Efficient and lasting squeezed light due to constructive two-photon interference

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We demonstrate numerically that efficient and lasting squeezed light can be achieved due to a constructive two-photon interference. Even if large phase fluctuations occur, the dispersion in one of the two quadrature components of the light field can still be reduced below the standard quantum limit set by the symmetric distribution of the vacuum state. Experimental observation of squeezed light in this system is possible by generalizing the recent techniques of Ou's group [L. J. Lu and Z. Y. Ou, Phys. Rev. Lett. **88**, 023601 (2002)].

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Over the past 20 years, particular attention has been focused on squeezed states of light [1,2]. Since the squeezed light can reduce quantum fluctuations in one of the two quadrature components of the field below the standard quantum limit (SQL) set by the symmetric distribution of the vacuum state, the squeezed states of light would offer possibilities of improving performance of many optical devices and optical communication networks. It has been shown that squeezed states of light can be generated by a number of nonlinear optical processes [3–11], such as four-wave mixing, two-photon laser, parametric amplifiers, and squeezed atom. The reader can consult three recent books [12–14] for more complete information about the experimental generation of squeezed states of light. Recently, squeezed light has attracted renewed interest due to its practical applications [12–15] in optical communication, gravity wave detection, high-resolution laser spectroscopy, and quantum information theory. For example, squeezed light has been used in spectroscopic measurement of atomic cesium [16], in a power-recycled interferometer [17], and in a phase-modulated signal-recycled interferometer [18], aiming to improve significantly the sensitivity of these devices. Squeezed light has also been applied in quantum information theory, for example, quantum teleporation [19,20], cryptography [21,22], dense coding [20], quantum nondemolition measurement [23], and quantum images [24]. In this respect, the security in quantum cryptography relies on the uncertainty relation for field quadrature components of these states. Furthermore, experiments on quantum teleporation have been successfully performed by means of two-mode squeezed vacuum state [25].

Very recently, Lu and Ou [26] have reported an experimental observation of a number of quite different features in photon statistical distribution (e.g., photon antibunching, photon bunching, and a novel nonclassical phenomenon that exhibits more complex structure) in a cw system as they change the relative phase between the coherent field and the down converted field. Lu and Ou [26] pointed out that the different features in photon statistics are attributed to a two-photon interference between a coherent field and a narrow-band two-photon field. Surely, their observation is interesting and significant in quantum optics. Especially, because their cw system is not wavelength dependent so long as to meet phase matching condition in parametric down conversion and more directional because of the phase matching condition, it has the advantage over other atomic transition based fluorescent systems [26]. Hence, it would be interesting and significant to examine further the squeezing properties of the quantum fluctuations in one of the two quadrature components of the light field in this system. This is the purpose of our present paper.

Following the experimental scheme proposed by Stoler [27] (also see Fig. 1 of Ref. [26]), we consider a mixing of a coherent state and a two-photon state and treat both of them with a simple single-mode model. In the experiment of Lu and Ou [26], the mixing is realized by injecting a coherent field into a parametric down converter. Then, the coherent state, which is taken as an initial state for the down-converted mode, interacts with a pump field via the following interaction Hamiltonian:

$$H = i\hbar\lambda V_P a^\dagger a^\dagger + \text{H.c.}, \tag{1}$$

where $a^\dagger$ is the creation operator of the field, V_P is the amplitude of the pump field, and λ is some constant proportional to a nonlinear coefficient of a nonlinear medium. As addressed in Ref. [26], we emphasize the role of quantum two-photon interference. Hence, given a weak coherent state $|\alpha\rangle$ [26,28,29],

$$|\alpha\rangle \approx |0\rangle + \alpha|1\rangle + (\alpha^2/\sqrt{2})\,|2\rangle \qquad (|\alpha| \ll 1), \tag{2}$$

one can obtain the output state [26]

$$|\Psi(t)\rangle \approx |0\rangle + \alpha|1\rangle + (\sqrt{2}\lambda V_p t + \alpha^2/\sqrt{2})|2\rangle \tag{3}$$

by assuming $|\lambda V_p t| \sim |\alpha|^2 \ll 1$, where t is the interaction time that is proportional to the length of the nonlinear medium.

