Perturbation approach to the Casimir force between two bodies made of different real metals

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The Casimir force acting between two test bodies made of different metals is considered. The finiteness of the conductivity of the metals is taken into account perturbatively up to the fourth order of the relative penetration depths of electromagnetic zero-point oscillations into the metals. The influence of nonzero temperature is computed explicitly for separate orders of perturbation and found to be important in the zeroth and first orders only. The configurations of two parallel plates and a sphere (spherical lens) above a plate are considered made of Au and Cr. The obtained results can be used also to take into account the surface roughness. Thus, the total amount of the Casimir force between different metals with all correction factors is determined. This may be useful in various applications.

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I. INTRODUCTION

Recently the Casimir effect attracted much attention as a macroscopic quantum phenomenon caused by the existence of zero-point oscillations of the electromagnetic field. Casimir [1] first theoretically proposed that the change of the zero-point oscillation spectrum in the presence of metallic boundaries as compared to the case of empty space leads to a finite force acting onto these boundaries. The Casimir force can be considered as the relativistic limit of the van der Waals force under the condition that the spatial separations between the surfaces of the macrobodies are so large that the retardation effects become essential. This was demonstrated at first qualitatively by Sparnaay [2]. During the last time a lot of precision experiments on measuring the Casimir force have been performed [3–10]. The increasing interest in the Casimir effect is caused by the fact that it found both fundamental as well as technological applications. Thus, the precise measurements of the Casimir force and the extent of their agreement with theory have been used [11–14] to set the strongest constraints on hypothetical long-range forces predicted by many extensions of the standard model of elementary particles. Concerning technological applications the first microelectromechanical machines were created being driven by the Casimir force [15,16].

Increased precision and important applications of the Casimir force measurements call for the elaboration of new computational methods taking real experimental conditions into account. During the last years different corrections to the ideal Casimir force were computed due to surface roughness, finite conductivity of the boundary metal and nonzero temperature (see, e.g., Refs. [17–29]). Also the combined effect of these influential factors was investigated for the case of two boundary bodies being made of one and the same metal. It was shown that at separations smaller than 1 μm the surface roughness and the finite conductivity of the boundary metal can contribute up to several tens percent of the ideal Casimir force. At the same time, at separations of order of several micrometers the temperature corrections can achieve the value of the main contribution and even become larger. In a transition range of separations all the above corrections play an important role and their combined effect must be considered. However, the case of boundary bodies made of different metals was not investigated up to now.

Here, we present a perturbative approach to the calculation of the Casimir force acting between two bodies made of different real metals. We start from the famous Lifshitz formula [30] and describe metals in the framework of the plasma model. Both the configurations of two plane parallel plates and a sphere (spherical lens) above a plate are considered. The combined effect of finite conductivity and nonzero temperature is found on the basis of a perturbation expansion in powers of the relative penetration depth of electromagnetic zero-point oscillations into the metals under consideration. The coefficients of this expansion up to the fourth order are calculated explicitly. The temperature effect is shown to be essential only in the zeroth- and first-order terms. The obtained results are generalizations of those, which previously have been obtained in Refs. [22,24,26], for the case of boundary bodies made of one and the same metal. The case of different metals considered here is of special importance for the nanotechnology where one of the plates (playing the role of an active element) and the underlying substrate are typically made of different materials. The perturbation formulas given below are very simple in their application and open the possibility to compute the combined effect of finite conductivity and nonzero temperature with an error not larger than 1–2 % within a wide separation range that is usually adequate for any practical purpose. In doing so, one avoids labor-intensive numerical computations based on the use of optical tabulated data for the complex refraction index (compare with Refs. [20,21] where such computations were performed for two bodies made of one and the same metal). As an example, the test bodies covered by Au and Cr layers are considered. The obtained formulas can simply be modified to take into account the surface roughness

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(see, e.g., Refs. [26,29] for the special averaging procedure). Thus, they can be used for a complete description of the Casimir force with all essential corrections.

The paper is organized as follows. In Sec. II the perturbation expansion for the Casimir force at zero temperature acting between plates made of different metals is derived. Section III is devoted to the consideration of temperature corrections in configurations of two different plates. The configuration of a sphere (spherical lens) above a plate made of different metals is considered in Sec. IV. Section V contains conclusions and discussion.

