Scheme for preparation of the *W* state via cavity quantum electrodynamics

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(Received 26 December 2001; published 3 May 2002)

We present an experimental scheme of preparing the tripartite *W* state via cavity quantum electrodynamics. The discussion of this scheme indicates that it can be realized by current technologies. Also, this scheme can be directly generalized to prepare the general *n*-qubit *W* states.

DOI: 10.1103/PhysRevA.65.054302 PACS number(s): 03.67.−a, 03.65.Ud, 42.50.Dv

Entanglement is at the heart of quantum information theory (QIT) which can bring secure schemes of cryptography [1], super fast computation [2], and other applications [3]. In recent years, there have been many efforts to characterize qualitatively and quantitatively the entanglement of multiparticle systems. Particularly, in case of tripartite entanglement, Dür *et al.* [4] have shown that there are two inequivalent classes of tripartite entanglement states, the Greenberger-Horne-Zeilinger (GHZ) [5] class and the *W* class, under stochastic local operation and classical communication. Especially, the *W* state has some interesting properties [4]; for example, it retains bipartite entanglement when any one of the three qubits is traced out.

On the other hand, generation and manipulation of entanglement states are also important tasks of QIT. The technology to produce entangled photons by spontaneous parametric down-conversion [6] is quite mature, and bipartite entanglement, i.e., the Einstein-Podolsky-Rosen (EPR) -type entanglement has been realized in some systems [7]. Several experiments to produce GHZ-type tripartite entanglement have been reported [8]. However, although some theoretical schemes have been proposed [9], there is no report of experimental realization of the *W* state. In this paper, we present a scheme that is feasible with current cavity quantum electrodynamics technology [10,11] to produce the *W* state.

The *W* state can be written in this form [4],

$$|\Psi\rangle=\frac{1}{\sqrt{3}}(|001\rangle+|010\rangle+|100\rangle). \tag{1}$$

It is such a superposition state that only one of the three particles is excited.

Our proposed implementations consist of microwave cavities, Ramsey zones [10–12], and a set of two-level atoms. The atoms are described by the basis states $|g\rangle$ (ground state) and $|e\rangle$ (excited state), which correspond to the logic qubit states $|0\rangle$ and $|1\rangle$, respectively. There is never more than one photon in a cavity so that the state of cavity field can be described in terms of the basis states $|0\rangle$ and $|1\rangle$. The evolution of the cavity field and the atom in the cavity is described by the Jaynes and Cummings Hamiltonian [14,10]

$$H=\hbar\omega_{eg}\sigma_z+\hbar\omega\left(a^\dagger a+\frac{1}{2}\right)-i\frac{\hbar\Omega}{2}f(x)(\sigma_+a-\sigma_-a^\dagger), \tag{2}$$

where a and $a^\dagger$ are the photon annihilation and creation operators in the cavity mode, and σ_z, σ_+, and σ_- are Pauli matrices of the atomic pseudospin. The cavity mode frequency ω is equal to the $e\Leftrightarrow g$ transition frequency ω_{eg}. $\Omega/2$ is the atom-field coupling constant which depends on the properties of the system of atom and cavity. In this scheme, we consider the case $f(x)=1$ in Eq. (2) [10].

Under domination of the Hamiltonian in Eq. (2), the atom-field system evolutes as following:

$$U(t)=\begin{pmatrix}\cos(\Omega t/2) & \sin(\Omega t/2)\\ -\sin(\Omega t/2) & \cos(\Omega t/2)\end{pmatrix}, \tag{3}$$

where the basis states are

$$|g,1\rangle=\begin{pmatrix}1\\0\end{pmatrix},\quad |e,0\rangle=\begin{pmatrix}0\\1\end{pmatrix}.$$

This process is known as the Rabi rotation. Here, we note that the state $|g,0\rangle$ will not be changed and the state $|e,1\rangle$ will not appear in this process.

We now first outline the process (depicted in Fig. 1) of preparing three atoms into a *W* state. The cavity *C* is initially set in state $|0\rangle$. The first atom A_1 in excited state is sent into

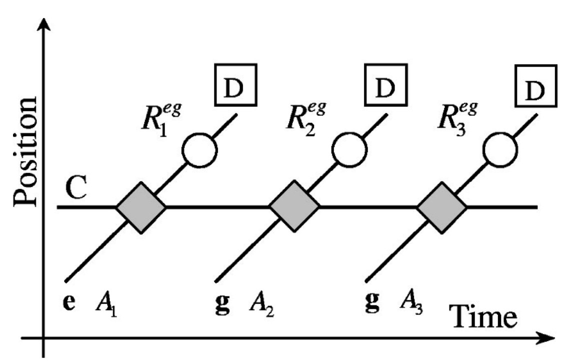

FIG. 1. Temporal sequence of the *W* state preparation. Space lines of A_1, A_2, A_3, and *C* in a position versus time diagram. The gray diamonds indicate a resonant Rabi rotation (with the angles written in the text). The open squares represent the detection events. The open circles represent classical Ramsey pulses.

