is dependent upon the squeezing parameter and the reflectivity of the beam splitter,

$$P(\theta,\lambda) = \theta^4 \frac{1+\lambda^2}{(1-\lambda^2)^3}. \tag{4}$$

The main drawback of this conditioning technique is the small probability of the coincidences occurring. This is offset by the current experimental feasibility of detecting single-photon coincidences, the knowledge of when to teleport the input (as given by coincidence events), and the realization that given finite resources — such as squeezing — teleportation can be improved.

The photon number distribution for the photon-subtracted EPR state has a higher weighting for large photon numbers than the standard EPR state (Fig. 2). This suggests that the conditioning procedure behaves similarly to entanglement distillation. To support this intuition we use the fact that the resource states are pure and calculate the von Neumann entropy $S = -\mathrm{Tr}(\rho \ln \rho)$ as a function of the squeezing parameter λ. It is known that the von Neumann entropy is a good measure of entanglement for bipartite pure states [11], hence we can analyze the difference in entanglement between the standard and conditional EPR states. We show in Fig. 3 the von Neumann entropy as a function of squeezing parameter for the photon-subtracted EPR state (dashed line) and the standard EPR state (solid line). Note that the entropy of the conditioned state is higher for a given level of squeezing. This result shows that the entanglement in the conditional resource is higher than that in the standard resource, hence conditioning procedure seems to have had the effect of distilling entanglement out of the initial EPR state.

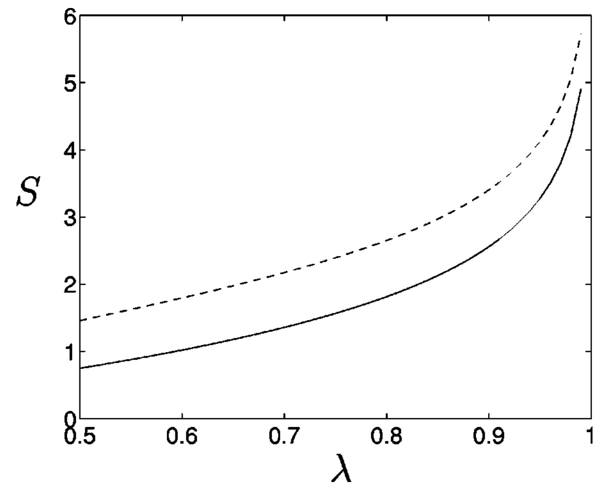

FIG. 3. von Neumann entropy S versus squeezing parameter λ. The standard EPR state gives the solid curve and the photon-subtracted EPR state the dashed curve. The figure shows a higher entanglement content in the photon-subtracted EPR state relative to the standard EPR state.

It is interesting to note that the conditional EPR states do not exhibit EPR correlations in the usual sense. That is, the variance of the amplitude sum and phase difference quadrature amplitudes are above shot noise for large values of squeezing. This is contrary to the standard definition of EPR correlations in which quadrature amplitude variances exhibit sub shot noise correlations [14]. However, fourth-order moments do decrease with increasing squeezing (and it is expected that higher order moments also show this behavior) indicating that EPR-like nonlocal correlations exist in the higher order but not necessarily the second-order moments. That this novel situation can arise is a consequence of the non-Gaussian nature of the conditioned states.

III. POSITION-DIFFERENCE, MOMENTUM-SUM TELEPORTATION

Opatrný, Kurizki, and Welsch [9] arrive at expressions for the teleportation fidelity, measurement probability, and average fidelity (the fidelity averaged over all measurements and weighted by the measurement probability) by calculating the effect of the teleportation operations on the relevant wave functions and then transforming into the Fock basis. In this paper we use the formalism of Hofmann *et al.* [12] to calculate these parameters. The fidelity is defined as the overlap between the input state $|\psi\rangle_T$ and the output state ρ_{out},