II. PERTURBATION EXPANSION FOR TWO PARALLEL PLATES MADE OF DIFFERENT METALS AT ZERO TEMPERATURE

We consider first the configuration of two metallic semispaces marked by an index ss and described by the dielectric permittivities $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$, respectively. Let these semispaces be separated by a plane parallel gap of width a. The Casimir force between two different metals at zero temperature is given by the Lifshitz formula [30–32]

$$F_{ss}^{(\delta,0)}(a) = -\frac{\hbar c}{32\pi^2 a^4}\int_0^\infty x^3\,dx\int_1^\infty \frac{dp}{p^2} \times [X_1(p,x)+X_2(p,x)], \tag{1}$$

where

$$X_1(p,x) = \left[\frac{(s_1+p\varepsilon_1)(s_2+p\varepsilon_2)}{(s_1-p\varepsilon_1)(s_2-p\varepsilon_2)}e^x - 1\right]^{-1},$$
$$X_2(p,x) = \left[\frac{(s_1+p)(s_2+p)}{(s_1-p)(s_2-p)}e^x - 1\right]^{-1}. \tag{2}$$

The quantities s_k $(k=1,2)$ are given by $s_k=\sqrt{\varepsilon_k - 1 + p^2}$, and the dielectric permittivities are computed on the imaginary frequency axis $\varepsilon_k \equiv \varepsilon_k(i\xi) = \varepsilon_k[icx/(2pa)]$. The upper index δ in Eq. (1) marks the account of the effect of finite conductivity, and the second upper index 0 is the value of temperature.

For separation ranges of practical interest, namely, from a few tens of a micrometer to ten micrometers, relaxation processes can be neglected and the dielectric permittivities of the metals are given by the free-electron plasma model,

$$\varepsilon_k(i\xi) = 1 + \frac{\omega_{pk}^2}{\xi^2}, \tag{3}$$

where ω_{pk} are the plasma frequencies of the metals under consideration. The perturbative approach for calculating the effect of finite conductivity is based on the use of small parameters,

$$\alpha_k = \frac{\xi}{\omega_{pk}} = \frac{\delta_k}{a}\,\frac{x}{2p}, \tag{4}$$

where $\delta_k = \lambda_{pk}/2\pi$ are the effective penetration depths of the electromagnetic zero-point oscillations into the metals and $\lambda_{pk} = 2\pi c/\omega_{pk}$ are the plasma wavelengths. For the case of plates made of one and the same metal this approach was used in Refs. [31,33,34] (up to the first order), in Ref. [35] (up to the second order) and in Refs. [22,36] up to the fourth and the sixth orders, respectively (see also the detailed explanations in the monographs [37–39]). The obtained results were compared with the numerical computations using the tabulated data for the complex refractive index and were found to be in agreement with an error of only 1–2 % at all separation distances larger than the plasma wavelength [21,29]. Here, we apply this approach to the case of different metals up to the fourth perturbative order that is sufficient for practical purposes.

The quantities ε_k and s_k entering Eq. (2) can be represented in terms of the small parameters (4) as

$$\varepsilon_k(i\xi) = 1 + \frac{1}{\alpha_k^2}, \qquad s_k = \sqrt{p^2 + \frac{1}{\alpha_k^2}}. \tag{5}$$

Expanding X_1 from Eq. (2) upto the fourth power in α_k one obtains

$$X_1 = \frac{1}{e^x-1}\Big\{1 - 2\frac{A}{p}(\alpha_1+\alpha_2) + 2\frac{A}{p^2}(2A-1)(\alpha_1+\alpha_2)^2 - \frac{A}{p^3}(2-8A+8A^2-2p^2+p^4)(\alpha_1^3+\alpha_2^3) - 4\frac{A}{p^3}(1-6A+6A^2)$$
$$\times(\alpha_1^2\alpha_2+\alpha_1\alpha_2^2) + 2\frac{A(2A-1)}{p^4}[(2A-1)^2-2p^2+p^4](\alpha_1^4+\alpha_2^4) + 2\frac{A(2A-1)}{p^4}(2-16A+16A^2-2p^2+p^4)$$
$$\times(\alpha_1^3\alpha_2+\alpha_1\alpha_2^3) + 4\frac{A(2A-1)}{p^4}(1-12A+12A^2)\alpha_1^2\alpha_2^2\Big\}, \tag{6}$$

where $A \equiv e^x/(e^x-1)$.