It is known that a single-mode electric field operator E can be expressed as

$$E = 2E_0(a_1 \cos \omega t + a_2 \sin \omega t), \tag{4}$$

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where ω is the frequency of the light field and a_1 and a_2 are a pair of conjugate quadrature operators given by

$$a_1=(1/2)[a+a^{\dagger}], \tag{5}$$

$$a_2=(1/2i)[a-a^{\dagger}] \tag{6}$$

in terms of the familiar photon annihilation (a) and creation ($a^{\dagger}$) operators. Both conjugate quadrature operators satisfy a commutation relation

$$[a_1,a_2]=i/2, \tag{7}$$

and correspondingly, their variances obey an uncertainty relation

$$(\Delta a_1)^2(\Delta a_2)^2 \geqslant L^2, \tag{8}$$

where the parameter $L=1/4$ is the SQL. It is known that light in a coherent state (approximated by a single-mode laser) or in a vacuum state is in a minimal-uncertainty state, i.e., $(\Delta a_1)^2(\Delta a_2)^2=L^2$ with an equal variance for each quadrature component of the light field. However, squeezed states may or may not be minimal-uncertainty states, but are such that the variance in one quadrature component a_j of the light field is less than the SQL, i.e., $(\Delta a_j)^2<L$. For convenience, we define the following functions

$$h_j=(\Delta a_j)^2-L \qquad (j=1,2). \tag{9}$$

Then, if $h_j<0$ ($j=1$, or 2), the quantum fluctuations in a_j are reduced below the SQL. This is the general definition of field squeezing [1,2]. Using Eq. (3), we can easily arrive at the functions h_j, which are given by

$$h_1=I_0+|\alpha|^2/2-I_1^2+\Gamma_2, \tag{10}$$

$$h_2=-I_0+|\alpha|^2/2-I_2^2+\Gamma_2, \tag{11}$$

with

$$I_0=|\alpha|^2[T\cos\phi+\tfrac{1}{2}\cos(2\Phi)], \tag{12}$$

$$I_1=|\alpha|^2\left[\frac{2T\cos(\phi-\Phi)}{|\alpha|}+\left(\frac{1}{|\alpha|}+|\alpha|\right)\cos\Phi\right], \tag{13}$$

$$I_2=|\alpha|^2\left[\frac{2T\sin(\phi-\Phi)}{|\alpha|}+\left(\frac{1}{|\alpha|}+|\alpha|\right)\sin\Phi\right], \tag{14}$$

$$\Gamma_2=(|\alpha|^4/2)[4T^2+4T\cos(\phi-2\Phi)+1], \tag{15}$$

where $T=|\lambda V_p|t/|\alpha|^2$ is the scaled interaction time, ϕ and Φ are the phases of the pump and coherent fields, respectively, and Γ_2 is the two-photon rate in the combined field. Taking the quantum fluctuations in the quadrature component a_1 of the combined field as an example, we examine the squeezing properties of the light field with the selection of the relative phase between the pump and coherent fields in the system introduced above.

First, we examine numerically the dynamics of a modified function $H_1 \equiv h_1/|\alpha|^2$ as the relative phase between the coherent and pump fields varies. From the experimental point of view, the dynamics of a long time scale is not practical. Hence, for experimentally relevant time systems, we only consider the case of a short time scale. On the other hand, we set $|\alpha|=0.01$ to satisfy the assumption condition of $|\lambda V_p t| \sim |\alpha|^2 \ll 1$, which is made to get the output state $|\Psi(t)\rangle$ given by Eq. (3). In detail, Fig. 1 and Fig. 2 present three-dimensional (3D) contour plots for the time evolution of H_1 with a different choice of the relative phase between the pump and coherent fields. As we hold the phase of the pump field at $\phi=\pi/2$, it is seen from Fig. 1(a) that the fluctuations in a_1 could be reduced below the SQL at $\pi<\Phi<3\pi/2$ or $0<\Phi<\pi/2$, and the field squeezing is lasting for a long time. Then, if we hold the phase of the pump field at $\phi=3\pi/2$, as shown in Fig. 1(b), we find that the squeezing effect occurs at $\pi/2<\Phi<\pi$ or $3\pi/2<\Phi<2\pi$. Comparing Fig. 1(b) with Fig. 1(a), we see that the squeezing region in the $\Phi-T$ parameter space is exchanged almost totally. Similar results are obtained in Fig. 2(a) and 2(b) when we hold the phase of the coherent field at $\Phi=\pi/2,\pi$, respectively, but vary the phase ϕ of the pump field from 0 to 2π. From both Fig. 1 and Fig. 2, we also notice that the depth of the field squeezing is enhanced with the evolution of the interaction time. In order to have a look at the squeezing effect in the ϕ-Φ parameter space, Fig. 3 presents a 3D contour plot for H_1, ϕ, and Φ at a fixed interaction time $T=0.8$, where the squeezing regions are clearly displayed in the ϕ-Φ parameter space. Hence, with a proper choice of the relative phase between the pump and coherent fields, the dispersion in the quadrature component a_1 of the combined field could be reduced below the SQL set by the symmetric distribution of the vacuum state.