$$F = {}_T\langle\psi|\rho_{out}|\psi\rangle_T. \tag{5}$$

Teleportation in this formalism proceeds as per normal for continuous variables [3–5,7]; Alice has one component of an entangled pair of states and Bob the other. She mixes her entangled state with the state she wishes to teleport to Bob on a beam splitter, and measures the position difference (x_-) and momentum sum (p_+). Alice sends these results to Bob via a classical channel, who now displaces his state by an amount $\beta = x_- + ip_+$ to recreate the input state at his location. The entire teleportation process can be described by a transfer operator $\hat{T}(\beta)$ such that

$$|\psi(\beta)\rangle_{out} = \hat{T}(\beta)|\psi\rangle_T \tag{6}$$

is the output state, normalized to the probability of measuring the result β,

$$P(\beta) = {}_{out}\langle\psi(\beta)|\psi(\beta)\rangle_{out}. \tag{7}$$

One is able to describe the probability of measuring a given β, the fidelity of teleportation $F(\beta)$, and the average fidelity $\overline{F}$, in terms of the transfer operator as follows:

$$P(\beta) = {}_T\langle\psi|\hat{T}^\dagger(\beta)\hat{T}(\beta)|\psi\rangle_T, \tag{8}$$

$$F(\beta) = \frac{1}{P(\beta)}|{}_T\langle\psi|\hat{T}(\beta)|\psi\rangle_T|^2, \tag{9}$$

$$\overline{F} = \int d^2\beta P(\beta)F(\beta) = \int d^2\beta |{}_T\langle\psi|\hat{T}(\beta)|\psi\rangle_T|^2. \tag{10}$$

Following this formalism one merely needs to calculate the transfer operator for the given entanglement resource in order to obtain the parameters of interest. Hofmann *et al.* [13] showed for the standard EPR state that the transfer operator is

$$\hat{T}(\beta) = \sqrt{\frac{1-\lambda^2}{\pi}}\sum_{n=0}^{\infty}\lambda^n\hat{D}_T(g\beta)|n\rangle\langle n|\hat{D}_T(-\beta). \tag{11}$$

Here $\hat{D}_T(\beta)$ is a displacement of amount β and g is the gain of the teleporter. By noting correspondences between the standard EPR state and the photon-subtracted EPR state one can write expressions for the transfer operator for each. The photon-subtracted EPR state transfer operator is

$$\hat{T}(\beta) = \sqrt{\frac{(1-\lambda^2)^3}{\pi(1+\lambda^2)}}\sum_{n=0}^{\infty}(n+1)\lambda^n\hat{D}_T(g\beta)|n\rangle\langle n|\hat{D}_T(-\beta). \tag{12}$$

We briefly note that if one has an entanglement resource of the form

$$|\psi\rangle = \mathcal{N}\sum_{n=0}^{\infty}c_n|n,n\rangle, \tag{13}$$

where $\mathcal{N}$ is the normalization of the entangled state and the c_n are the coefficients that describe its photon number distribution, one can generalize the transfer operator to

$$\hat{T}(\beta) = \sqrt{\frac{\mathcal{N}}{\pi}}\sum_{n=0}^{\infty}c_n\hat{D}_T(g\beta)|n\rangle\langle n|\hat{D}_T(-\beta). \tag{14}$$

It is easy to see that the entanglement resource discussed in this paper is of this form. The significance of this result is that one has some freedom to choose an entanglement resource applying directly to the given situation. Doing so may help to enhance the teleportation fidelity or ease of implementation of the protocol. For example, in the current experimental setup (Fig. 1) another possible resource, a photon-added conditional EPR state can be obtained by having one-photon Fock states $(|\mathcal{M}\rangle = |1\rangle)$ incident at the spare port of the beam splitters and measuring the vacuum $(\mathcal{N}=0)$ at the detectors. This state is represented in the Fock basis by

$$|\psi\rangle_{PA} = \sqrt{\frac{(1-\lambda^2)^3}{1+\lambda^2}}\sum_{n=0}^{\infty}(n+1)\lambda^n|n+1,n+1\rangle_{AB}, \tag{15}$$

where *PA* denotes that this is the photon-added resource, and its transfer operator is

$$\hat{T}(\beta) = \sqrt{\frac{(1-\lambda^2)^3}{\pi(1+\lambda^2)}}\sum_{n=0}^{\infty}(n+1)\lambda^n\hat{D}_T(g\beta)|n+1\rangle \times\langle n+1|\hat{D}_T(-\beta). \tag{16}$$

This transfer operator also fits into the general form mentioned above. This alternative entanglement resource is not discussed further here since it gives identical results to that of the photon-subtracted EPR state and would be more difficult to realize experimentally.