In the same way the expansion of X_2 is

$$X_2=\frac{1}{e^x-1}[1-2Ap(\alpha_1+\alpha_2)+2Ap^2(2A-1)(\alpha_1+\alpha_2)^2-Ap^3(1-8A+8A^2)(\alpha_1^3+\alpha_2^3)-4Ap^3(1-6A+6A^2)$$
$$\times(\alpha_1^2\alpha_2+\alpha_1\alpha_2^2)+8A^2p^4(1-3A+3A^2)(\alpha_1^4+\alpha_2^4)-2Ap^4(1-18A+48A^2-32A^3)(\alpha_1^3\alpha_2+\alpha_1\alpha_2^3)$$
$$-4Ap^4(1-14A+36A^2-24A^3)\alpha_1^2\alpha_2^2]. \tag{7}$$

Substituting expressions (6) and (7) into Eq. (1) and performing the integrations with respect to p and x one finally obtains

$$F_{ss}^{(\delta,0)}(a)=F_{ss}^{(0,0)}(a)\left\{1-\frac{16}{3}\frac{\delta}{a}+24\frac{\delta^2}{a^2}-\frac{640}{7}\frac{\delta^3}{a^3}\left[1-\frac{2\pi^2}{105}\right.\right.$$
$$\left.\left.\times(1-3\kappa)\right]+\frac{2800}{9}\frac{\delta^4}{a^4}\left[1-\frac{326\pi^2}{3675}(1-3\kappa)\right]\right\}, \tag{8}$$

where

$$\delta\equiv\frac{\delta_1+\delta_2}{2},\qquad \kappa\equiv\frac{\delta_1\delta_2}{(\delta_1+\delta_2)^2}, \tag{9}$$

and $F_{ss}^{(0,0)}(a)=\pi^2\hbar c/240a^4$ is the ideal Casimir force per unit area of plates made of perfect metal. If $\delta_1=\delta_2=\delta_0$, i.e., when the plates are made of one and the same metal, Eq. (8) coincides with the result obtained earlier in Refs. [22,36].

III. TWO PARALLEL PLATES MADE OF DIFFERENT METALS AT NONZERO TEMPERATURE

Now let us consider the case of nonzero temperature. The Lifshitz formula at $T\neq 0$ is obtained from Eq. (1) by changing the integration with respect to x into a summation over the discrete Matsubara frequencies $\xi_l=cx_l/2ap=2\pi l k_B T/\hbar$, where $l=0,\pm 1,\pm 2,\ldots$ and k_B is the Boltzmann constant. It is convenient also to introduce a new variable $k_\perp=\xi_l\sqrt{p^2-1}/c$ (Ref. [29]). The result is

$$F_{ss}^{(\delta,T)}(a)=-\frac{k_BT}{2\pi}\sum_{l=-\infty}^{\infty}\int_0^\infty k_\perp\,dk_\perp\sqrt{\frac{\xi_l^2}{c^2}+k_\perp^2}$$
$$\times[X_1(k_\perp,\xi_l)+X_1(k_\perp,\xi_l)]. \tag{10}$$

Using the Poisson summation formula this expression can be represented as the sum of the zero-temperature result (1) and a temperature correction.

It can be easily checked that the temperature corrections of the first expansion coefficients of the perturbation result (8) following from Eq. (1) are independent of the materials, i.e., they are the same for plates made of one and the same metal and of different metals. In the framework of the plasma model these corrections can be calculated analytically in a closed form without using any perturbation expansion (see Refs. [40,23,26] where the corrections to the zeroth-, first-, and second-order coefficients, respectively, were found for the plates made of one and the same metal). It was proved in Refs. [25,26,28] that the plasma model is well adapted to the Lifshitz formula (10) at nonzero temperature and that it avoids all problems and contradictions arising in the case of the Drude dielectric function. As it was shown in Ref. [26], in the temperature range from 0 K to 1000 K the temperature corrections to the expansion coefficients starting from the second-order one are not essential. The reason is that at small surface separations the temperature effect itself is negligible, whereas at large separations the contribution of all the terms of the order of $(\delta/a)^k$ with $k\geqslant 2$ is smaller than 1%. This opens the opportunity to modify Eq. (8) by the use of temperature corrections, calculated in Refs. [23,26,40], in order to obtain the approximate expression for the Casimir force acting between different metals with account of both finite conductivity and nonzero temperature. The final result can be represented in the form

