Phase-modulation bistability and threshold self-start of laser passive mode locking

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A mechanism for operation bistability of passive mode-locked lasers is discovered. It explains why self-starting requires a critical minimum strength of seed fluctuations. There are two contributing factors: (a) a small-scale structure of the spectral loss curve due to a parasitic frequency-dependent loss, (b) phase modulation of the resulting pulses due to nonlinearity and frequency dispersion of the refractive index of intracavity elements. We discuss applications to actual experimental systems.

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I. INTRODUCTION

At the present time the shortest light pulses with duration of 4–5 fs are generated in titanium-doped sapphire lasers (see the reviews in [1,2]). The ultrafast laser technology has been revolutionized by gain media with a broadband electron-phonon lasing transition and by the Kerr-lens mode-locking (KLM) process related to the practically inertialess nonlinearity of the refractive index. The Kerr nonlinearity of the laser medium by self-focusing induces an intensity-dependent aperture of the intracavity radiation beam and correspondingly an ultrafast saturable-absorber-like intensity-dependent diffraction loss.

Kerr nonlinearity is weak and as a result the KLM process is not usually self-starting. This means that when initially switched on, the laser will usually operate in a continuous-wave (cw) mode, with modes unlocked, and some additional means of initiating the self-mode-locked process has to be utilized. A sufficiently strong initial seed fluctuation must be induced by the additional mean for initiating KLM [3]. The simplest method to start KLM in a laboratory setup is to slightly tap the table or one of the cavity mirrors. Once initiated, the mode-locking operation could be retained for periods of as much as several hours or days depending on the degree of physical perturbation or vibration in the surrounding environment.

This bistability of KLM lasers and the threshold dependence of self-start for the passive mode-locking process on an intensity of initial seed fluctuation pulse were discussed in many experimental and theoretical papers (see monograph in [3] and references therein). In Ref. [4] it was attributed to a longitudinal spatial inhomogeneity of the gain saturation in the active medium due to a passing pulse. In Refs. [5,6] it was explained by the relation of pulse buildup time and phenomenological lifetime of the seed fluctuation. Strong seed fluctuations can induce pulses more quickly, and the chance is enhanced that mode locking starts. For subthreshold seed fluctuations, the laser displays fluctuating continuous emission. This means that the whole cavity is filled with radiation, but the intensity is not steady. We may call this turbulent cw emission (see below).

Experimental data led the authors of Ref. [6] to conclude that spurious reflections from interfaces inside the laser cavity reduce the lifetime of the seed fluctuations considerably and thus prevent the self-starting of passive mode locking. Such spurious reflections are unavoidable; even inside the gain medium there may be small spatial inhomogeneities due to technical peculiarities in their manufacturing. These spurious reflections produce a parasitic frequency-dependent loss. Results of this phenomenological model agree very well with experimental results.

We are here going to explain why such minute perturbations are so enormously effective in influencing the startup of mode locking. In this paper we show how the small-scale structure of the spectral loss curve caused by parasitic frequency-dependent loss can produce the above-mentioned threshold behavior of self-starting of passive mode locking. Our model is different from one of Ref. [6] by two features. First, in our model a lifetime of pulses is not a phenomenological parameter. It is determined by a frequency dispersion of the intracavity medium. Second, for our bistability the phase modulation due to the nonlinear refractive index is of fundamental importance.

Our theoretical and numerical investigation is based on equations close to the complex Ginzburg-Landau equation. The latter is extensively used in descriptions of diversified nonlinear systems and phenomena: instability in Poiseuille flow [7], Rayleigh-Bénard instability in binary fluid mixtures [8], electroconvection in nematics [9], and so on. Varied states of these systems are described by plane-wave, turbulent [10], collapse [11,12], stable quasiperiodic, and pulse [13,14] solutions. In Ref. [15] in the framework of an analogous equation a multiple-pulse operation, multistability and hysteresis phenomena connected to multiple-pulse regimes in passive mode-locked lasers were described. The bistability discussed in this paper is supplemented by an assortment of possible properties of nonlinear systems described by an equation close to the complex Ginzburg-Landau one.

