Stabilization dynamics in an intense circularly polarized laser field

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We study quantum and classical dynamics of the electron in ionization stabilization of the two-dimensional hydrogen atom irradiated by an intense circularly polarized laser. When the quantum time evolution of the wave packet is stable but the corresponding classical time evolution is diffusive, stabilization can be understood to arise from the effect of quantum localization that suppresses classically chaotic diffusion. When both the quantum and classical time evolutions exhibit a stable behavior, stabilization can be regarded as originating from stable classical orbits generated by nonlinear resonance. We also investigate the effect of the bare Coulomb singularity on ionization stabilization.

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The strong interaction of atoms with intense laser fields produces many interesting nonlinear atomic phenomena such as multiphoton ionization, above-threshold ionization, and high-harmonic generation [1,2]. Another nonlinear phenomenon under intensive research for the last ten years or so is the stabilization of ionization [3–5], a phenomenon referring to a decrease or saturation of the ionization rate with respect to an increase in the laser intensity. A conceptual understanding of the stabilization of ionization can be obtained if the problem is viewed in the Kramers-Henneberger (KH) frame [6].

While much of the attention has been given to the case of a linearly polarized laser field, it was realized that a circularly polarized laser field may provide a more favorable condition for stabilization to occur than a linearly polarized laser field [4,7–13]. Pont and Gavrila [4] in 1990 have already shown that stabilization of ionization can occur when the laser field is circularly polarized. There exist theoretical [7] and experimental [8] work in which the dependence of ionization and stabilization upon the polarization of the laser was investigated. Recently, Protopapas *et al.* [11] and Patel *et al.* [12] have found that a circularly polarized laser leads to a higher degree of stabilization than a linearly polarized laser. In addition, Chism *et al.* [13] observed that, when stabilization occurs in a circularly polarized laser field, the electron wave packet evolves without spreading in the form of a "Trojan wave packet."

In this paper, we investigate stabilization of ionization in a two-dimensional model hydrogen atom in the presence of a circularly polarized laser. We wish to study the question, "to what extent quantum and classical dynamics agree in describing the phenomenon of ionization stabilization?" Several earlier works [9,10,14–16] have indicated that classical analysis provides an adequate description of stabilization of ionization. Another issue we wish to consider is the role of Coulomb singularity in stabilization. Some earlier works [14,15] have indicated that stabilization occurs more easily with the soft-core potential than with the Coulomb potential.

The system we consider is a two-dimensional model hydrogen atom irradiated by a circularly polarized laser field. Under the dipole approximation the time evolution of the system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi\rangle=\left(\frac{1}{2}p^2+V(r)+F(t)(x\cos\omega t+y\sin\omega t)\right)|\psi\rangle, \tag{1}$$

where atomic units are used and $F(t)$ represents the time-varying amplitude of the laser field being treated classically. The potential $V(r)$ is given by $-1/r=-1/\sqrt{x^2+y^2}$ for the Coulombic model and is taken to be $-1/\sqrt{r^2+a^2}$ ($a=0.8$ a.u.) for the soft-core model. The atom is assumed to be prepared initially in an eigenstate ψ_{nlm} of the two-dimensional hydrogen atom [17]. We assume that the laser is turned on according to the relation $F(t)=F_0\sin^2(\pi t/2T_{switch})$ for $0\leqslant t\leqslant T_{switch}$. For $t\geqslant T_{switch}$, the constant amplitude F_0 is maintained. The parameter values need to be chosen in accordance with the condition for stabilization that twice the pondermotive energy, $F_0^2/2\omega^2$, be greater than the photon energy $\hbar\omega$ and that the photon energy be greater than the energy needed to ionize the atom from the initial state [2]. Thus, we set the frequency of the laser $\omega=2.2$ a.u. (the ionization energy of the two-dimensional hydrogen atom is 2 a.u.$=54.4$ eV [17]) and vary the field strength over the range $0<F_0<30.0$ a.u., which corresponds, in term of the laser intensity I, to $0<I<2.853\times10^{19}$ W/cm^2. The laser switch time T_{switch} is chosen to be four laser cycles, $T_{switch}=4\times2\pi/\omega=4T$.