$$F_{ss}^{(\delta,T)}(a)=F_{ss}^{(0,0)}(a)\left\{1+\frac{30}{\pi^4}\sum_{n=1}^{\infty}\left[\frac{1}{(nt)^4}-\frac{\pi^3}{nt}\frac{\coth(\pi nt)}{\sinh^2(\pi nt)}\right]-2\frac{\delta}{a}\left[\frac{8}{3}-\frac{15}{\pi}\sum_{n=1}^{\infty}\frac{1}{nt\sinh^2(\pi nt)}\right.\right.$$
$$\left.\times\left(\frac{1}{(\pi nt)^2}\sinh(\pi nt)\cosh(\pi nt)+4\coth(\pi nt)+2\pi nt-6\pi nt\coth^2(\pi nt)+\frac{1}{\pi nt}\right)\right]$$
$$\left.+24\frac{\delta^2}{a^2}-\frac{640}{7}\frac{\delta^3}{a^3}\left[1-\frac{2\pi^2}{105}(1-3\kappa)\right]+\frac{2800}{9}\frac{\delta^4}{a^4}\left[1-\frac{326\pi^2}{3675}(1-3\kappa)\right]\right\}, \tag{11}$$

TABLE I. The relative Casimir force between two parallel plates taking account of finite conductivity and temperature corrections versus separation for different pairs of metals.

Separation a (μm)	$F_{ss}^{(\delta,T)}/F_{ss}^{(0,0)}$ for Au-Au $T=0$ K	$T=300$ K	$F_{ss}^{(\delta,T)}/F_{ss}^{(0,0)}$ for Au-Cr $T=0$ K	$T=300$ K	$F_{ss}^{(\delta,T)}/F_{ss}^{(0,0)}$ for Cr-Cr $T=0$ K	$T=300$ K	$F_{ss}^{(0,0)}(a)$ (nN/mm^2)
0.35	0.745	0.745	0.637	0.637	0.575	0.575	86.6
0.4	0.770	0.770	0.667	0.667	0.597	0.597	50.8
0.6	0.835	0.835	0.752	0.752	0.684	0.684	10.0
0.8	0.872	0.873	0.803	0.804	0.743	0.744	3.17
1	0.895	0.897	0.836	0.838	0.784	0.786	1.30
3	0.963	1.083	0.940	1.062	0.917	1.042	1.60×10^{-2}
5	0.977	1.531	0.963	1.518	0.949	1.505	2.08×10^{-3}
7	0.984	2.116	0.973	2.104	0.963	2.091	5.41×10^{-4}
10	0.988	3.027	0.981	3.015	0.974	3.002	3.05×10^{-4}

where $t\equiv T_{eff}/T$, and $k_BT_{eff}\equiv\hbar c/2a$.

One can easily find the asymptotic behavior of Eq. (11) at low ($T\ll T_{eff}$) and high ($T\gg T_{eff}$) temperatures (which also means small, respectively, large separations when taking into account the definition of T_{eff}). At low temperatures ($t\gg1$) it holds

$$F_{ss}^{(\delta,T)}(a)\approx F_{ss}^{(0,0)}(a)\left\{1+\frac{1}{3t^4}-2\frac{\delta}{a}\left[\frac{8}{3}-\frac{15}{\pi^3t^3}\zeta(3)\right]+24\frac{\delta^2}{a^2}-\frac{640}{7}\frac{\delta^3}{a^3}\left[1-\frac{2\pi^2}{105}(1-3\kappa)\right]+\frac{2800}{9}\frac{\delta^4}{a^4}\left[1-\frac{326\pi^2}{3675}(1-3\kappa)\right]\right\}, \tag{12}$$

where $\zeta(z)$ is the Riemann zeta function. At high temperatures ($t\ll1$) the result is given by

$$F_{ss}^{(\delta,T)}(a)\approx F_{ss}^{(0,0)}(a)\frac{30\zeta(3)}{\pi^3t}\left(1-3\frac{\delta}{a}\right). \tag{13}$$

All the results (11)–(13) take into account that the metals of both the plates are different.

For example, in Table I some numerical data obtained by Eq. (11) are presented for the pairs of plates Au-Au, Au-Cr, and Cr-Cr. Note that both Au- and Cr-covered test bodies are widely used in the measurements of the Casimir force (see, e.g., Refs. [3,7–10,15,16,41]). For Au the value of the plasma wavelength $\lambda_{p1}=136$ nm was used [20] and for Cr $\lambda_{p2}=314$ nm [41]. The separation range of 0.35–10 μm is covered including both the cases of low and high temperatures. The smallest separation of 0.35 μm is chosen to be larger than both plasma wavelengths in order to respect the application range of the fourth-order perturbation expansion of Eq. (11). In Table I the ratio of the Casimir force acting between real metals at zero and room temperatures relative to the ideal value (i.e. to a force between perfect metals at zero temperature) is given. In the last column the absolute values of the ideal Casimir force in units of force per unit area are presented. As is seen from Table I, the effect of finite conductivity is especially important at the smallest separations. The results for the pair of different metals (Au-Cr) differ significantly from both the cases of Au-Au and Cr-Cr. At small separations the temperature effect is negligible. With an increase of the separation distance the temperature effect also increases in all cases, and for separations larger than 3 μm it becomes larger than the ideal Casimir force. However, even at largest separations under consideration the effects of finite conductivity influence the value of the temperature force. Note that the asymptotics of low temperatures (12) is applicable at separations smaller than 2 μm and the asymptotics of high temperatures (13) works good starting from 4 μm.