II. BASIC EQUATIONS AND NUMERICAL SIMULATION RESULTS

In a coordinate system moving with the pulses, the change of intracavity radiation during the process of its interaction with intracavity nonlinear elements is described by the following normalized equation:

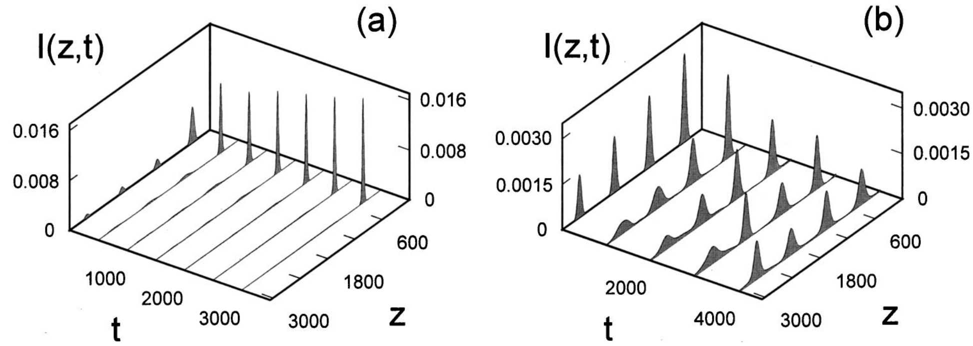

FIG. 1. Illustration of the dependence of the established operation on initial conditions. The threshold for single-pulse formation is an initial pulse with equilibrium duration and chirp, and power of 0.011. (a) Single-pulse operation when one pulse is above threshold, grows, and suppresses all others. (b) Turbulent cw operation when all pulses are below threshold. The initial pulses are unchirped Gaussians. $\theta=-15,\ p=0.2,\ q=-7,\ b=3.5,\ a=3,\ h=10^{-3}$, and $\Gamma=10^2$.

$$\frac{\partial}{\partial t}E(z,t)=\frac{1}{2}\left(\frac{1+a}{1+b\int|E(z,t)|^2dz}-1+p|E|^2+iq|E|^2\right)E(z,t). \tag{1}$$

The dimensionless variables and parameters from Eq. (1) are obtained from the corresponding real-world quantities (written here with a prime) through the following relations.

The slowly varying field amplitude is $E=E'/\sqrt{I_a}$, where I_a is the intensity of the field saturating the nonlinear losses. The time variable is $t=t'\sigma_0$, where σ_0 is the net linear resonator loss including the linear loss in the saturable absorber σ_1 (or the linear diffraction losses). The coordinate is $z=z'\sqrt{\sigma_0/D_{\mathrm{Im}}}$, where D_{Im} is the frequency dispersion for the gain ($D_{\mathrm{Im}}\approx 0.5\sigma_0 v_{\mathrm{gr}}^2/\Gamma_g^2$; here, v_{gr} and Γ_g are the group velocity and the spectral half-width of the gain in radians per second, respectively).

The first term in the parentheses of Eq. (1) describes the saturable gain. The saturation is determined by the total intracavity radiation energy, and accordingly integration is carried out over the whole cavity volume; a is the relative excess of the pumping rate above lasing threshold; $b=I_a\sqrt{D_{\mathrm{Im}}/\sigma_0}/(LI_g)$, where L is the cavity length; I_g is the intensity of the monochromatic radiation saturating the gain (the carrier frequency of the radiation corresponds to the center of the frequency gain band). The second term on the right-hand side of Eq. (1) is related to the net linear resonator losses. The third term $p|E|^2$ describes the nonlinear losses ($p=\sigma_1/\sigma_0$). The last term is associated with the nonlinear refractive index $q=-2\omega n_2 I_a l/(n_0\sigma_0 L)$, where ω is the carrier frequency of the laser radiation; n_2 is the nonlinearity determining the nonlinear refractive index $\delta n'$ in absolute units for dimensional intensity $\delta n'=n_2|E'|^2$; n_0 is the linear refractive index, and l is the length of the nonlinear medium.