In Fig. 1 we show the computed ionization probability after eight laser cycles ($t=8\cdot2\pi/\omega=8T$), for both the Coulombic atom and the soft-core model atom. The ionization probability is determined by subtracting from one the probability that the wave packet remains inside the absorbing boundary [18]. For the Coulombic hydrogen atom, the ionization probability is seen to increase as the field strength is increased to $F_0\approx12$ a.u. but decrease as the field strength is increased beyond $F_0\approx12$ a.u. For the soft-core model atom, the ionization probability reaches a maximum at $F_0\approx18$ a.u. and then decreases as the field strength is further

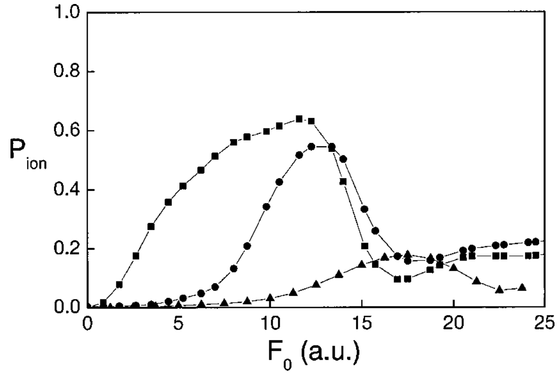

FIG. 1. Ionization probability at $t=8T$ vs the laser amplitude F_0. Squares and circles are for the case of the initial ground-state ψ_{100} and for the case of the initial excited state ψ_{322}, respectively, for the Coulombic atom, and triangles are for the case of the initial ground state for the soft-core model atom.

increased. The ionization probability is generally smaller for the soft-core model atom than for the Coulombic atom.

The quantum dynamical behavior of the electron in the stabilization regime can be studied by looking at the time evolution of the wave packet. We have previously reported results of our computation for the quantum time evolution of the electron wave packet for the case of the Coulombic atom [19]. It was found that, both for the initial ground-state ψ_{100} and for the initial excited-state ψ_{322}, the initial wave packet spreads out smoothly during the turn on of the laser and develops into a structure localized around a circular orbit with its center drifted away from the core. The circular orbit has a radius greater, by a factor of 1.7 for the case of the initial ground state and of 1.4 for the case of the initial excited state, than the maximum displacement $\alpha_0=F_0/\omega^2=3.5$ a.u. in the KH frame. The wave packet does not spread for a long time, which is consistent with the earlier findings [12,13,15,20] that stabilization occurs when localized and nondispersive wave packets are formed. In Fig. 2 we show the quantum time evolution of the wave packet for the soft-core model atom prepared initially in its ground state for the case $F_0=17$ a.u. It can be seen that a nondispersive wave packet localized around a circular orbit appears also for the soft-core model atom. It is interesting to see that, in the case of the soft-core potential, the radius of the circular orbit is 3.5 a.u., the same as the maximum displacement α_0 in the KH frame.

The classical dynamical behavior of the electron is studied by observing the classical motion of an ensemble of initial points distributed in accordance with the initial quantum wave packet. Figures 3(a)–3(d) report our computational results for the Coulombic atom. The ensemble of initial points of Fig. 3(a) centered at the atomic core with energy $E_0=-2.0$ a.u. and angular momentum $L_z=0$, which represents the initial ground-state wave packet, spreads out to give Fig. 3(b) at the end of the laser pulse. There is no indication of any localized behavior here and thus the classical dynamical behavior differs significantly from the corresponding quantum dynamical behavior. On the other hand, the ensemble of initial points of Fig. 3(c) localized around a circle