IV. CONFIGURATION OF A SPHERE ABOVE A PLATE MADE OF DIFFERENT METALS

The configuration of two plane parallel plates was used only in two experiments [2,9]. More often the configuration of a sphere (spherical lens) above a plate was employed [3–8,10,15,16,41]. By this reason it is expedient to modify the obtained results for this configuration. This can be achieved by the application of the proximity force theorem [42]. According to this theorem the Casimir force acting between a semispace and a lens $F_{sl}(a)=2\pi RE_{ss}(a)$, where R is the lens (sphere) radius and $E_{ss}(a)$ is the energy per unit area of two parallel plates, which is related to the force of Eqs. (1) and (8) by the equality $F_{ss}(a)=-\partial E_{ss}(a)/\partial a$. Although the proximity force theorem is an approximation, its accuracy is very high (it leads to an error of the order of a/R, which is much smaller than 1% for configurations used in the experiments [43,44]).

Applying the proximity force theorem to Eq. (8) one obtains the Casimir force acting between a plate and a lens (sphere) made of different real metals at zero temperature,

$$F_{sl}^{(\delta,0)}(a)=F_{sl}^{(0,0)}(a)\left\{1-4\frac{\delta}{a}+\frac{72}{5}\frac{\delta^2}{a^2}-\frac{320}{7}\frac{\delta^3}{a^3}\left[1-\frac{2\pi^2}{105}\times(1-3\kappa)\right]+\frac{400}{3}\frac{\delta^4}{a^4}\left[1-\frac{326\pi^2}{3675}(1-3\kappa)\right]\right\}, \tag{14}$$

where $F_{sl}^{(0,0)}(a) = -\pi^3\hbar c R/360a^3$ is the ideal Casimir force. If one puts $\delta_1 = \delta_2 = \delta_0$ then Eq. (14) coincides with the earlier result for test bodies made of one and the same metal [22,36].

In the same way as it was done for the two parallel plates, Eq. (14) can be generalized to take into account nonzero temperature in the range from 0 K to 1000 K. Using the results of Ref. [26], one obtains

$$F_{sl}^{(\delta,T)}(a) = F_{sl}^{(0,0)}(a)\Bigg\{1 + \frac{90}{\pi^4}\sum_{n=1}^{\infty}\left[\frac{\pi}{2(nt)^3}\coth(\pi n t) - \frac{1}{(nt)^4} + \frac{\pi^2}{2(nt)^2}\frac{1}{\sinh^2(\pi n t)}\right] - 2\frac{\delta}{a}\Bigg[2 - \frac{45}{\pi^4}\sum_{n=1}^{\infty}\Bigg(\frac{\pi}{(nt)^3}\coth(\pi n t) - \frac{4}{(nt)^4} + \frac{\pi^2}{(nt)^2}\frac{1}{\sinh^2(\pi n t)} + \frac{2\pi^3}{nt}\frac{\coth(\pi n t)}{\sinh^2(\pi n t)}\Bigg)\Bigg] + \frac{72}{5}\frac{\delta^2}{a^2} - \frac{320}{7}\frac{\delta^3}{a^3}\left[1 - \frac{2\pi^2}{105}(1-3\kappa)\right] + \frac{400}{3}\frac{\delta^4}{a^4}\left[1 - \frac{326\pi^2}{3675}(1-3\kappa)\right]\Bigg\}. \tag{15}$$

The asymptotic behavior of Eq. (15) at low temperatures (separations) is

$$F_{sl}^{(\delta,T)}(a) \approx F_{sl}^{(0,0)}(a)\Bigg\{1 + \frac{45\zeta(3)}{\pi^3 t^3} - \frac{1}{t^4} - 2\frac{\delta}{a}\left[2 - \frac{45\zeta(3)}{\pi^3 t^3} + \frac{2}{t^4}\right] + \frac{72}{5}\frac{\delta^2}{a^2} - \frac{320}{7}\frac{\delta^3}{a^3}\left[1 - \frac{2\pi^2}{105}(1-3\kappa)\right] + \frac{400}{3}\frac{\delta^4}{a^4}\left[1 - \frac{326\pi^2}{3675}(1-3\kappa)\right]\Bigg\}. \tag{16}$$