The change of spectral components of field $E(k,t)$ during the process of its interaction with intracavity dispersive elements is described by the following equation:

$$\frac{\partial}{\partial t}E(k,t)=\left[-k^2(1+i\theta)+h\left(\frac{1}{1+(\Gamma k)^2}-1\right)\right]E(k,t), \tag{2}$$

where $E(k,t)$ is the Fourier transform of $E(z,t)$; $-k^2(1+i\theta)$ describes the frequency dispersion of both the gain and refractive index. The parameter θ is ratio of the real and imaginary parts of the frequency dispersion of the permittivity for the intracavity distributed medium ($\theta=D_{\mathrm{Re}}/D_{\mathrm{Im}}$, where the dispersion $D_{\mathrm{Re}}\approx-0.5v_{\mathrm{gr}}dv_{\mathrm{gr}}/d\omega$ accounts for the frequency dispersion of the group velocity of all intracavity elements, among them the ones used for compensation of the frequency chirp of the resulting pulses). The narrow-band Lorentzian models one of spectral peaks of parasitic losses in which the generation is born: h is the modulation depth of frequency-dependent parasitic losses, and Γ is the ratio of the spectral widths of the gain and the parasitic peak. Since the dispersion of refractive index connected with the parasitic loss peak and arising from Kramers-Kronig relation is small, we may neglect it here.

Our numerical simulation is based on the split-step Fourier method. For each temporal step the total action of intracavity elements to the field is split into nonlinear and dispersion components [16] described by Eqs. (1) and (2), respectively. These equations describe the formation of ultrashort pulses owing to both the nonlinear losses and the soliton mechanism related to the nonlinearity and the frequency dispersion of the refractive index [17].

Figure 1 shows the bistability between two regimes: that of the single pulse vs that of turbulent cw emission as described above. In both parts of the figure, all parameters are just the same. The only difference is in initial conditions.

Different initial conditions are represented by several different pulses; this reflects the spread in seed fluctuation strength. Of the four pulses shown in Fig. 1(a), only the largest one is above threshold. It grows at the expense of the other three, and mode locking builds up. In Fig. 1(b), all pulses are subthreshold, fluctuate in power, width, and chirp, and none wins over the others. This is the turbulent cw operation alluded to above.

III. COMPLEX GINZBURG-LANDAU EQUATION ANALYSIS

In the cases of sufficiently long pulses $\Gamma k \ll 1$ and sufficiently short pulses $\Gamma k \gg 1$ the frequency dispersion of intracavity elements is described in Eq. (2) by a quadratic dependence and this equation becomes equivalent to the following one for $E(z,t)$:

$$\frac{\partial}{\partial t}E(z,t)=(d_{\mathrm{eff}}+i\theta)\frac{\partial^2}{\partial z^2}E(z,t); \tag{3}$$

here, $d_{\mathrm{eff}}=1$ for short pulses $\Gamma k \gg 1$ and $d_{\mathrm{eff}}=1+h\Gamma^2$ for long pulses $\Gamma k \ll 1$. The combination of Eq. (3) with Eq. (1) gives

$$\frac{\partial}{\partial t}E(z,t)=(d_{\mathrm{eff}}+i\theta)\frac{\partial^2}{\partial z^2}E(z,t)+\frac{1}{2}(g+p|E|^2+iq|E|^2)E(z,t), \tag{4}$$

where $g=(1+a)/[1+b\int|E(z,t)|^2dz]-1$ is the net gain including the linear losses. Notice that sufficiently powerful pulses remain or become sufficiently short and are described by Eq. (4) with $d_{\mathrm{eff}}=1$. If initially pulse is short but weak, then it quickly spreads and after that is described by Eq. (4) with $d_{\mathrm{eff}}=1+h\Gamma^2$.