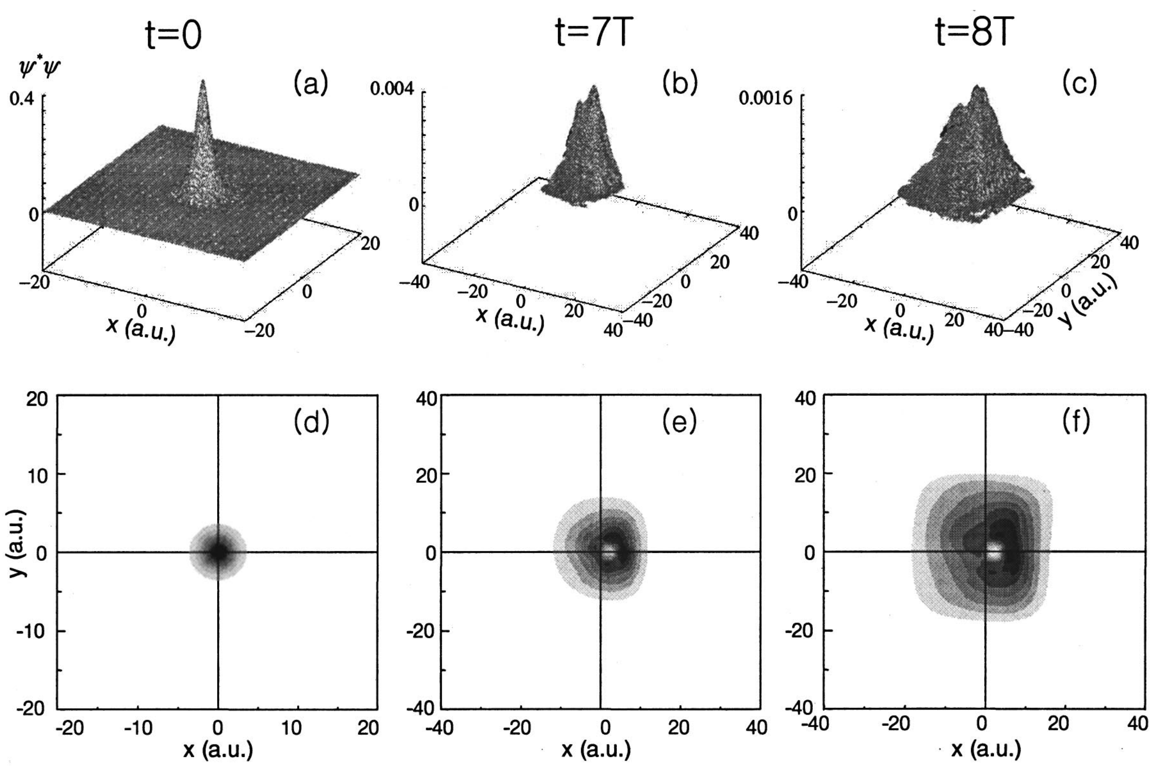

FIG. 2. Quantum time evolution of the wave packet. The peak amplitude of the laser field is taken to be $F_0=17$ a.u. (a), (b), and (c) show the time evolution of the wave packet prepared initially in the ground state of the soft-core model atom; the wave packet at (a) $t=0$, (b) $t=7T$, and (c) $t=8T$. (d), (e) and (f) are, respectively, the corresponding contour plots at (d) $t=0$, (e) $t=7T$, and (f) $t=8T$.

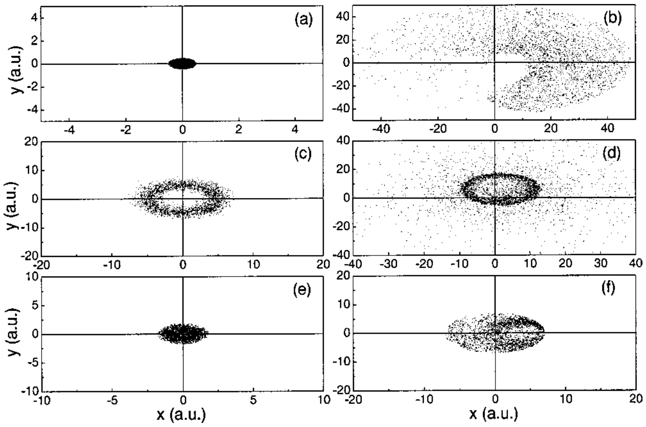

FIG. 3. Classical time evolution of the ensemble of points. The peak amplitude of the laser field is taken to be $F_0=17$ a.u. (a) and (b) show the time evolution of the ensemble distributed initially to mimic the ground-state ψ_{100} of the Coulombic atom; ensemble of points at (a) $t=0$, (b) $t=8T$. (c) and (d) show the time evolution of the ensemble distributed initially to mimic the excited state ψ_{322} of the Coulombic atom; ensemble of points at (c) $t=0$, (d) $t=8T$. (e) and (f) show the time evolution of the ensemble distributed initially to mimic the ground state of the soft-core model atom; ensemble of points at (e) $t=0$, (f) $t=8T$.