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the cavity and we let $\Omega t_1 = 2\pi - 2\arcsin(\sqrt{2/3})$. When A_1 flies out of the cavity, the second atom A_2 in ground state is sent into the cavity with $\Omega t_2 = \pi/2$. The third atom A_3 in ground state is sent into the cavity after A_2 flies out with $\Omega t_3 = \pi$. The evolution of the atom-field system can be written as follows:

$$|e\rangle_1|g\rangle_2|g\rangle_3|0\rangle_C \to \left(\sqrt{\frac{2}{3}}|g\rangle|1\rangle - \frac{1}{\sqrt{3}}|e\rangle|0\rangle\right)_{1,C}|g\rangle_2|g\rangle_3$$
$$\to \frac{1}{\sqrt{3}}(|g\rangle|g\rangle|1\rangle - |g\rangle|e\rangle|0\rangle - |e\rangle|g\rangle|0\rangle)_{1,2,C}|g\rangle_3$$
$$\to \frac{-1}{\sqrt{3}}(|g,g,e\rangle + |g,e,g\rangle + |e,g,g\rangle)_{1,2,3}|0\rangle_C . \quad (4)$$

Then the three atoms are entangled in a W state. After proper Ramsey rotations (between $|g\rangle$ and $|e\rangle$) [12] on each atom, respectively, the three atoms can be detected by detectors.

In the final detection we use the basis states

$$|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

which are eigenstates of Pauli operator σ_z with eigenvalues $+1$ and -1, respectively. Cabello has discussed the nonlocal correlations of three particles in the W state [13]. It is shown in [13] that there exists a Bell theorem without inequalities

$$P(z_i = -1, z_j = -1) = 1, \quad (5)$$

$$P(x_j = x_k | z_i = -1) = 1, \quad (6)$$

$$P(x_i = x_k | z_j = -1) = 1, \quad (7)$$

$$P(x_i = x_j = x_k) = \frac{3}{4}, \quad (8)$$

where $P(z_i = -1, z_j = -1)$ means the probability of two qubits (although we cannot tell which one) giving the result -1 when measuring σ_z on all three qubits, and $P(x_j = x_k | z_i = -1)$ $(i \neq j, j \neq k$ and $k \neq i)$ is the conditional probability of σ_{xj} and σ_{xk} having the same result given that the result of σ_{zi} is -1. However, the hidden-variable theory predicts that $P(x_i = x_j = x_k) = 1$ contradicts the quantum prediction in Eq. (8). To test the quantum predictions in Eqs. (5)–(8), the three atoms' pseudospins can be detected along z direction or x direction. When an atom is measured in z direction, the Ramsey rotation is $R^{eg} = 0$, and $R^{eg} = \pi/2$ if the atom is measured along x direction. To be specific, the $\pi/2$ pulse can perform the transformations $|g\rangle \to (|g\rangle + |e\rangle)/\sqrt{2}$ and $|e\rangle \to (-|g\rangle + |e\rangle)/\sqrt{2}$.

Cabello has also given the Bell inequality involving tri- and bipartite correlations for the W state [13]

$$-8 \leqslant \langle A_1A_2A_3\rangle - \langle A_1B_2B_3\rangle - \langle B_1A_2B_3\rangle - \langle B_1B_2A_3\rangle$$
$$-2\langle A_1A_2\rangle - 2\langle A_2A_3\rangle - 2\langle A_1A_3\rangle \leqslant 4, \quad (9)$$

which is violated by the W state [for instance, by choosing $A_i = \sigma_{zi}$ and $B_i = \sigma_{xi}$, state (1) gives the value 5 for the middle term in Eq. (9)] but *not* by the GHZ state. To test this inequality, two Ramsey rotations $R^{eg} = 0$ and $R^{eg} = \pi/2$ are needed for measurements of σ_z and σ_x, respectively.