With $d_{\mathrm{eff}}=0$, $g=0$, $p=0$ the complex Ginzburg-Landau equation (4) becomes the nonlinear Schrödinger one. With $\theta=0$, $q=0$, Eq. (4) is the diffusion equation. For it, there is the Lyapunov functional [18]

$$\frac{d}{dt}\left\{\int_V\left[2d\left|\frac{\partial E}{\partial z}\right|^2+(1-p_0)|E|^2+p_0\ln(1+|E|^2)\right]dz-\frac{1+a}{b}\ln\left(1+b\int_V|E|^2dz\right)\right\}=-4\int_V\left|\frac{\partial E}{\partial t}\right|^2dz; \tag{5}$$

here, V is the cavity volume, the dispersion is assumed as constant $d_{\mathrm{eff}}=d$, and the nonlinearity p falls with a rise in intensity $p=p_0/(1+|E|^2)$, $p_0<1$. This relation allows us to solve the problem of the transient evolution: from any initial conditions the system passes into the steady-state single-pulse mode or the steady-state cw regime. After the transient process any change must stop ($|\dot{E}|^2$) when the Lyapunov functional having the sign of the derivative with respect to time reaches its minimal value (finally it must happen due to the functional being bounded below and it can only decrease during the system evolution). The functional's minimum is realized only either by a single steady-state pulse $E(z)$ (when passive mode-locking criteria have been fulfilled) or monochromatic radiation spatially uniformly filling the cavity (E does not depend on a coordinate z). With $a<0$, $p_0+a>0$ (the generation condition is fulfilled for bleaching nonlinear absorber and is not fulfilled for nonbleaching one) the Lyapunov functional has two local minima: $E(z)=0$ and $E(z)\neq 0$. That is, in this case the system is bistable: a single-pulse passive mode locking vs an absence of generation. With $a>0$, $a \ll p_0$ the bistablity between a single-pulse regime and a cw operation is realized. But for this bistability the pump a must be within the narrow range $a \ll p_0$ and it cannot explain the bistability of KLM Ti:sapphire lasers.

If the dispersion d_{eff} depends on a duration pulses as in Eq. (4) the range of such bistability ($\theta=0, q=0$) is essentially broader. In this case the threshold of passive mode locking for sufficiently short powerful pulses is less than for long ones by a factor of $(1+h\Gamma^2)$. This mechanism of a bistability can be realized in experiments with passive mode-locked lasers.

Our paper is devoted to a different mechanism for the bistability between the passive mode locking and cw operation. This mechanism is related to the phase modulation of pulses due to the dispersion and nonlinearity of the refractive index. It has been known that the phase-modulation instability prevents passive mode locking. As a consequence, a cw operation is realized instead of a single-pulse regime. The dependence of these regimes on the parameters of the Ginzburg-Landau equation (4), q, p, and θ, was analyzed in detail in Ref. [19]. The areas for cw operation and passive mode locking on the plane $\xi=q/p$ and θ are shown in Fig. 2 [19] (here and further p does not depend on an intensity $|E|^2$). For Eq. (4) with $d_{\mathrm{eff}}=1+h\Gamma^2$ (long weak pulses) area 1 corresponds the single-pulse operation and areas 2 and 3 the cw operation. In the case $d_{\mathrm{eff}}=1$ (short powerful pulses) areas 1 and 2 correspond a passive mode locking and area 3 a cw operation. Therefore, the laser operation with parameters from area 2 is bistable. That is, with a powerful initial seed pulse passive mode locking is achieved. In alternative case the cw operation is realized. In such a way we interpret our numerical simulation results on the bistability shown in Fig. 1.

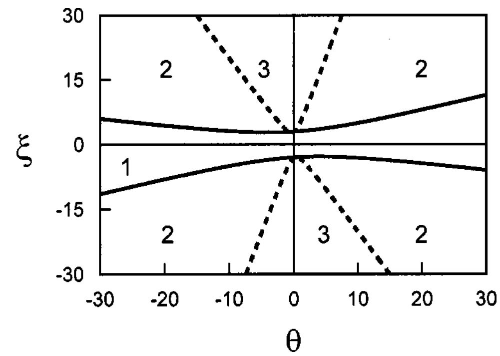

FIG. 2. Dependence of the established operation on dispersive and nonlinear parameters θ and $\xi=q/p$. With any initial conditions for ξ and θ from area 1 passive mode locking is always established. For the parameters from area 3 the cw operation is always realized. For area 2 the former is established with a sufficiently short powerful seed pulse in initial radiation; in the opposite case, the latter is realized. The boundary curves are determined for h and Γ from Fig. 1.