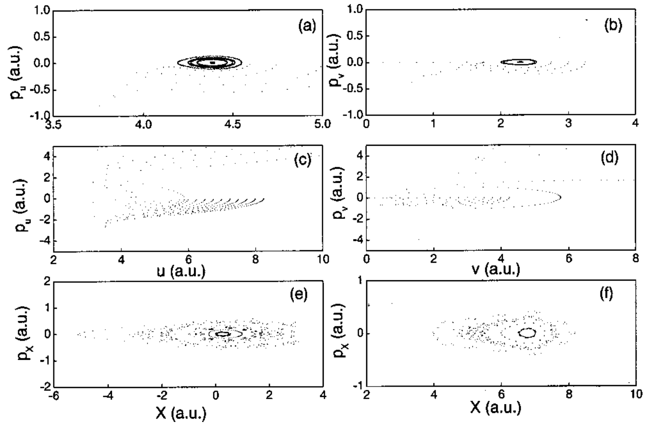

FIG. 4. Poincaré surfaces of section. The laser amplitude is assumed to be constant, $F=F_0=17$ a.u. (a) and (b) are Poincaré surfaces of section at quasienergy $K=23.46$ a.u. for the Coulombic atom. (c) and (d) are Poincaré surfaces of section at quasienergy $K=25.36$ a.u. for the Coulombic atom. (e) and (f) are Poincaré surfaces of section at quasienergy $K=29.86$ a.u. for the soft-core model atom.

of radius 5.0 a.u. with $E_0=-2/25$ a.u. and $L_z=2.0$ a.u., which corresponds to the initial excited-state ψ_{322}, evolves to remain localized around a circular orbit as shown in Fig. 3(d). Apparently, in this case of the initial excited state, there is a good quantum-classical agreement. The classical behavior of the electron in the soft-core model atom is seen in Figs. 3(e) and 3(f), where the initial distribution of points representing the ground state of the soft-core model atom with $E_0=-0.5$ a.u. and $L_z=0$ and the distribution of these points at the end of the laser pulse ($t=8\times 2\pi/\omega=8T$), respectively, are shown. We see from Fig. 3(f) that the ensemble of the initial points remain localized around a circular orbit of radius ~ 3.5 a.u. There is thus a good agreement between the quantum and classical dynamical behavior in this case.

The reason for different classical dynamical behaviors depending on the initial state and the atomic potential can be studied by looking at the Poincaré surfaces of section for the electron motion occurring at the constant maximum field strength, i.e., with the Hamiltonian given by $H=\frac{1}{2}p^2+V(r)+F_0(x\cos\omega t+y\sin\omega t)$. In Figs. 4(a)–4(d) we show Poincaré surfaces of section for the Coulombic atom for two different values of quasienergy $K\approx E_0-\omega L_z+F_0^2/2\omega^2$ [10]. For our computation we use the semiparabolic coordinates u and $v[x=(u^2-v^2)/2,\ y=uv]$ to remove the $1/r$ singularity [21]. Figures 4(a) and 4(c) show the Poincaré surfaces of section in the (u,p_u) space, and Figs. 4(b) and 4(d) in the (v,p_v) space. At the quasienergy $K=23.46$ a.u. stable elliptic periodic points of nonlinear resonance islands lie at ($x=u^2/2\approx 9.8,\ y=0.0$) and at ($x=-v^2/2\approx -2.5,\ y=0.0$). A single trajectory for an initial condition chosen at one stable elliptic point is shown in Fig. 5. In the rotating frame, the electron moves clockwise along a circular orbit with the frequency slightly smaller than the laser frequency. The circular orbit passes the two stable elliptic points and has the radius of about 6.0 a.u. as shown in Fig. 5(a). In the laboratory frame, the electron rotates fast with the laser frequency ω around a circle of radius α_0, as it is driven by a circularly polarized laser, while rotating counterclockwise slowly around the core to balance the Coulomb attraction, as shown in Fig. 5(b). Nonlinear resonance occurs between the two rotating motions. At the quasienergy $K=25.36$ a.u. the elliptic points become unstable as seen from Figs. 4(c) and 4(d).