Second, we describe the process (depicted in Fig. 2) of preparing the three-cavity entangled W state. The three cavities C_1, C_2, and C_3 are all initially in state $|0\rangle$ while atom A is in state $|e\rangle$. Sequentially atom A passes through the three cavities C_1, C_2, and C_3, and the Rabi rotation angles are $\Omega t_1 = 2\arcsin(1/\sqrt{3})$, $\Omega t_2 = \pi/2$, and $\Omega t_3 = \pi$, respectively. After the atom flies out from C_3, the three cavities are entangled in a W state. This evolution can be written as follows:

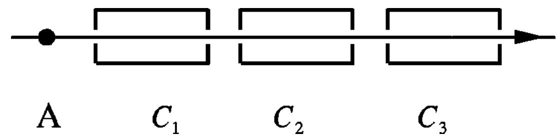

FIG. 2. Proposed experimental arrangement for generating a three-cavity W state. Cavities C_1, C_2, and C_3 each support the same mode ω_{eg}.

$$|e\rangle_A|0\rangle_1|0\rangle_2|0\rangle_3 \to \left(\sqrt{\frac{2}{3}}|e\rangle|0\rangle + \frac{1}{\sqrt{3}}|g\rangle|1\rangle\right)_{A,1}|0\rangle_2|0\rangle_3$$
$$\to \frac{1}{\sqrt{3}}(|e\rangle|0\rangle|0\rangle + |g\rangle|0\rangle|1\rangle + |g\rangle|1\rangle|0\rangle)_{A,1,2}|0\rangle_3$$
$$\to \frac{1}{\sqrt{3}}|g\rangle_A(|001\rangle + |010\rangle + |100\rangle)_{1,2,3} . \quad (10)$$

It is necessary to address the practical matters regarding the experimental realization of the proposed scheme. To prepare the three-atom W state, we only need to preserve the coherence of the cavity field before the third atom flying out of the cavity. The length of the cavity is of the order of centimeter and the velocity of the atom is of the order of 100 m/s, so the staying time of the atom in the cavity is about 0.1 ms. In fact, the atom-cavity interaction time is only a few tens of μs. It is reported that the cavity can have a photon storage time of $T_r \simeq 1$ ms (corresponding to $Q = 3 \times 10^8$) and the radiative lifetime of the circular Rydberg state can reach 30 ms [10]. All these parameters satisfy the requirements of our scheme.

The current technology also satisfies the requirement for preparing the three-cavity W state similarly. However, one obvious disadvantage of three-cavity entanglement is that it is a little difficult to detect the state. Auxiliary atoms are needed to interact with the three cavities, respectively.

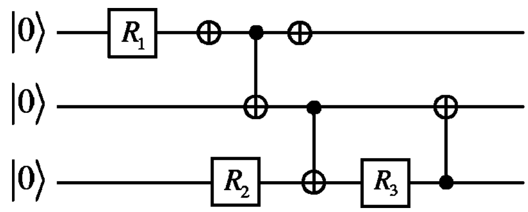

FIG. 3. Quantum logic circuit for preparation of the W state from direct product state $|0\rangle|0\rangle|0\rangle$. The denotations are the same as in Ref. [15].

It has been shown that any unitary operation on multiqubit can be fulfilled by arrangement of element gates [15], e.g., two-bit quantum gates such as controlled-NOT (CNOT) and single-qubit rotation gate. We can construct the network (depicted in Fig. 3) composed of element gates to produce the W state.

R_1, R_2, and R_3 in Fig. 3 are such unitary transformations

$$R_1=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2} & -1\\ 1 & \sqrt{2}\end{pmatrix},$$

$$R_2=\begin{pmatrix}\cos(\pi/8) & -\sin(\pi/8)\\ \sin(\pi/8) & \cos(\pi/8)\end{pmatrix},$$

$$R_3=\begin{pmatrix}\cos(\pi/8) & \sin(\pi/8)\\ -\sin(\pi/8) & \cos(\pi/8)\end{pmatrix}.$$

It is obvious that this logic circuit is rather difficult to be realized by current technology in most systems for QIT. However, in above atom-cavity system, the auxiliary state $|g,0\rangle$ is not changed in Rabi oscillation. So the scheme is simplified in a degree.

In summary, we have proposed a scheme to prepare three-atom and three-cavity entangled W states, and it can be realized experimentally. Furthermore, it is not difficult to generalize this scheme to prepare general n-qubit W states [4].

This work was supported by the National Natural Science Foundation of China, the national ‘‘973’’ project, the Innovation Funds from Chinese Academy of Sciences (CAS), and was also funded by the outstanding Ph.D thesis award and the CAS's talented scientist award entitled to Lu-Ming Duan.

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