IV. COMPETITION OF PHASE-MODULATED PULSES

We turn to a discussion of the underlying mechanism for this bistability. In our numerical simulation the transient process consists of three stages: In a first fast stage there is the establishment of equilibrium between the gain and the total radiation energy $\int |E|^2 dz$. In a following slower stage equilibrium pulse duration and equilibrium frequency chirp are established. They are determined by the balance between mechanisms that either shorten or spread out the pulses and between mechanisms increasing or reducing the pulse's phase modulation. In a final, very slow stage there is competition between several pulses of different amplitude, all of which have found their equilibrium chirp and duration.

We analyze the third stage with the use of the following model approximation. In Eq. (2) let us switch from $E(k,t)$ to $E(z,t)$:

$$\frac{\partial}{\partial t}E(z,t)=\left(1+\frac{h\Gamma^2}{1-\Gamma^2\frac{\partial^2}{\partial z^2}}+i\theta\right)\frac{\partial^2}{\partial z^2}E(z,t), \qquad (6)$$

where the derivative in the denominator must be treated as the expansion of the fraction into Taylor series in powers of $\Gamma^2\partial^2/\partial z^2$. Then this derivative is replaced by $-\hat{\beta}^2$, where $\hat{\beta}$ is equal to an inverse duration of the resulting pulses β. The combination of the obtained equation with Eq. (1) gives

$$\frac{\partial}{\partial t}E(z,t)=\left(1+\frac{h\Gamma^2}{1+(\Gamma\hat{\beta})^2}+i\theta\right)\frac{\partial^2}{\partial z^2}E(z,t)+\frac{1}{2}\left(\frac{1+a}{1+b\int|E(z,t)|^2dz}-1+p|E|^2+iq|E|^2\right)E(z,t). \qquad (7)$$

Equation (7) differs from Eq. (4) often used for the description of a formation of ultrashort pulses in passive mode-locked lasers [15,19] by the dependence of the frequency dispersion of the gain-loss on the pulse parameter: namely, its inverse duration β. Notice that for Eq. (7) with $\hat{\beta}^2=-\partial^2/\partial z^2$ and $\theta=0$, $q=0$, there is the relation of the type Eq. (5), which allows us to analyze the transient evolution and the above-mentioned bistability with $\theta=0$, $q=0$.

As in Ref. [15], the solution of Eq. (7) is sought in the form of several pulses with various amplitudes and with equilibrium duration and frequency chirp:

$$E(t,z)=\sum_k E_{0k}\frac{e^{(\lambda_k+i\delta\omega_k)t}}{\cosh^{1+i\alpha_k}(\beta_k z)}, \qquad (8)$$

where E_{0k}, α_k, β_k, $\delta\omega_k$, and λ_k are the peak amplitude, the equilibrium frequency chirp, the equilibrium inverse duration, the frequency shift, and the growth rate for k-th pulse, respectively. Substituting Eq. (8) into Eq. (7) gives

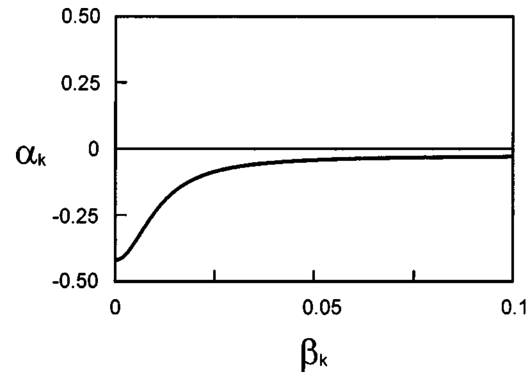

FIG. 3. Dependence of equilibrium frequency chirp α_k on the equilibrium inverse duration β_k for dissimilar pulses formed in laser cavity. The parameters θ, p, q, h, and Γ are the same as in Fig. 1.

$$\lambda_k+i\delta\omega_k=d_{\rm eff}(1+i\theta_{\rm eff})(1+i\alpha_k)^2\beta_k^2+0.5g, \qquad (9)$$

$$0.5(p+iq)I_{0k}=d_{\rm eff}(1+i\theta_{\rm eff})(1+i\alpha_k)(2+i\alpha_k)\beta_k^2; \qquad (10)$$

here, $d_{\rm eff}=1+h\Gamma^2/[1+(\Gamma\beta_k)^2]$ is the effective dispersion of the gain-loss and

$$\theta_{\rm eff}=\frac{\theta}{d_{\rm eff}}. \qquad (11)$$

In writing Eq. (10) we replaced a factor of $\exp(2\lambda_k t)$ by 1, so the solution (8) of Eq. (7) with parameters determined from Eqs. (9) and (10) will be correct to the extent that $\lambda_k t \ll 1$.