We consider teleportation of a coherent state to gauge the ability of the conditional EPR state relative to the standard EPR state. The fidelity is calculated including a variable gain g and output coherent amplitude γ so as to include the possibility of the output state being an attenuated or amplified version of the input state. Choosing a coherent state of amplitude $\alpha = 3$, an output amplitude $\gamma = 3$, and unity gain, we calculate the average fidelity of teleportation $\overline{F}$ as a function of squeezing parameter λ for both the standard and photon-subtracted EPR states. Figure 4 shows both of these functions; the photon-subtracted EPR state (dashed curve) teleporting better than the standard EPR state (solid curve), since its average fidelity is higher for all values of the squeezing parameter. This result means that the conditioning technique has improved the efficacy of teleportation.

Polkinghorne and Ralph [15] identified a particular gain for which the output of teleportation using the standard EPR resource exactly corresponds to an attenuated version of the input. A similar, though not as ideal effect also occurs for the conditional EPR resource. In Fig. 5 we show the average

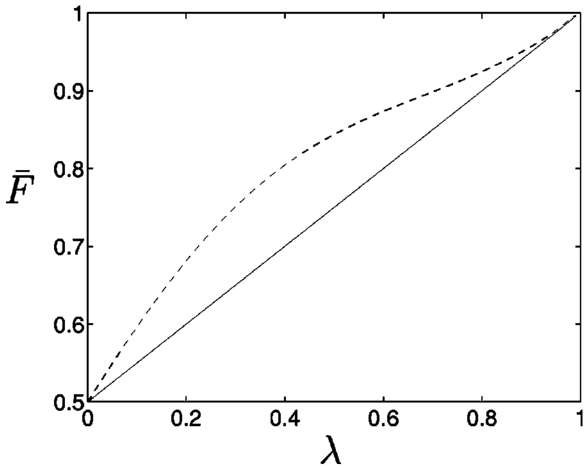

FIG. 4. Average fidelity $\overline{F}$ as a function of squeezing parameter λ for teleportation of a coherent state of amplitude $\alpha=3$ using the position-difference, momentum-sum continuous variable teleportation protocol. The dashed curve is the average fidelity for the photon-subtracted EPR state and the solid curve is for the standard EPR state. The dashed curve is above the solid curve for all values of the squeezing parameter, showing that the photon-subtracted EPR state performs better than the standard EPR state as an entanglement resource for teleportation.

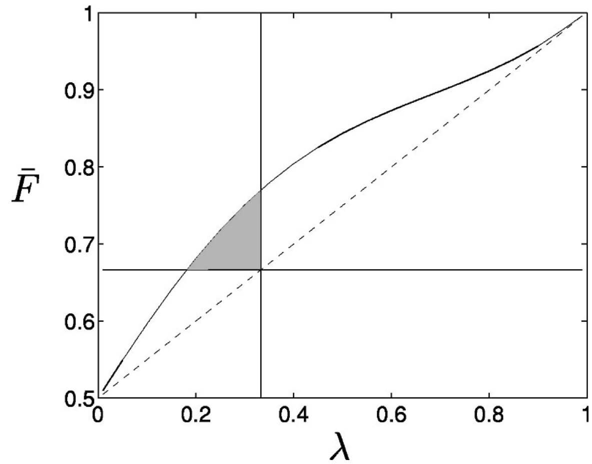

FIG. 6. Average fidelity $\overline{F}$ as a function of squeezing parameter λ for teleportation of a coherent state of amplitude $\alpha=3$ using the standard EPR state (dashed line) and the photon-subtracted EPR state (solid line). The gray shaded region denotes where the photon-subtracted EPR state beats the 2/3 successful quantum teleportation limit, whereas the standard EPR state does not. The horizontal line denotes the $\overline{F}=2/3$ boundary and the vertical line gives the right-hand edge of the shaded region and is where the standard EPR state lies on the boundary.