We see that the single-photon rate calculated from Eq. (3) is the same as that derived from Eq. (2). This means that the mixed field is dominated by the coherent field [26]. Meanwhile, the two-photon rate derived from Eq. (3) is Γ_2 given by Eq. (15), while the two-photon rate Γ_0 $(=|\alpha|^4/2)$ in the coherent field can be easily obtained from Eq. (2). Defining the ratio $R=\Gamma_2/\Gamma_0$ between both two-photon rates, we have

$$R=4T^2+4T\cos(\phi-2\Phi)+1. \tag{16}$$

Obviously, if we properly select the relative phase between the pump and coherent fields, the ratio R can be bigger (or smaller) than 1 and a constructive (or destructive) two-photon interference can be shown. Especially, a completely destructive two-photon interference can be observed for the case of $R=0$. As an example, holding the phase of the coherent field at $\Phi=\pi/2$, we present in Fig. 4 a 3D contour plot for the two-photon rate ratio R, the phase ϕ of the pump field and the interaction time T. We find that a constructive two-photon interference is observed at $\pi/2<\phi<3\pi/2$, while a destructive two-photon interference occurs at $0<\phi<\pi/2$ or $3\pi/2<\phi<2\pi$. Moreover, comparing Fig. 4 with Fig. 2(a), we conclude that a constructive two-photon interference could lead to an efficient and lasting squeezed light.

In the experiment of Lu and Ou [26], the relative phase between the pump and coherent fields is adjusted through the change of the path of the pump field. Surely, phase fluctua-

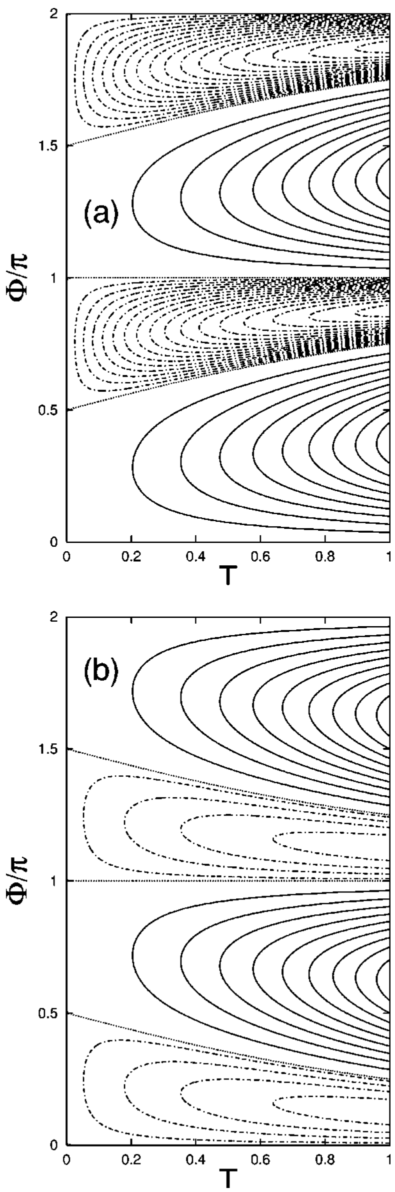

FIG. 1. 3D contour plots of the modified function H_1, the phase Φ of the coherent field, and the scaled interaction time T for $|\alpha|=0.01$ at (a) $\phi=\pi/2$, where the solid line is for $H_1<0$ (-0.5 to -4.5 with an interval of -0.5), the dotted line for $H_1=0$, and the dot-dashed line for $H_1>0$ (0.05 to 0.8 with an interval of 0.05); (b) $\phi=3\pi/2$, where the solid line is for $H_1<0$ (-0.5 to -4.5 with an interval of -0.5), the dotted line for $H_1=0$, and the dot-dashed line for $H_1>0$ (0.1 to 0.7 with an interval of 0.2).