In the opposite case of high temperatures (large separations) the asymptotic behavior is

$$F_{sl}^{(\delta,T)}(a) \approx F_{sl}^{(0,0)}(a)\frac{45\zeta(3)}{\pi^3 t}\left(1 - 2\frac{\delta}{a}\right). \tag{17}$$

As an example, in Table II the numerical results obtained by Eq. (15) are presented for the case of a plate and a sphere made of Au-Au, Au-Cr, and Cr-Cr. Table II is organized in the same manner as Table I—only the configuration of the test bodies is different. The last column of Table II contains the values of the ideal Casimir force for a sphere of radius $R = 1$ mm. The data demonstrate almost the same behavior with the increase of the separation as in the case of two plane parallel plates. It is seen, however, that the effect of nonzero temperature becomes noticeable at smaller separations. The asymptotics of low temperatures (16) works good at separations smaller than 2 μm and the asymptotics of high temperatures (17) is applicable for $a > 4$ μm.

V. CONCLUSIONS AND DISCUSSION

To conclude, we have developed a perturbative approach to the calculation of the Casimir force acting between two parallel plates or a sphere (spherical lens) above a plate made of different real metals. The coefficients of the perturbation expansion in powers of two small parameters were found up to the fourth order. These parameters have the meaning of the effective penetration depth of electromagnetic zero-point oscillations into both metals. The effect of nonzero temperature was taken into account explicitly in the coefficients of perturbation expansions of zeroth and first orders. The temperature dependence of the higher-order expansion coeffi-

TABLE II. The relative Casimir force between a lens and a plate taking account of finite conductivity and temperature corrections versus separation for different pairs of metals.

Separation	$F_{sl}^{(\delta,T)}/F_{sl}^{(0,0)}$ for Au-Au		$F_{sl}^{(\delta,T)}/F_{sl}^{(0,0)}$ for Au-Cr		$F_{sl}^{(\delta,T)}/F_{sl}^{(0,0)}$ for Cr-Cr		$F_{sl}^{(0,0)}(a)$
a (μm)	$T=0$ K	$T=300$ K	$T=0$ K	$T=300$ K	$T=0$ K	$T=300$ K	(nN)
0.35	0.799	0.800	0.706	0.707	0.639	0.640	6.45×10^{-2}
0.4	0.820	0.822	0.732	0.735	0.665	0.668	4.25×10^{-2}
0.6	0.872	0.879	0.805	0.811	0.746	0.754	1.26×10^{-2}
0.8	0.902	0.916	0.846	0.862	0.797	0.813	5.32×10^{-3}
1	0.920	0.947	0.873	0.902	0.831	0.860	2.72×10^{-3}
3	0.972	1.443	0.954	1.427	0.937	1.411	1.01×10^{-4}
5	0.983	2.275	0.972	2.262	0.961	2.249	2.18×10^{-5}
7	0.988	3.181	0.980	3.168	0.972	3.155	7.93×10^{-6}
10	0.991	4.551	0.986	4.538	0.980	4.526	2.72×10^{-6}

cients is negligible in the temperature range from 0 K to 1000 K and thereby it is of no practical interest. The asymptotic behavior of the explicit temperature dependences at low and high temperatures is also given.

The obtained formulas are simple in application and give the possibility to calculate the Casimir force with account of both finite conductivity and nonzero temperature between the test bodies made of different metals. They can be applied in a wide range of separations and temperatures quite sufficient for all practical purposes. The error of the results obtained in such a way is only 1–2 % and is in fact caused by the error in the values of plasma wavelengths. The much more complicated alternative approach using the optical tabulated data for the complex refractive index and the exact Lifshitz formula does not lead to more exact results because of the errors in optical data and the necessity to use some interpolation and extrapolation procedures [20]. It is notable also that the above perturbative approach is very convenient to take into account the surface roughness. This can be done by averaging of the obtained results over all possible separation distances and it leads to a perfect agreement between experiment and theory [10,17]. Thus, the suggested perturbative approach presents a complete description of the Casimir force acting between different metals with all important corrections and can be used in various applications of the Casimir effect.

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