fidelity $\overline{F}$ maximized over the gain for a given comparison coherent amplitude [21], in the example of teleportation of a coherent state of amplitude $\alpha=5$, with a squeezing parameter of $\lambda=0.5$. Both the standard (solid curve) and the photon-subtracted (dashed curve) EPR resources are shown. According to Polkinghorne and Ralph, the average fidelity goes to unity when the gain is equal to the squeezing parameter for the standard EPR resource. This is evident in the figure since the solid curve reaches its maximum value of unity at $\gamma=2.5$, which is the expected comparison amplitude for a gain of 0.5 ($=\lambda$). The photon-subtracted average fidelity does not go to unity as the standard EPR resource does, reaching a maximum of 0.976. However, it reaches this maximum at a higher comparison coherent amplitude of $\gamma=3.7$. This implies that our conditioning technique may be improving the efficiency of the protocol. The conditional resource also beats the standard resource at this comparison amplitude and at unity gain (when $\gamma=\alpha=5$), implying that the conditional resource does better than the standard resource in the region of interest.

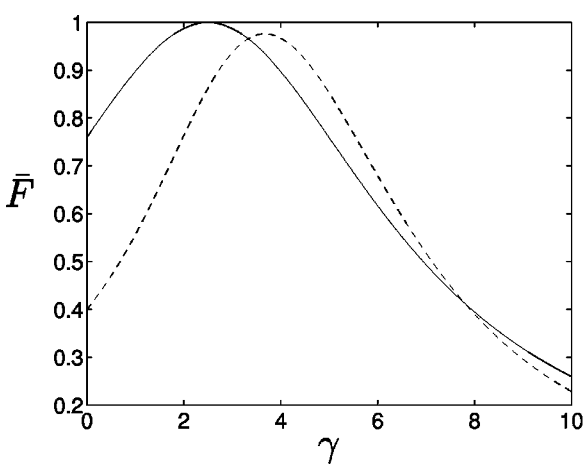

FIG. 5. Average fidelity $\overline{F}$ maximized over the gain as a function of comparison coherent amplitude γ for teleportation of a coherent state of amplitude $\alpha=5$, with a squeezing parameter of $\lambda=0.5$. The standard EPR resource is the solid curve; the photon-subtracted EPR resource is the dashed curve. Note that although the average fidelity for the photon-subtracted resource does not reach unity as the standard resource does, it reaches its maximum of 0.976 at a larger comparison amplitude of $\gamma=3.7$, implying that there has been an efficiency increase due to using the conditional entanglement resource.

The boundary beyond which entanglement is required in continuous variable teleportation of coherent states was found by Furusawa *et al.* [3] to be $\overline{F}=0.5$. On the other hand a qualitatively different boundary, beyond which the state reproduction is unambiguously quantum was found by Ralph and Lam [7] and has recently been the source of considerable discussion [16–18]. The criterion for beating this second boundary at unity gain is $\overline{F}>2/3$ [7,16,17]. Consider the average fidelity of both the standard EPR state and the photon-subtracted EPR state as functions of the squeezing parameter λ, shown in Fig. 6, where we teleport a coherent state of amplitude $\alpha=3$ with the teleporter at unity gain. We can find a region where the conditional resource beats the 2/3 successful quantum teleportation limit while the standard resource does not; this region is shaded gray in the figure. The horizontal line denotes the $\overline{F}=2/3$ boundary and the vertical line gives the upper edge of the shaded region, occurring where the average fidelity of the standard EPR state equals 2/3. The significance of this boundary is that above the $\overline{F}=2/3$ there exists no other better copy of the state that Bob