From Eq. (10) we find the expression which together with Eq. (11) determines the equilibrium frequency chirp α_k as the function of the equilibrium inverse duration β_k,

$$\frac{\alpha_k}{2-\alpha_k^2}=\frac{\xi-\theta_{\rm eff}}{3(1+\xi\theta_{\rm eff})}, \qquad (12)$$

and the relation between the peak intensity I_{0k} and β_k:

$$I_{0k}=\frac{2d_{\rm eff}\beta_k^2}{p}(2-\alpha_k^2-3\alpha_k\theta_{\rm eff}). \qquad (13)$$

Finally from Eq. (9) we find the growth rate as the function of β_k

$$\Lambda_k=d_{\rm eff}\beta_k^2(1-\alpha_k^2-2\alpha_k\theta_{\rm eff}), \qquad (14)$$

where $\Lambda_k=\lambda_k-0.5g$. Notice that with the use of Eq. (13) the parameters Λ_k and α_k can be expressed as functions of peak intensities of pulses $\Lambda_k=\Lambda_k(I_{0k})$, $\alpha_k=\alpha_k(I_{0k})$.

Figure 3 shows the dependence $\alpha_k=\alpha_k(\beta_k)$ obtained by a numerical calculation with laser parameters corresponding to Fig. 1. The important finding here is about the dependence of

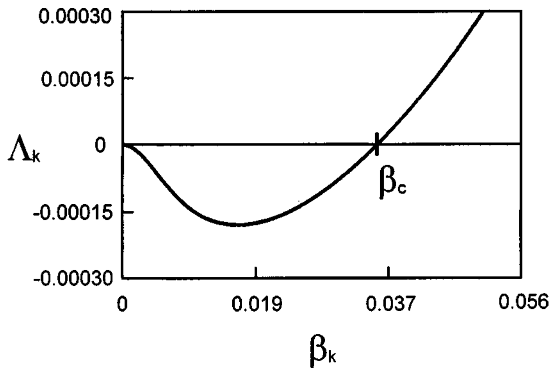

FIG. 4. Dependence of the amplification coefficient Λ_k for the pulse with equilibrium duration and frequency chirp on the equilibrium inverse duration β_k under the competition of pulses during the transient evolution. The parameters θ, p, q, h, and Γ are the same as in Fig. 1.

the dispersion of gain-loss, $d_{\mathrm{eff}}=1+h\Gamma^2/[1+(\Gamma\beta_k)^2]$, on the pulse width $1/\beta$. For very long pulses, i.e., $\beta\approx 0$, $d_{\mathrm{eff}}=1+h\Gamma^2=11$. On the other hand, for very short pulses $\beta\approx\infty$ and $d_{\mathrm{eff}}=1$. This shows that longer pulses sense stronger dispersion of the gain-loss. This requires stronger dispersion of the refractive index for compensation of the frequency chirp $\theta=d_{\mathrm{eff}}\xi$ [see the numerator on the right-hand side of Eq. (12)]. As a result longer pulses have stronger frequency chirp. Boundary curves $\xi=\xi(\theta)$ in Fig. 2 are obtained from Eq. (14) with $\Lambda_k=0$, $d_{\mathrm{eff}}=1+h\Gamma^2=11$ (the solid curve) and $d_{\mathrm{eff}}=1$ (the dashed curve).