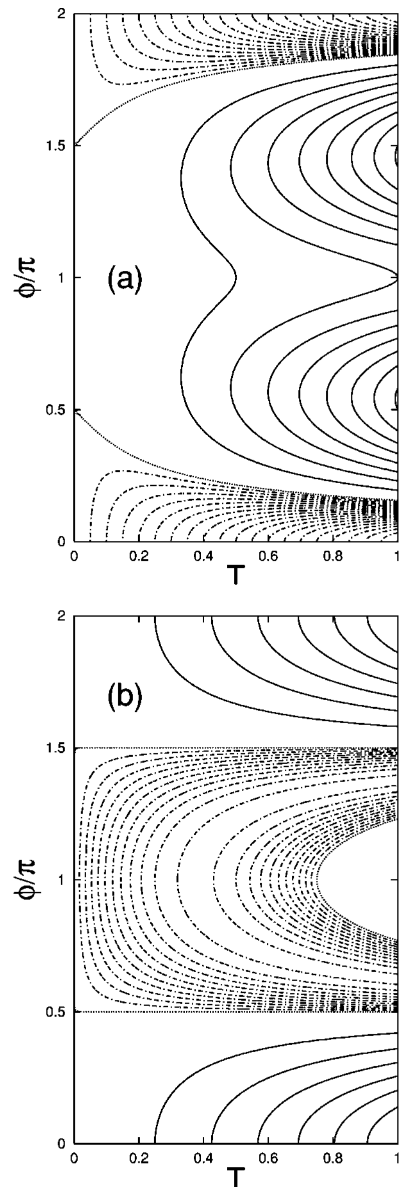

FIG. 2. 3D contour plots of the function H_1, the phase ϕ of the pump field, and the scaled interaction time T for $|\alpha|=0.01$ at (a) $\Phi=\pi/2$, where the solid line is for $H_1<0$ (-0.5 to -4 with an interval of -0.5), the dotted line for $H_1=0$, and the dot-dashed line for $H_1>0$ (0.05 to 0.95 with an interval of 0.05); (b) $\Phi=\pi$, where the solid line is for $H_1<0$ (-1 to -7 with an interval of -1), the dotted line for $H_1=0$, and the dot-dashed line for $H_1>0$ (0.05 to 0.55 with an interval of 0.05).

tions may occur due to, for example, mechanical vibration and air flow when the pump field and the injected coherent field travel through different paths via multiple mirror sets [26]. In the experiment, Lu and Ou found that a swing of the relative phase from π to $\pi/2$ will cause the combined light field to go from antibunching to bunching. This means that the photon correlation function is extremely sensitive to the phases of the pump and coherent fields. However, for the squeezed light predicted in our numerical calculation, it is not so. For example, Fig. 2(a) shows that squeezed light is still observed even if the relative phase $|\phi-\Phi|$ swings from 0 to π (note that Φ is held at the value of $\pi/2$). Certainly, if there is an active servo system, which can hold the phase at a desired value, this system could generate much more efficient squeezed light.

In conclusion, we have demonstrated numerically that efficient and lasting squeezed light could be generated due to a constructive two-photon interference. Even if large phase fluctuations occur, the dispersion in one of the two quadrature components of the light field could still be reduced below the SQL set by the symmetric distribution of the vacuumstate. The relevance of this work lies in the

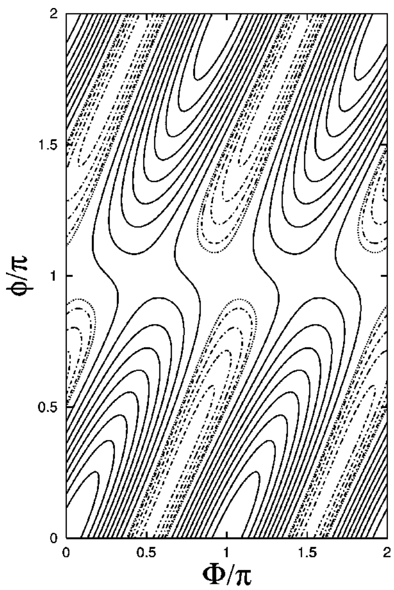

FIG. 3. 3D contour plot of the function H_1, the phase Φ of the coherent field, and the phase ϕ of the pump field for $|\alpha|=0.01$ at the interaction time $T=0.8$, where the solid line is for $H_1<0$ (-0.5 to -4.5 with an interval of -0.5), the dotted line for $H_1=0$, and the dot-dashed lines for $H_1>0$ (0.1 to 0.7 with an interval of 0.2).