It is now clear why the ensemble of initial points representing the ground-state ψ_{100} spreads out while the ensemble of initial points representing the excited-state ψ_{322} remains localized. The ensemble of initial points representing ψ_{100} evolves in time and, at $t=T_{swtich}$, covers the range of quasienergy $K\approx 25.36\pm 2.0$ a.u. at which the classical motions are mostly unstable. On the other hand, the ensemble of

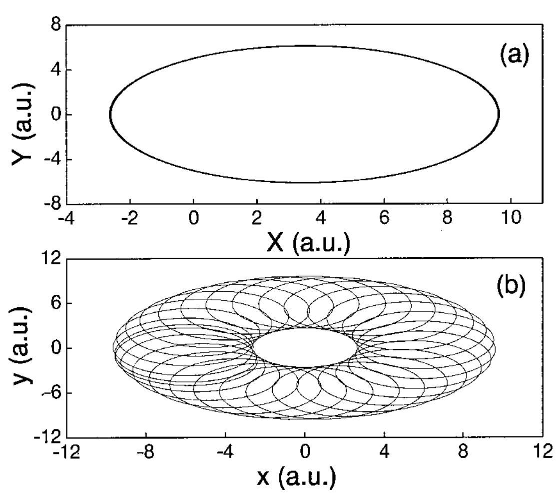

FIG. 5. A trajectory starting from a stable elliptic point viewed in (a) the rotating frame and (b) the laboratory frame.

initial points representing ψ_{322} evolves to cover, at $t = T_{switch}$, the range of quasienergy $K \approx 23.46 \pm 3.0$ a.u. at which the classical motions are sufficiently stable. When the initial state is the ground-state ψ_{100}, the quantum and classical dynamical behaviors do not agree. The disagreement is a consequence of quantum localization, the suppression of classically chaotic diffusion.

Shown for comparison in Figs. 4(e) and 4(f) are the Poincaré surfaces of section for the soft-core model atom at quasienergy $K = 29.86$ a.u. Clearly, there exists a classical stable structure. A similar stable structure was observed by Chism *et al.* [13] in their investigation of the soft-core model atom. Since the ensemble of initial points representing the ground state of the soft-core model atom covers the region of quasienergy around $K = 29.86$ a.u., the classical dynamical behavior of the electron starting from the ground state is stable. This is in contrast to the diffusive behavior exhibited by the ensemble of initial points representing the ground state of the Coulombic atom. This difference in classical dynamical behavior is reflected in the ionization probability shown in Fig. 1; the ionization probability from the ground state is generally higher for the Coulombic atom than for the soft-core model atom.

In summary, we have investigated quantum wave-packet dynamics and classical ensemble motion of the electron in the two-dimensional model hydrogen atom irradiated by a circularly polarized laser, for the case when the laser intensity is sufficiently high that stabilization of ionization occurs. For the Coulombic potential and when the initial state is the ground-state ψ_{100}, the quantum wave-packet dynamics exhibits a stable behavior and the wave packet remains localized around a circular orbit, while the corresponding classical motion is diffusive. This difference in the quantum and classical dynamical behavior can be understood in terms of the phenomenon of quantum localization. For the Coulombic atom and when the initial state is the excited-state ψ_{322}, not only the quantum wave packet but also the ensemble of classical points exhibit a tendency to remain localized. In this case, stabilization of ionization may be considered to originate from stable classical orbits generated by nonlinear resonance of the atom with the intense laser.

If the potential is taken to be the soft-core potential instead of the Coulomb potential, we observe a stable classical dynamics even when the initial state is the ground state. As a result, the ionization probability is smaller for the soft-core model atom than for the Coulombic atom. The ground state of the soft-core model atom is more strongly resistive to laser excitation than that of the Coulombic atom, i.e., stabilization occurs more easily with the soft-core potential than with the Coulomb potential. This is in agreement with the earlier observations [14,15] made for the case of a linearly polarized laser field.

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