Figure 4 demonstrates the dependence $\Lambda_k=\Lambda_k(\beta_k)$ with laser parameters corresponding to Figs. 1 and 3. The decrease of Λ_k at small β_k is connected to a spectral broadening of the pulses and selective parasitic losses described by the narrow-band Lorentzian in Eq. (2). When the spectral width of pulses becomes more than the width of the narrow Lorentzian, then Λ_k increases with increasing β_k (greater β_k corresponds to greater peak intensity I_{0k} and to less nonlinear losses). This dependence $\Lambda_k=\Lambda_k(\beta_k)$ explains the bistability between pulsing and cw solution described above. If the equilibrium inverse durations β_k for all pulses are less than β_c, noise pulses with long durations (small β_k) will grow in amplitudes, and the whole cavity will be filled with radiation [see Fig. 1(b)]. To the contrary, if one of the pulses has the equilibrium inverse duration $\beta_k>\beta_c$, the pulse with the greatest inverse duration will have the greatest growth rate Λ_k. This pulse will survive and grow in the cavity, while all other pulses will decay. Thus, single-pulse operation will be realized [see Fig. 1(a)].

V. APPLICATION TO KLM Ti:SAPPHIRE LASERS

We apply our numerical simulations and calculations to the practical case of a Ti:sapphire laser. We use the following laser parameters: linear resonator length $L=1.5$ m, length of Ti:sapphire rod $l=15$ mm. Net linear losses $\sigma_0=6\times 10^6$ s^{-1} (6% per round-trip time), linear diffraction losses $\sigma_1=0.2\sigma_0$ $(p=0.2)$. Half-width of the spectral gain band $\Gamma_g=2\times 10^{14}$ rad/s; group velocity $v_{\mathrm{gr}}=3\times 10^8$ m/s; intensity saturating the gain, $I_g=10^9$ W/m^2. Refractive index of the laser rod $n_0=1.7$, refractive index nonlinearity coefficient $n_2=3.0\times 10^{-20}$ m^2/W. Intensity saturating the nonlinear losses $I_a=5\times 10^{13}$ W/m^2 [19]. From these values we determine the frequency dispersion for the gain, $D_{\mathrm{Im}}=0.07$ cm^2/s (or 0.8 fs^2), $q=-7$, and the parameter $b=3.5\times 10^{-3}$. The relation of dimensional and dimensionless durations is $z'=z\times 10^{-6}$ m. In numerical simulation presented in Fig. 1, one-thousandth of the length of the resonator round-trip is involved. Correspondingly we must use $b_{\mathrm{eff}}=10^3 b=3.5$. As a result, Fig. 1 faithfully represents the initial stages of transient evolution of actual laser systems. The value of $\theta=-15$ used here corresponds to a group velocity dispersion of -12 fs^2. Note that the frequency chirp for long pulses with $h=10^{-3}$ and $\Gamma=10^2$ can be compensated only by a group velocity dispersion equal to -300 fs^2. The value $\beta_c=0.036$ corresponds to the equilibrium pulse width $\tau_c=1.76\times 10^{-4}\beta_c^{-1}/v_{\mathrm{gr}}\approx 200$ fs and peak intensity $I_{0c}=5\times 10^{11}$ W/m^2. Thus the parasitic frequency-dependent loss with spectral peaks having a frequency width equal to 10^{-2} of the gain spectral bandwidth and the height equal to 10^{-3} of the total linear losses suffices to produce bistability and to prevent self-starting of passive mode locking. Self-starting can be obtained by initial fluctuations of equilibrium durations smaller than 0.2 ps (of peak intensities greater than 5×10^{11} W/m^2).

Notice that the above-mentioned bistability with $q=0$, $\theta=0$ for used parameters $h=10^{-3}$, $\Gamma=10^2$ is not realized (as our numerical simulation has shown the passive mode locking with a single pulse in the cavity is established with any initial conditions). This means that the phase-modulation bistability is more serious obstacle to passive mode locking than the bistability with $q=0$, $\theta=0$.

VI. CONCLUSION

In this study, we have found a mechanism resulting in a bistable operation of passively mode-locked lasers with parasitic frequency-dependent losses. This bistability is realized as either a cw operation or passive mode locking with a single steady-state ultrashort pulse in cavity. The mechanism found here is connected with phase modulation of the resulting pulses due to nonlinearity and frequency dispersion of the refractive index of the intracavity elements. In consequence of this bistability a self-starting of mode locking is obtained only with specific initial conditions.

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