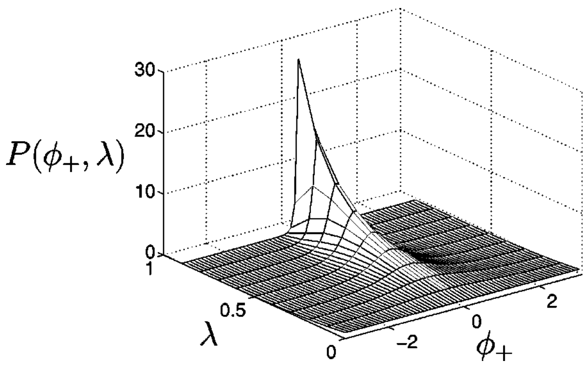

FIG. 7. Joint phase probability distribution as a function of both phase sum ϕ_+ and squeezing parameter λ. The distribution becomes sharply peaked with increasing λ, indicating that the photon-subtracted EPR state is tending towards eigenstates of phase sum.

gets [17]. Below this fidelity it is possible for Alice to have cheated, keeping a better copy for herself, or an eavesdropper to have obtained a duplicate, possibly better copy. Using our conditioning technique, Alice and Bob are able to improve the teleportation output and ensure that Bob obtains the best copy from an entanglement resource initially unable to do so.

IV. NUMBER-DIFFERENCE, PHASE-SUM TELEPORTATION

Milburn and Braunstein [10] introduced a teleportation protocol using number-difference and phase-sum measurements on the standard EPR state. Their protocol has the same structure as the more usual teleportation scheme involving the two-mode squeezed vacuum but the measurements made by Alice are of number difference and phase sum. Clausen, Opatrný, and Welsch [19] proposed a variation on the scheme of Milburn and Braunstein avoiding problems associated with the measurement of phase. Their teleportation scheme is conditional on Alice making certain measurements, whereas the scheme discussed here uses an *entanglement resource* that is conditional on making certain measurements and follows the protocol of Milburn and Braunstein (and later by Cochrane, Milburn, and Munro [8]) exactly. Because Clausen, Opatrný, and Welsch's scheme is conditional upon Alice's measurements, it does not work for every run of the experiment. However, our scheme works for every run since we wait until the resource is improved before executing the protocol.

We now show that making photon-subtracted conditional measurements on the standard entanglement resource improves the number-difference, phase-sum protocol also. The usual EPR state is an eigenstate of number difference and a near eigenstate of phase sum for sufficiently large squeezing [8,10]. To see that the photon-subtracted EPR state also fulfils these criteria, we calculate the joint phase probability density for the photon-subtracted EPR state. In general the joint phase probability is given by

$$P(\phi_1,\phi_2)=|\langle\phi_1|\langle\phi_2|\psi\rangle_{AB}|^2, \tag{17}$$

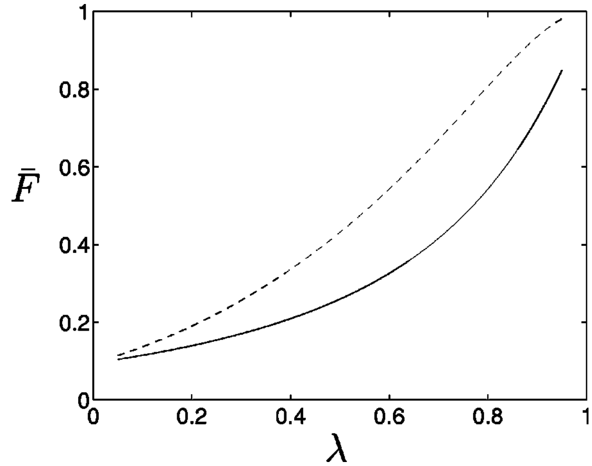

FIG. 8. Average fidelity $\overline{F}$ as a function of squeezing parameter λ for teleportation of a coherent state of amplitude $\alpha=3$ using the number-difference, phase-sum teleportation protocol. The dashed curve is the average fidelity of the photon-subtracted EPR state and the solid curve is the average fidelity of the standard EPR state. The dashed curve lies above the solid curve for all values of the squeezing parameter, showing that the photon-subtracted EPR state teleports better than the standard EPR state.