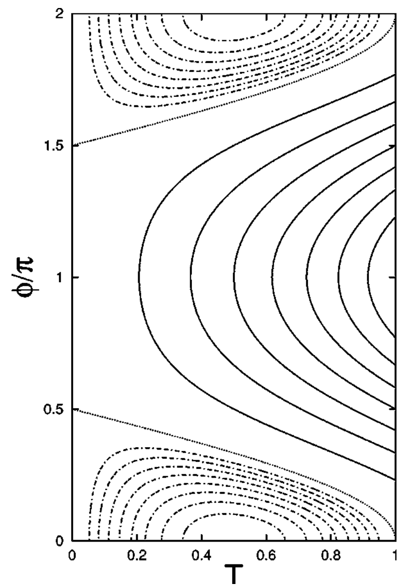

FIG. 4. 3D contour plot of the two-photon rate ratio R, the phase ϕ of the pump field, and the scaled interaction time T for $|\alpha|=0.01$ at $\Phi=\pi/2$, where the solid line is for $R>1$ (2 to 8 with an interval of 1), the dotted line for $R=1$, and the dot-dashed line for $R<1$ (0.1 to 0.8 with an interval of 0.1).

coupling of our prediction of squeezed light with other nonclassical phenomena such as photon bunching and antibunching within the same experimental apparatus. Also, experimental observation of squeezed light in this system is possible by generalizing the experimental techniques of Lu and Ou [26].

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[1] J. Mod. Opt. **34(67)**, 709 (1987), special issue on squeezed light, edited by R. Loudon and P. L. Knight.
[2] J. Opt. Soc. Am. B **4** (10), 1453 (1987), special issue on squeezed states of the electromagnetic field, edited by H. J. Kimble and D. F. Walls.
[3] D.F. Walls, Nature (London) **306**, 141 (1983).
[4] B. Yurke, Phys. Rev. A **29**, R408 (1984).
[5] L.A. Wu *et al.*, Phys. Rev. Lett. **57**, 2520 (1987).
[6] R.H. Xie, Phys. Rev. A **53**, 2897 (1996).
[7] A.B. Bhattacherjee, Phys. Rev. A **56**, 796 (1997).
[8] Y. Zhang *et al.*, Phys. Rev. A **62**, 023813 (2000).
[9] U.V. Poulsen and K. Mølmer, Phys. Rev. Lett. **87**, 123601 (2001).
[10] K.S. Zhang *et al.*, Phys. Rev. A **64**, 033815 (2001).
[11] K.G. Köprülü and O. Aytür, J. Opt. Soc. Am. B **18**, 846 (2001), and related references therein.
[12] H. A. Bachor, *A Guide to Experiments in Quantum Optics* (Wiley, Weinheim, 1998).
[13] H. A. Haus, *Electromagnetic Noise and Quantum Optical Measurements* (Springer-Verlag, Berlin, 2000).
[14] J. Kim *et al.*, *Nonclassical Light from Semiconductor Lasers and LEDs* (Springer-Verlag, Berlin, 2001).
[15] Appl. Phys B: Photophys. Laser Chem. **55**, 189 (1992), special issue on quantum noise reduction in optical systems/experiments, edited by C. Fabre and E. Giacobino.
[16] E.S. Polzik *et al.*, Phys. Rev. Lett. **68**, 3020 (1992).
[17] V. Chickarmane and S.V. Dhurandhar, Phys. Rev. A **54**, 786 (1996).
[18] V. Chickarmane *et al.*, Phys. Rev. A **57**, 3898 (1998).
[19] G.J. Milburn and S.L. Braunstein, Phys. Rev. A **60**, 937 (1999).
[20] J. Zhang and K.C. Peng, Phys. Rev. A **62**, 064302 (2000).
[21] T.C. Ralph, Phys. Rev. A **61**, 010303(R) (2000).
[22] M. Hillery, Phys. Rev. A **61**, 022309 (2000).
[23] J.X. Zhang *et al.*, Appl. Opt. **40**, 5949 (2001).
[24] J. Jeffers and G.L. Oppo, Phys. Rev. A **60**, 2393 (1999).
[25] A. Furusawa *et al.*, Science **282**, 706 (1998).
[26] L.J. Lu and Z.Y. Ou, Phys. Rev. Lett. **88**, 023601 (2002).
[27] D. Stoler, Phys. Rev. Lett. **33**, 1397 (1974).
[28] P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, Tucson, 1990), pp. 327–330.
[29] K.J. Resch *et al.*, Phys. Rev. Lett. **88**, 113601 (2002).