where the $|\phi_j\rangle$ are the phase states [14],

$$|\phi_j\rangle=\sum_{n=0}^{\infty} e^{-i\phi_j n}|n\rangle. \tag{18}$$

The joint phase probability density for the photon-subtracted EPR state is

$$P(\phi_+,\lambda)=\frac{(1-\lambda^2)^3}{1+\lambda^2}\left|\sum_{n=0}^{\infty} e^{in\phi_+}(n+1)\lambda^n\right|^2, \tag{19}$$

where $\phi_+=\phi_1+\phi_2$ is the phase sum. As $\lambda\to 1$ this distribution becomes more peaked about $\phi_+=0$ on the range $[-\pi,\pi]$, implying that the phase is highly correlated and the state is close to an eigenstate of phase sum. This is shown in Fig. 7.

Teleportation proceeds as follows [8,10]: Alice has one component of the two-mode squeezed vacuum, Bob the other. Alice makes joint number-difference and phase-sum measurements on the target state and her component of the entanglement resource, obtaining the results k and ϕ_+, respectively. She sends these results to Bob via a classical channel who performs the phase shift $e^{in\phi_+}$ and the amplification $|n+k\rangle\to|n\rangle$, where n is an index, to obtain the target state at his location. We find for a given number-difference measurement between Alice's mode and the input state k that for the photon-subtracted EPR state the teleportation fidelity is

$$F(k)=\begin{cases}\dfrac{(1-\lambda^2)^3}{(1+\lambda^2)P(k)}\left|\sum_{n=0}^{\infty}|c_{n+k}|^2(n+1)\lambda^n\right|^2, & k\geqslant 0\\ \dfrac{(1-\lambda^2)^3}{(1+\lambda^2)P(k')}\left|\sum_{n=0}^{\infty}|c_n|^2(n+k'+1)\lambda^{(n+k')}\right|^2, & k'=-k>0,\end{cases} \tag{20}$$

where the c_n are the coefficients describing the photon number distribution of the input state and

$$P(k)=\begin{cases}\dfrac{(1-\lambda^2)^3}{1+\lambda^2}\sum_{n=0}^{\infty}|c_{n+k}|^2(n+1)^2\lambda^{2n}, & k\geqslant 0\\ \dfrac{(1-\lambda^2)^3}{1+\lambda^2}\sum_{n=0}^{\infty}|c_n|^2(n+k'+1)^2\lambda^{2(n+k')}, & k'=-k>0\end{cases} \tag{21}$$

is the probability of measuring the number difference k.

To illustrate the relative performance of the entanglement resources we consider teleportation of a coherent state of amplitude $\alpha=3$. The average fidelity as a function of squeezing parameter for both the standard and photon-subtracted EPR states is shown in Fig. 8; the conditional state (dashed curve) outperforming the standard resource (solid curve) over all values of the squeezing parameter. Again the conditioning procedure has improved the teleportation protocol output.

V. DISCUSSION

By following the ideas of Opatrný, Kurizki, and Welsch [9] we have shown how to improve the efficacy of teleportation by making conditional measurements on the two-mode squeezed vacuum for both the position-difference, momentum-sum, and number-difference, phase-sum continuous variable teleportation protocols. The conditional measurements only require single photon coincidence detection which, although a challenging task, is currently feasible in the laboratory. The coincidence events also indicate when it is best to teleport. We have also shown that the conditional EPR state gives a resource able to provide unambiguously quantum teleportation for a large range of squeezing.

ACKNOWLEDGMENT

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[19] J. Clausen, T. Opatrný, and D.-G. Welsch, Phys. Rev. A **62**, 042308 (2000).
[20] Strictly speaking, the two-mode squeezed vacuum is not an EPR state, however, in the limit of infinite squeezing it tends to the EPR state. This limit is unphysical since infinite squeezing requires infinite energy, but for moderate squeezing the two-mode squeezed vacuum displays EPR *correlations* and for this reason is often referred to in the literature as an EPR state. We shall follow this convention here.
[21] The variable γ is the amplitude of the state with which Victor (the verifier) uses to compare the output state of the protocol.